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# Service Control in a Retrial Service Facility System with Two-type of Customer-Semi MDP

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**Abstract**

In the literature on service facility system, it is common to assume fixed service rate, such as flexible retrial arrival, this assumption is not realistic. In contrast retrial demands model rarely define the mechanism through which arrival of customers is classified into two types say priority customers and ordinary customers. For the given values of maximum inventory, maximum waiting space, reorder level and lead times, we determine the optimal ordering policy at various instants of time. The system is formulated as a Semi-Markov Decision Process and the optimum policy to be employed is found using linear programming method. Numerical examples are provided to illustrate the model.

## 1. Introduction:

In this model, we discuss the problem of optimally controlling the admission of two type of customers (priority and non - priority) to a service facility system with inventory for service completion. We consider a service facility system having finite waiting space. For the given values of maximum inventory and reorder level s, the service times and lead times are assumed to be exponentially distributed.

For example, in real life situation where customer service is becoming more and more important. An important aspect of efficiency in service is product availability, which is related to replenishment policy and quick service. In many cases, not all customer demand for a single product requires the same service level. The type of inventory that, we wish to study are spare parts inventory in the airline or shipping industries and spare parts for refinery equipment.

In all these cases, equipment is categorized in to many different classes and different service levels are defined for each type. Some of the equipment is very critical for the smooth running of the operations and needs to be serviced on a priority basis, while other equipment is less critical and will have lower priority. Typically, the equipment is ranked as vital, essential or auxiliary. Hence spare parts are rationed and when inventory levels are low at supply status. Only the vital equipment is serviced and the other equipment has to wait for a fresh supply of spare parts.

We conclude that from previous models an integrated approach like Markov Decision Process model is most appropriate to study service facility system (Queues-Inventory) and Maintenance systems. Sapna, K.P., and Berman, O., [1] studied one such system under MDP structure using LPP method to control the service rates. So for in the literature only admission control and service rate control problems are studied under MDP regime. Hild Mohamed et. al [3] analyzed a Markov decision problem: Optimal control of servers in a service facility holding perishable inventory with impatient customers.

Dekker, R., Hill, R.M., and Kleijn, M.J., [2] considered a lost sales (S - 1, S) inventory system with priority demand classes. Sapna, K.P. [10] considered a lost-sales (s, Q) inventory system with demand classes - ordinary and priority. The demands of these two classes arrive according to

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independent Poisson processes with different parameters. The distribution of the lead time is assumed to be exponential. This model was extended by Sivakumar, B. and Arivarignan, G. [8] by assuming MAP for two types of customers.

Recently Karthick, T. et.al [4] considered a (s, S) inventory system with two types of arriving customers (Type-1 or Type-2). The arrival of customers is assumed to follow a MAP and the lead time is assumed to have a phase-type distribution. Maheswari, P. et. al [6] analyze discrete time service facility system with two types of customers. Krishnakumar, S. et. al [7] considered retrial service facility system with ordinary customers and priority customers. Poisson streams and service times are assumed to follow an exponential distribution.

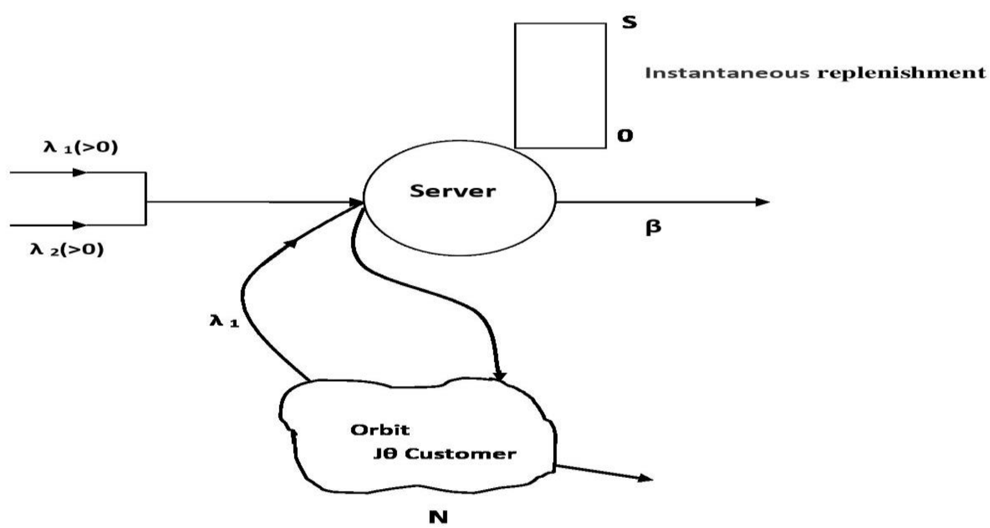
Veinott, A.F., [11] was the first to consider the problems of several demand classes in inventory systems. He analyzed a periodic review inventory model with multiple demand classes and zero lead time and introduced the concept of a critical level policy.

Nahmias, S. and Demmy, S., [9] introduced multiple demand classes. They considered two demand classes, Poisson arrival for demands, back-ordering and fixed lead time for supply of orders and derived approximate expressions for total cost.

Kim, E., [5] considered the admission control and the inventory management problem of a make-to-order facility with a common component, which is purchased from a supplier under stochastic lead time processes and setup costs.

The rest of the model is organized as follows. We provide a formulation of our Markov Decision model in the section 2. In section 3 Analysis of system and in 4, MDP formulation and in 5, steady state analysis is done. System performance measures are computed in section 6. In section 7, we present a procedure to prove the existence of a stationary optimal control policy and solve it by employing LP technique.

## 2. **Problem Formulation**




We consider a single service Queueing system, in which two type of customers arrive the service facility according to independent Poisson process with parameter $\lambda_1(>0)$ for priority customer and $\lambda_2(>0)$ for ordinary customers and served according to a FCFS queue discipline.

- One unit (item) from inventory is used upto serve one customer.

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- The capacity of the system is finite $N < \infty$ any customer who see N (when N is finite) customer in the system has to leave the system compulsory.
- The service times follow an exponential distribution with parameter $\beta_r$, r =0,1,2,…K
- The maximum capacity of inventory is S (Instantaneous replenishment is done at inventory level is zero) and the service rate $\beta_r$ may be chosen from given set of K values $\{\beta_0, \beta_1, \beta_2, \beta_3, ...\beta_K\}$
- Each $\beta_r$ $(1 \leq \text{r} \leq K)$ can be chosen from given set of N values $\{\beta_0, \beta_1, \beta_2, \beta_3, ...\beta_K\}$ where $\beta_r$ depends on number of customer in orbit $\beta_0$ ,when Z(t)=0, the service rate become $\beta_0$ =0

**3. Analysis**

Let X(t), Z(t) and I(t) denotes the status of the server, number of customer in the orbit and inventory level at time t, respectively.

Then $\{(X(t), \text{Z(t)}, \text{I}(t)) : t \geq 0\}$ is a three dimensional continuous time stochastic process with state space, where $E_1$= $\{0,1\}$ (0 denotes the idle server and 1 denotes the busy server)

$E_2$= {0,1,2,…,N}and $E_3$= {1,2,…,S}.

The infinitesimal generator A of the Markov process has entry of the form $(a_{(i,j,k)}^{(l,m,n)})$ ).

Some of the state transitions are given below:

From state (0,j,k) only transitions into the following states are possible:

(i) (1,j,k) with rate $\lambda_1 + \lambda_2$ for $0 \leq j \leq N$ ; $1 \leq k \leq S$ (customer arrival)

(ii) (1,j-1,k) with rate $(\lambda_1 + \lambda_2 + j\theta_1)$ forj=1, 2,…N; $1 \leq k \leq S$ (Customer arrive from an orbit).

From state (1,j,k) only transitions into the following states are possible:

(i) (1,j+1,k)with rate $\lambda_1$ for $0 \leq j \leq N-1$ ; $1 \leq k \leq S$ (customer arrival).

(ii) (0,j-1,k-1) with rate $\beta$ for $1 \leq j \leq N; 1 \leq k \leq S$ (Service completion)

No impatient customer. Customer leaves from system (or) orbit .The service of the system is finite, Arrival source to the system is finite. ie) The orbit size is always finite.

Now, we have to convert this Markov process into continuous time MDP by considering the following five components,

**4. MDP Formulation**

(i)**Decision epochs:** The decision epochs are random points as time line at each service Completion.

(ii)**State space**:

$$E = \{e = (\text{i, j, k}) : \text{i} = 0,1; 0 \leq \text{j} \leq \text{N}, \text{N} < \infty; 1 \leq \text{k} \leq \text{S}\}$$

(iii)**Action set**:

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$$A = \{a : 0 \le a = r \le k\}$$
$$A = \bigcup_{e \in E} A_e \ .$$

A decision rule from the class of rules $\Pi$ is equivalent to a function $\pi : E \to A$ and is given by

$$\pi(\mathrm{i,j,k}) = \{a : (\mathrm{i,j,k}) \in \mathrm{E}, \mathrm{a} \in \mathrm{A}\} \ .$$

(iv)**Transition probability** :

$p_{(i,j,k)}^{(\mathrm{l,m,n})}(a)$ -a transition probability from state (i,j,k) to the state (l,m,n).

(v) **Cost**:

Cost occurred when action 'a' is taken at state (i,j,k) is given by $C^{\pi}\left((l,m,n) | (i,j,k), a\right)$.

The long-run expected (average) cost rate when policy $\pi$ is adopted is given by

$$C^{\pi} = c_1 \overline{I}^{\pi} + c_2 \overline{W}^{\pi} + c_3 \overline{\alpha}^{\pi} + g \overline{B}^{\pi} \qquad (1)$$

when in the steady state , for given policy $\pi$ is given by $r^{\pi}$ is the expected reorder rate, $\overline{W}^{\pi}$ is the average waiting time for a customer, $\overline{\alpha}^{\pi}$ is the service expected cost due to different service rate due to d rate, $c_1$ denotes the holding cost / unit time/ unit item, $c_2$ denotes the waiting cost /customer / unit time , $c_3$ denotes the service cost / customer and g denotes the balking cost/customer associated with using the different service rates.

5. **Steady state Analysis:**

Let f denote the stationary policy, which is deterministic time invariant and Markovian Policy (MD). From the assumptions made in our system, it can be seen that $\{(X(t), \mathrm{Z(t)}, \mathrm{I}(t)) : t \ge 0\}$ is the controlled process $\{(X^f(t), \mathrm{Z}^f\mathrm{(t)}, \mathrm{I}^f(t)) : t \ge 0\}$ when policy $f$ is adopted. Since the process $\{(X^f(t), \mathrm{Z}^f\mathrm{(t)}, \mathrm{I}^f(t)) : t \ge 0\}$ is a Markov Process with finite state space E. The process is completely Ergodic, if every stationary policy gives rise to an irreducible Markov chain. It can be seen that for every stationary policy $\pi$ the Markov process is completely Ergodic and also the optimal stationary policy $\pi^*$ exists, because the state and action spaces are finite

Our objective is to find an optimal policy $\pi^*$ for which $C^{\pi^*} \le C^{\pi}$ for every MD policy in $\Pi^{MD}$ .

For any fixed MD policy $\Pi^{MD}$ and (i,j,k),(l,m,n)$\in E$ ,define

$$p_{ij}^{\pi}(\mathrm{l,m,n,t}) = \mathrm{pr}\{\mathrm{X}^{\pi}(\mathrm{t}) = l, \mathrm{Z}^{\pi}(\mathrm{t}) = \mathrm{m}, \mathrm{I}^{\pi}(\mathrm{t}) / \mathrm{X}^{\pi}(0) = \mathrm{i}, \mathrm{Z}^{\pi}(0) = \mathrm{j}, \mathrm{I}^{\pi}(0) = \mathrm{k}\}; (\mathrm{i,j,k}), (\mathrm{l,m,n}) \in \mathrm{E}$$

Now $p_{ij}^{\Pi}(\mathrm{l,m,t})$ satisfies the Kolmogorov forward differential equation. $\mathrm{P}'(\mathrm{t}) = \mathrm{P}(\mathrm{t})\mathrm{A}$, where A is an infinitesimal generator of the Markov process $\{(\mathrm{X}^R(\mathrm{t}), \mathrm{Z}^R(\mathrm{t}), \mathrm{I}^R(\mathrm{t})) : \mathrm{t} \ge 0\}$

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For each MD policy $\Pi$, we get a Markov chain with state space E and action set A which are finite,

$p^{\pi}(\mathrm{l,m,n}) = \lim_{t\to\infty} p_{ij}^{\pi}(\mathrm{l,m,n;t})$ exists and is independent of initial state (i,j,k)conditions.

The balance equations are obtained by using the fact that transition out of a state is equal to transition into a state

$$(\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,0,\mathrm{S}) = \beta P^{\pi}(1,0,1) \qquad (2)$$

$$(\lambda_1+\lambda_2+\mathrm{j}\theta_1)\,\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{S}) = \beta P^{\pi}(1,\mathrm{j},1), \qquad \mathrm{j}=1,2,\ldots,\mathrm{N} \qquad (3)$$

$$(\lambda_1+\beta)\,\mathrm{P}^{\pi}(1,0,\mathrm{S}) = (\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,0,\mathrm{S}) + \theta_1 P^{\pi}(0,1,\mathrm{S}) \qquad (4)$$

$$\begin{aligned}(\lambda_1+\beta)\,\mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{S}) &= (\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{S}) + (\mathrm{j}+1)\theta_1 P^{\pi}(0,\mathrm{j}+1,\mathrm{S}) \\ &\quad + \lambda_1 P^{\pi}(1,\mathrm{j}-1,\mathrm{S}) \qquad (5)\end{aligned}$$

$$\beta\,\mathrm{P}^{\pi}(1,\mathrm{N},\mathrm{S}) = (\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,\mathrm{N},\mathrm{S}) + \lambda_1 P^{\pi}(1,\mathrm{N}-1,\mathrm{S}) \qquad (6)$$

$$(\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,0,\mathrm{k}) = \beta P^{\pi}(1,0,\mathrm{k}+1), \quad 1\le \mathrm{k} \le \mathrm{S}-1 \qquad (7)$$

$$\begin{aligned}(\lambda_1+\lambda_2+\mathrm{j}\theta_1)\,\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{k}) &= \beta P^{\pi}(1,\mathrm{j},\mathrm{k}+1); \\ &\qquad \mathrm{j}=1,2,\ldots\mathrm{N}, 1\le \mathrm{k} \le S-1 \qquad (8)\end{aligned}$$

$$(\lambda_1+\beta)\,\mathrm{P}^{\pi}(1,0,\mathrm{k}) = (\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,0,\mathrm{k}) + \theta_1 P^{\pi}(0,1,\mathrm{k}), \quad 1\le \mathrm{k} \le S-1 \qquad (9)$$

$$\begin{aligned}(\lambda_1+\beta)\,\mathrm{P}^{\pi}(1,\mathrm{j},\mathrm{k}) &= (\lambda_1+\lambda_2)\,\mathrm{P}^{\pi}(0,\mathrm{j},\mathrm{k}) + (\mathrm{j}+1)\theta_1 P^{\pi}(0,\mathrm{j}+1,\mathrm{k}) \\ &\quad + \lambda_1 P^{\pi}(1,\mathrm{j}-1,\mathrm{k}), \quad 1\le \mathrm{k} \le S-1 \qquad (10)\end{aligned}$$

Together with the above set of equations, the total probability condition

$$\sum_{(j,r)\in E} P^{\pi}(\mathrm{j,r,k}) = 1 \qquad (11)$$

We get the steady state probabilities $\{P^{\pi}(i,j,k), (i,j,k) \in E\}$ uniquely.

**6. System Performance Measures.**

(i) The average inventory level in the system is given by

$$\bar{I}^{\pi} = \sum_{i=0}^{1}\sum_{k=1}^{S} k \sum_{j=0}^{N} P^{\pi}(\mathrm{i,j,k}) \qquad (12)$$

(ii) Mean waiting time in the system is given by

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$$\bar{W}^{\pi} = \sum_{j=1}^{N}\sum_{k=1}^{S}\left(\frac{\lambda_1 + j\theta_1}{\beta^{\pi}}\right)P^{\pi}(1,\mathrm{j},\mathrm{k}) + \sum_{j=1}^{N}\sum_{k=1}^{S}\left(\frac{\lambda_1 + \lambda_2 + j\theta_1}{\beta^{\pi}}\right)P^{\pi}(0,\mathrm{j},\mathrm{k}). \qquad (13)$$

(iii)The expected service rate is given by

$$\overline{\alpha}^{\pi} = \sum_{j=0}^{N}\sum_{k=1}^{S}\beta^{\pi}P^{\pi}(1,\mathrm{j},\mathrm{k}) \qquad (14)$$

(iv) The mean Balking rate is given by

$$\overline{B}^{\pi} = (\lambda_1 + \lambda_2)\sum_{k=1}^{S}P^{\pi}(1,\mathrm{N},\mathrm{k}) \qquad (15)$$

The long run expected cost rate is given by

$$C^{\Pi} = c_1\sum_{i=0}^{1}\sum_{k=1}^{S}k\sum_{j=0}^{N}P^{\pi}(\mathrm{i},\mathrm{j},\mathrm{k}) + c_2\left(\sum_{j=1}^{N}\sum_{k=1}^{S}\left(\frac{\lambda_1 + j\theta_1}{\beta^{\pi}}\right)P^{\pi}(1,\mathrm{j},\mathrm{k}) + \sum_{j=1}^{N}\sum_{k=1}^{S}\left(\frac{\lambda_1 + \lambda_2 + j\theta_1}{\beta^{\pi}}\right)P^{\pi}(0,\mathrm{j},\mathrm{k})\right) + \mathrm{c}_3\sum_{j=0}^{N}\sum_{k=1}^{S}\beta^{\pi}P^{\pi}(1,\mathrm{j},\mathrm{k})$$
$$+ g(\lambda_1 + \lambda_2)\sum_{k=1}^{S}P^{\pi}(1,\mathrm{N},\mathrm{k}) \qquad (16)$$

**7. Linear Programming Problem:**

**7.1 Formulation of LPP**

In this section we propose a LPP model within a MDP framework. First we define the variables, D(i,j,a) as a conditional probability expression

$$D(\mathrm{i},\mathrm{j},\mathrm{k},\mathrm{a}) = \Pr\left\{decisionis\,'a'/\,\mathrm{stateis}\,(\mathrm{i},\mathrm{j},\mathrm{k})\right\}$$

Since $0 \le$ D (i,j,k,a)$\le 1$, this is compatible with Randomized time invariant Markovian policies. Here, the Semi-Markovian decision problem can be formulated as a linear programming problem.

Hence

$$0 \le D(i,j,k) \le 1 \;\; and \;\; \sum_{a\in A}D(i,j,\mathrm{k},a) = 1, \mathrm{i} = 0,1; 0 \le j \le N; 0 \le k \le S.$$

For the reformulation of the MDP as LPP, we define another variable y(i,j,k,a) as follows.

$$\mathrm{y}(i,j,\mathrm{k},a) = D(\mathrm{i},\mathrm{j},\mathrm{k},\mathrm{a})P^{\Pi}(\mathrm{i},\mathrm{j},\mathrm{k}) \qquad (17)$$

From the above definition of the transition probabilities

$$P^{\pi}(i,j,k) = \sum_{a\in A}y(\mathrm{i},\mathrm{j},\mathrm{k},\mathrm{a}), (\mathrm{i},\mathrm{j},\mathrm{k}) \in \mathrm{E}, \mathrm{a} \in \mathrm{A} \qquad (18)$$

Expressing $P^{\Pi}$(i,j) in terms of y(i,j,a), the expected total cost rate functions(33) is

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Obtained and the LPP formulation is of the form

Minimize

$$C^{\Pi} = c_1 \sum_{i=0}^{1}\sum_{k=1}^{S} k \sum_{j=0}^{N} P^{\pi}(\mathrm{i,j,k}) + c_2 \left( \sum_{j=1}^{N}\sum_{k=1}^{S} \left( \frac{\lambda_1 + j\theta_1}{\beta^{\pi}} \right) P^{\pi}(1,\mathrm{j,k}) + \sum_{j=1}^{N}\sum_{k=1}^{S} \left( \frac{\lambda_1 + \lambda_2 + j\theta_1}{\beta^{\pi}} \right) P^{\pi}(0,\mathrm{j,k}) \right) + \mathrm{c}_3 \sum_{j=0}^{N}\sum_{k=1}^{S} \beta^{\pi} P^{\pi}(1,\mathrm{j,k})$$

$$+ g(\lambda_1 + \lambda_2) \sum_{k=1}^{S} P^{\pi}(1,\mathrm{N,k}) \qquad (19)$$

Subject to the constraints,

(i) $\mathrm{y(i,j,k,a)} \geq 0; \mathrm{(i,j,k)} \in \mathrm{E}, \mathrm{a} \in \mathrm{A}$

$ii) \sum_{l=0,1} \sum_{\mathrm{(i,j)} \in \mathrm{E}} \sum_{a \in A} y(\mathrm{i,j,k,a}) = 1,$

And the balance equation (2)-(26) obtained by replacing

$P^{\pi}(\mathrm{i,j,k})$ with $\sum_{a \in A} y(\mathrm{i,j,k,a})$.

**7.1 Lemma:**

The optimal solution of the above Linear Programming Problem yield a deterministic policy.

Proof:

From the equations (34) and (35)

$$\mathrm{D(i,j,k,a)} = \frac{y(\mathrm{i,j,k,a})}{\sum_{k=0}^{N} y(\mathrm{i,j,k},\beta_r)}, a = \beta_r, \mathrm{r} = 0,1,2,\ldots\mathrm{k} \qquad (20)$$

$$P^{\pi}(\mathrm{i,j,k}) = \sum_{a \in A} y(\mathrm{i,j,k,a}), \mathrm{(i,j,k)} \in \mathrm{E} \qquad (21)$$

Since the decision process is completely ergodic every basic feasible solution to the above linear Programmingproblem has the property that for each $\mathrm{(i,j,k)} \in \mathrm{E}$, $\mathrm{y(i,j,k,a)} > 0$ for exactly are $a \in A$. Hence, for each $\mathrm{(i,j,k)} \in E,$ D (i, j,k,a)=1, for a unique and for other values of a Thus given the number of customers in the orbit, we have to choose the service rate $\beta$ for which D (i,j,k,a)=1 . Hence the basic feasible solution of the LPP yields a deterministic policy

**8. Numerical illustration and Discussion:**

In this section we consider a service facility system to illustrate the method described in section 4, through numerical examples. We implemented TORA software to solve LPP by simplex algorithm.

The following table describes the solution for LPP problem by varying the arrival (Poisson) rates from 1.5 for priority customer, 1 for ordinary customer and an exponential service rates from 4 to 9. The expected cost is computed by taking waiting cost per customer is 0.5 and the service cost per customer is 0.8

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| Arrival rate: → / Service rate: ↓ | $\lambda_1 = 1.5$ $\lambda_2 = 1$ | $\lambda_1 = 1.5$ $\lambda_2 = 1$ | $\lambda_1 = 1.5$ $\lambda_2 = 1$ | $\lambda_1 = 1.5$ $\lambda_2 = 1$ |
|---|---|---|---|---|
| $\beta = 4$ | 6.1209 | 6.1965 | 7.0213 | 7.3453 |
| $\beta = 5$ | **7.4301** | 8.2861 | 8.2134 | 8.4036 |
| $\beta = 6$ | 9.4342 | **9.4013** | **9.3976** | **9.2317** |
| $\beta = 7$ | 10.1759 | 10.1211 | 10.0759 | 10.0643 |
| $\beta = 8$ | 11.8934 | 11.6450 | 10.5512 | 11.9654 |
| $\beta = 9$ | 12.6430 | 12.7182 | 12.5407 | 12.3742 |

**Table 1: The Expected total cost**

From the above table,

(a) The minimum expected cost for arrival rates $\lambda_1 = 1.5$, $\lambda_2 = 1$ will be obtained by adjusting the service rate as $\beta = 5$ per unit time.

The minimum expected cost for arrival rate $\lambda_1 = 1.5$, $\lambda_2 = 1$will be obtained at service rate 6

| (waiting cost per customer, the service cost per customer) ($c_1$, $c_2$) | Expected total cost |
|---|---|
| (0.5, 0.8) | 7.4301 |
| (1, 0.8) | 8.6429 |
| (1.5, 0.8) | 9.6307 |
| (2, 0.8) | 10.1625 |

**Table 2: The Expected total cost for varying waiting cost per customer**

(a)

| (waiting cost per customer, the service cost per customer) ($c_1$, $c_2$) | Expected total cost |
|---|---|
| (0.5, 0.8) | 7.4301 |
| (0.5, 1.6) | 9.0832 |
| (0.5, 2.4) | 9.4581 |
| (0.5, 3.2) | 10.1126 |

## 6. Conclusions and future research:

Analysis of inventory control at service facility is fairly recent system study. Most of previous work determined optimal ordering policies or system performance measures. We approached the problem in a different way with two type of customers given a service rate we determine the optimal controlling the admission of customers to determine the optimal order quantity or reorder level to be employed to minimize the long – run expected cost rate. Thus the admission control using inventory in service facility is established. In future we may extend this model to multi type of customer with discrete time MDP.

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