

# Chinese Remainder Theorem based Performance Analysis of RSA Cryptosystem

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**Abstract**—Message security and authenticity is a very important issue which cannot neglect in wireless network. This paper explains how the RSA algorithm can be used to achieve both, over a wireless network. The RSA-CRT technique was premeditated for data decryption and operative illustration of cryptography using the Chinese Remainder Theorem (CRT) for message security which is nearly four times faster.

**Keywords**- Encryption, Decryption, RSA, and CRT

## I. INTRODUCTION (HEADING 1)

The RSA algorithm is predicated on the notion that factoring very big integers is inefficient. As a result, deducing an RSA key necessitates a significant amount of computer processing power and time [2].

### A. KEY GENERATION ALGORITHM

This algorithm finds extreme applications in cryptography. Some of the notations used in this algorithm are as follows.

- $n$  - modulus.
- $e$  - exponent.
- $d$  - furtive exponent

Key Generation Algorithm s as shown in Figure 1.

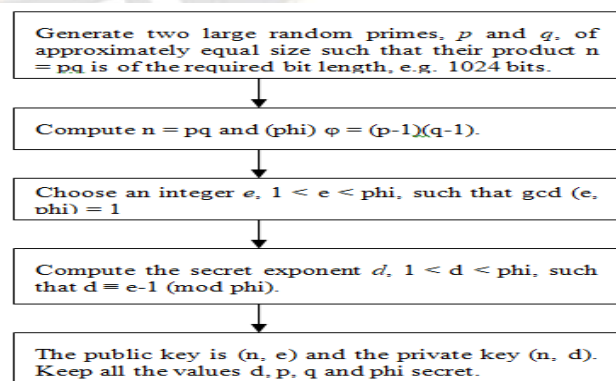


Figure 1. Key Generation Algorithm

## B. ENCRYPTION

Sender A follows the below steps:

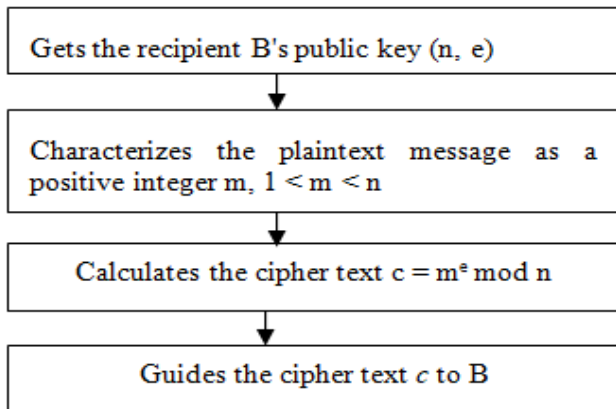


Figure 2: Encryption

## C. DECRYPTION

Recipient B does the succeeding steps:-

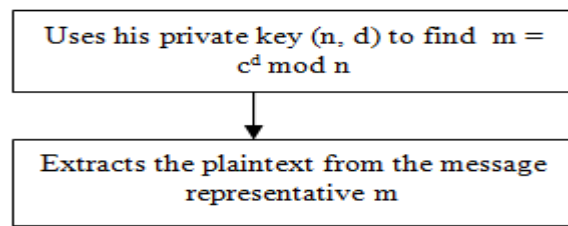


Figure 3: Decryption

## D. EXAMPLE

Figure 4 shows an example of RSA encryption and decryption.

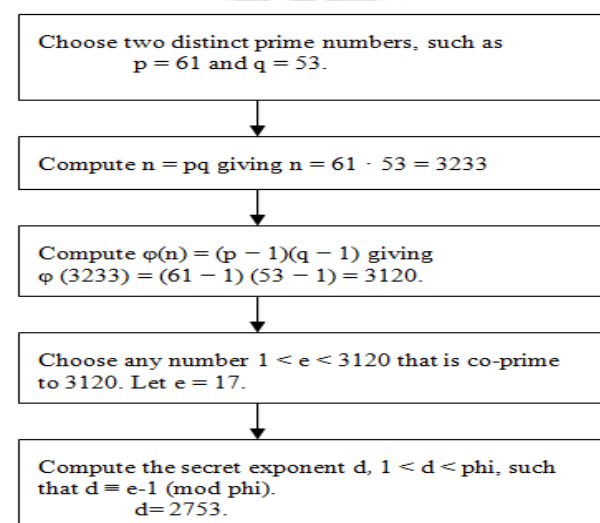


Figure 4: Example

## E. EXTENDED EUCLIDEAN ALGORITHM

Calculating the inverse of the modulus function of one integer to another is common in cryptography systems. As an example,  $x = y^{-1} \pmod{n}$ . Calculating  $x$  for any given  $y$  can be challenging, as one might anticipate.

It is easy to calculate the inverse function  $x$  by using the Extended Euclidean Algorithm. In this Algorithm, inverse of the modulus function have calculated if  $\gcd(x, n) = 1$ , then only inverse of  $x$ ,  $y$ , exists. In other words, there exists an integer  $y$  for which  $xy + sn = 1$ , where  $y$  is the modular inverse of  $x \pmod{n}$ , for which  $xy + sn = 1$ .

## F. WORKING EXAMPLE

To obtain  $d = e^{-1} \pmod{n}$ , first locate integers  $x$  and  $y$  such that  $xy + sn = 1$ , then calculate the inverse  $d$ .

We may determine  $d = 3^{-1} \pmod{20}$  using our technique above by finding  $\gcd(20, 3)$  and checking that it is one (otherwise, the inverse cannot be found).

$$20 = 3 \times 6 + 2$$

3 is equal to  $2 \times 1 + 1$  resulting in  $\gcd(20, 3) = 1$ . The numbers in this computation are then used to find  $xy + sn = 1$ .

$$1 = 3 - 1 \times 2$$

$$1 = 3 - 1 \times (20 - 6 \times 3)$$

$$1 = -1 \times 20 + 7 \times 3$$

As  $x$  (the coefficient of 3) is 7, the inverse is also 7..

Hence the secret exponent  $d$  has been computed using Extended Euclidean algorithm which has been explained in the above section. We can observe from above example as modulus is large, to decrypt original message using private key consumes large amount of time. So it decreases computational efficiency.

If you able to reduce computational time we can improve computational efficiency.

The computational time elapsed in RSA algorithm functioning can be reduced to almost four times If we modify RSA algorithm using Chinese Remainder Theorem.

The application of the Chinese Remainder Theorem to increase decryption speed is a critical performance booster. The Chinese Remainder Theorem is an algorithm in arithmetic computation that allows modular arithmetic to be performed efficiently when the modulus is big (typically several hundred bits). The brief idea of Chinese Remainder Theorem is given in the next section

## II. CHINESE REMAINDER THEOREM

The theorem's original version is a statement regarding modular arithmetic's simultaneous congruences. Integers of 1024, 2048, or 4096 bits are often utilised, which makes calculations take a long time. In this article, illustrate how using three extra values pre-computed from the prime factors

of  $n$ , the CRT representation of numbers in  $Z_n$  can be used to do modular exponentiation about four times faster.

#### A. RSA CALCULATIONS WITH CRT

The CRT provides a more efficient way to calculate

$$m = c^d \bmod n.$$

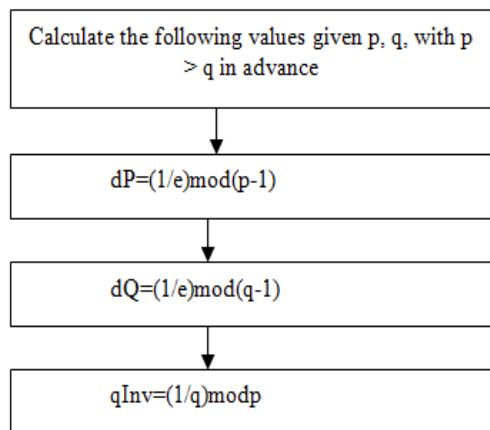


Figure 5: Steps in calculation

Do the following to compute the message  $m$  given  $c$

$$m_1 = c^{dP} \bmod p$$

$$m_2 = c^{dQ} \bmod q$$

$$h = qInv \cdot (m_1 - m_2) \bmod p$$

$$m = m_2 + h \cdot q$$

We keep the private key in a quintuple  $(p, q, dP, dQ, qInv)$ .

#### B. A SIMPLE EXAMPLE

This example comes from our RSA Algorithm.

Let  $p=137, q=131, n=137 \cdot 131, e=3, d=11787, m=513$

$$c = 513^3 \bmod n = 8363.$$

We might easily compute  $cd \bmod n$  to decode  $c$ :

$$m = 8363^{11787} \bmod 17947 = 513.$$

On a pocket calculator, this is quite difficult. Let's try the CRT way now. Take note of how much lower and more manageable the exponent and modulus numbers are. By breaking the RSA calculation down into two smaller ones, this simple (but insecure) example illustrates how easy it is to do the RSA calculation.

With a pocket calculator, we can compute this same result. Let us observe the results for 1024, 2048, 4096 bits RSA algorithm with and without CRT.

### III. RESULTS & DISCUSSIONS

TABLE 1. RSA ALGORITHM WITHOUT CRT

S.N	Key length (bits)	Time required for key generation (ms)		Generation time in ms		Verification time in ms	
		256R, p4	512R, p4	256R, p4	512R, p4	256R, p4	512R, p4
1	1024	32	16	0	0	16	16
2	2048	16	0	0	0	94	93
3	4096	16	31	0	0	609	625

TABLE 2. RSA ALGORITHM WITH CRT

S.N	Key length (bits)	Time required for key generation (ms)		generation time in ms		verification time in ms	
		256R, p4	512R, p4	256R, p4	512R, p4	256R, p4	512R, p4
1	1024	0	32	0	0	47	47
2	2048	16	16	0	16	297	297
3	4096	47	32	0	0	2297	2297

#### IV. CONCLUSION

It is evident from the results obtained for RSA algorithm with and without Chinese remainder theorem on 1024, 2048 and 4096 bits respectively. Under Chinese remainder theorem RSA algorithm performs better and becomes almost four times faster as the computational efficiency improves.

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