International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 11
Article Received:*10 September 2023* ***Revised:****20 October 2023* ***Accepted:****30 October 2023*

Applıcatıon of Laplace Transform in Scıence and Engıneerıng

Salisu Ibrahim[1]
[1]Mathematics Education, Tishk International University-Erbil, Kurdistan Region, Iraq,
Email: ibrahimsalisu46@yahoomail.com; salisu.ibrahim@tiu.edu.iq

Abstract: One reliable mathematical tool that is used extensively in many scientific and technical fields is the Laplace transform (LP). Similar to the application of transfer functions in solving ordinary differential equations (ODEs), LPs offer a simple method for tackling increasingly complex engineering problems. LPs are used in physics and engineering, and this research first looks at such uses before concentrating on how they are used in electric circuit analysis. The research also explores more sophisticated uses, such as load frequency control in power systems engineering.

Keywords: Ordinary Differential Equation**,** Laplace transforms, Electric circuit analysis, control engineering, quantum physics.

1. INTRODUCTION

The LP was created by Swiss mathematician Leonhard Euler in the 18th century to solve complex DEs. The LP is often taught only as a way to solve electrical circuits, DEs, but its use and impact are much broader than in the fields of electronics and communications (Poularikas, 2020 & Beerend, 2023, & Bellman, 1984, & Davies, 1979, & Dyke, 2001). The use of the LP has created the literature and traditions that underlie transient analysis. The conversion itself became widespread only when the renowned electrician Oliver Heaviside began to use the variation to solve electrical circuits. Most of the advances in electrical engineering were made in 1925, put into practice in 1935, and then subjected to mathematical analysis and scientific understanding in 1945. (Gopal, 1988, & Grubbström, 1967, & Hosono, 1981, & Honig, 1984).

The LP is an integral medium that is used to analyse linear ODEs. Physicists, opticians, electrical engineers, control engineers, mathematicians, signal processors, and probability theorists all use it (Kazem, 2013, & Khuri, 2001, & Roberts, 2006, & Charles, 2006). The Laplace transformation is a crucial idea from the department of arithmetic referred to as practical evaluation. It is an effective approach for reading linear time-invariant structures which include electric circuits, harmonic oscillators, mechanical structures, manipulate principle and optical gadgets the use of algebraic methods. Given an easy mathematical or practical expreession of an input or output to a gadget (Ridout, 2009, & Sheng, Chen, 2011, & Singhal, Vlach, Vlach, 1975), the Laplace rework affords an opportunity practical expreession that regularly analyse the system of reading the conduct of the gadget. "LP of some elementary functions. Sectional or piecewise continuity. Functions of exponential order. Sufficient conditions for existence of LPs. Some important properties of LPs. Linearity property". (Spiegel. 1965). The evaluation of electrical circuits and answer of linear DEs is analysed through use of LP rework. In real Physics structures the LP rework may be understood as a metamorphosis from the time area, wherein output and input are features of the frequency to time with inside the area, wherein output and input are features of complicated angular frequency. The primary system of reading a gadget the use of Laplace rework entails conversion of the gadget switch feature or differential equation into s –area, the use of s –area to transform enter features, locating an output feature through algebraically combing the enter and switch features, the use of partial features to lessen the output feature to less complicated additives and conversion of output equation again to time area. "The LP has been applied to various problems: to evaluation of payments, to reliability and maintenance strategies, to utility function analysis, to the choice of investments, to assembly line and queuing system problems, to the theory of systems and elements behavior, to the investigation of the dispatching aspect of job/shop scheduling, for assessing econometric models, to study dynamical economic systems" (Sudicky, 1989).

Many scientific and technological fields, including Control Engineering, Communication, Signal Analysis, and System Analysis, use the LP approach extensively. It is also used to convert a circuit for solution derivation from the time domain to the frequency domain. The acquired answer is then transformed back into the time domain using the inverse LP. The LP is a widely used integral transform in mathematics that is particularly useful in science and engineering. It may be used to convert time-dependent variables into complex angular frequency functions in the frequency domain. This shift is essential to the way that engineers analyze and build systems today, and it has a wider impact on disciplines like electric circuit analysis, communication engineering, control engineering, and nuclear physics (Watugala, 1993, & Widder, 2015).

One of the biggest challenges facing scientists has been solving complex differential equations. In order to address these challenges. Researchers like (Ibrahim, 2020; Ibrahim, & Isah, 2021; Isah, & Ibrahim, 2021; Ibrahim, & Isah, 2022; Salisu, 2022b) have all examined the use of numerical techniques to solve PDEs, fractional differential equations (FDEs), and ODEs. The practical significance of commutativity in this setting is emphasized. (Ibrahim, & Koksal, 2021a) examined how commutativity affected sensitivity when initial conditions (ICs) weren't zero. In two different publications (Salisu, 2022a; Salisu, 2022c), Salisu simultaneously investigated the implementation and decomposition of fourth-order Linear Time-Varying Systems (LTVSs) with non-zero ICs utilizing cascaded pairs of second-order commutative systems. On this issue, (Ibrahim, & Koksal, 2021b; Salisu, & Rababah, 2022) and Salisu did a parallel investigation. Furthermore, (Ibrahim, 2020; Ibrahim, & Isah, 2021; Isah, & Ibrahim, 2021; Ibrahim, & Isah, 2022; Salisu, 2022b) have devised a numerical approximation approach for the degree reduction of curves and surfaces. These methods provide viable ways to deal with intricate PDEs, FDEs, and ODEs. The analytical approached has been investigated by (Ibrahim, & et al, 2022; Ibrahim, & et al, 2023; Ibrahim, 2022a; Salisu, 2022b; Ibrahim, & et al, 2024; Ibrahim, & Baleanu, 2023)

This study's main goal is to investigate how well conversion techniques work for solving complex circuit problems. The study attempts to provide the reader with a solid understanding of circuit theory and LP concepts, clarifying answers to issues related to important applications in electrical circuits. Using a basic circuit analysis method in the transformation domain, the research attempts to create a time-domain differential equation in order to obtain the desired response function and use the transformation method to solve the desired variable.

2. PRELIMINARIES

We provide a list of definitions and relevant properties of the LP in this section. In later applications, these ideas will be critical to solving problems in a variety of engineering areas with the Laplace Transform.

2.1 *Definition:*

Consider a function f(t) that is defined for (t > 0), The Laplace Transform of the function f(t) is defined as the integral $\int_0^\infty e^{-st} f(t)dt$ *It is indicated as* $L\,[f\,(t)]$ *or* $F(s)$ *and is given below*

$$\mathrm{L[f(t)]} = \int_0^\infty \mathrm{e^{-st}\ f\,(t)dt} \ = \mathrm{F}(s). \tag{1}$$

2.2 Laplace Transformation Properties :

Linearity property: Let $f(t)$ and $g(t)$ be two functions of t and α, β be constants then,

$$[\alpha\, f(t) + \beta\, g(t)] = \alpha L\,[f(t)] + \beta L[g(t)]. \quad (2$$

Shifting Property:

If $L[f(t)] = F\,(s)$, then $L[e^{at}\, f(t)] = F(s\,-\,a)$. (3)

Multiplication by t^n Property:

if $L[f(t)] = F(s)$, then

$$\mathrm{L}[t^n f(\mathrm{t})] = (-1)^{\mathrm{n}} \frac{\mathrm{d^n}}{\mathrm{ds^n}} [\mathrm{F(s)}]. \tag{4}$$

2.3 LT of Derivative

The LT of derivative can be defined as

If $L\,[f(t)] = F(s)$, then

$$\mathrm{L}[f^n(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \cdots - f^{n-1}(0). \tag{5}$$

LT of Bessel's Function

Condition the LT of Bessel function as.

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 11
***Article Received:**10 September 2023 **Revised:**20 October 2023 **Accepted:**30 October 2023*

$$\mathrm{L}[J_O(t)] = \frac{1}{\sqrt{s^2+1}}. \quad (6)$$

where,

$$J_O(t) = \sum_{k=0}^{\infty}(-1)^k \frac{\left(\frac{1}{4}t^2\right)^k}{(k!)^2},$$ is called Bessel's function.

2.4 Inverse LT

The inverse LT of derivative can be defined as

If $L\,[f(t)] = F(s)$, then

$$L^{-1}[\mathrm{F(s)}] = \mathrm{f(t)},$$

is called inverse LT of $f(s)$. (7)

Inverse LT Theorem:

$$If\ L^{-1}[\emptyset_1(s)] = f_1(t);\ L^{-1}[\emptyset_2(s)] = f_2(t)\ \ then,$$

$$L^{-1}[\emptyset_1(s).\emptyset_2(s)] = \int_0^t f_1(u).f_2(t-u)du. \quad (9)$$

3. LT AND ITS APPLICATIONS IN ENGINEERING AND SCIENCE

In this section. We are going to discusses the LT and the application in various field of science and engineering. The LT is extensively used in the domains of engineering and science.

3.1 Laplace Transform

If a function does not approach infinity at any point and has a finite number of breaks, it is said to be piecewise continuous. The LP is utilized for the definition of a piecewise continuous function. Since it converts DE into algebraic problems, the LP of a function, represented as *L{f(t)}* or *F(s)*, is an essential tool in DE solution.

By applying an integral transform to the derivative function with the real variable *t*, the LT effectively turns it into a complex function with the variable s. Let *f(t)* be a function that satisfies a set of predetermined constraints for *t ≥ 0*. The LT formula formally defines the LT of *f(t)*, denoted as *L{f(t)}* or *F(s).*

Definition: Let us assume that f(t) is a function that is piecewise continuous. The definition of L{f(t)}, the LT of f(t), is:

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st}\ f(t)dt = F(s).$$

Theorem 1: (Existence)

As an example, suppose that the function f(t) is continuously defined for every t ≥ 0 and behaves exponentially as t tends to infinity. This can be expressed mathematically as |f(t)| ≤ M e^{at}(at) and f(t) ≥ C, where M, a, and C are constants. Then the LT

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt. \quad (10)$$

exists for s > a.

Proof:

As will be explained below, the integral included in the definition of F(s) can be split into two distinct integrals.

$$\int_0^{\infty} f(t)e^{-st}dt = \int_0^{C} f(t)e^{-st}dt + \int_C^{\infty} f(t)e^{-st}dt.$$

A = max{|f(t)| : 0 ≤ t ≤ C} we have.

$$\int_0^{C} f(t)e^{-st}dt \le A\int_0^{C} e^{-st}dt = A\left(\frac{1}{s} - \frac{e^{-sC}}{s}\right) < \infty \quad (11)$$

3.2 Inverse LT

we defined the LT of f by

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt.$$

Well also say that f is an inverse LT of *F*, and write

$$f(t) = \mathcal{L}^{-1}\{F(s)\} \quad (12)$$

Recovery of the original function f from its transformed counterpart F is a necessary step in applying the LT to solve DEs. There is a formula for this retrieval procedure, but it is not practicable to apply it without knowledge of complex variable functions. Luckily, there is another way that just uses the LT table to find the inverse transforms needed to solve the problem.

3.3 Applications in Science and Engineering Fields

3.3.1 Control engineering

Control system design heavily relies on the Laplace transformation. It is required to apply the LT to numerous time-dependent functions while analyzing a control system. An equally useful method for

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 11
***Article Received:**10 September 2023 **Revised:**20 October 2023 **Accepted:**30 October 2023*

determining the function f(t) from its Laplace representation is the inverse LT.

3.3.2 **System, Signal analysis and design**

Within the domain of transforms, which includes the LT, a signal is transformed using a system of equations or rules into a different signal. The Laplace transform, in particular, converts a signal from the time domain to the s-domain, which is also known as the s-plane.

3.3.3 **Electronic circuits analysis:**

The LT is a widely used tool by electronic engineers to effectively solve DEs that arise during the analysis of electronic circuits.

3.3.4 **Modeling of the system:**

The LT is used in system modeling to simplify calculations while handling a large number of DEs.

3.3.5 **Processing of digital signals:**

Applying the LT is essential for solving problems related to digital signal processing; it is a very useful tool.

3.3.6 **Nuclear Physics:**

The real features of radioactive decay are revealed through the use of the LT. It facilitates the investigation of nuclear physics' analytical side.

3.3.7 **Process Management:**

The LT is an essential tool for process management because it makes it easier to analyze the variables that, when changed, result in the desired changes in the end product. There are numerous examples of LT being used in science and engineering to solve DEs. The following examples demonstrate the application of the LT in a number of engineering fields.

3.3.8 **Automatic control**

A key technique in control theory, the LT is used in automatic control. It is crucial for describing linear time-invariant systems because it converts convolution operators into multiplication operators and makes it possible to express the transfer function of a system.

4. EXAMPLE

LP is used to convert time domian signal to s-domain for the frequency response analysis, which is used to look at the frequency response of the system. LT is utilized to expedite the examination of dynamic behavior or the development of a new system that complies with predetermined specifications. In contrast, signals can be broken down into their individual frequencies and oscillatory functions using the Fourier transform.

4.1 LP in Simple Electric Circuits

Consider an electric circuit containing a switch, and the following components: electromotive power (E) voltage, capacitor (C), inductance (L), and series resistance (R).

Kirchhoff's law allows us to deduce

$$\boldsymbol{L\frac{dI}{dt}+RI+\frac{Q}{c}=E.} \tag{13}$$

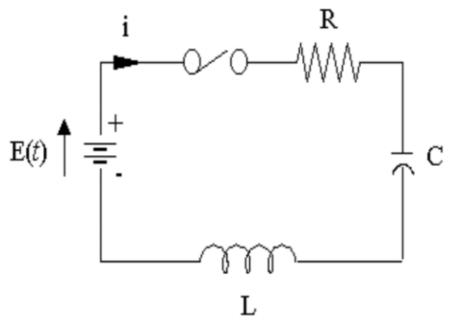

Figure 4.1: Electric Circuits.

Example: 1:

A 300-volt electromotive force (emf) is linked in series with a 0.02-farad capacitor, a 16-ohm resistor, and a 2-henry inductance to form a circuit. Both the circuit's current and the capacitor's charge are zero at time t. Assuming t > 0, find the charge and current at every given moment.

Solution:

The solution entails applying Kirchhoff's law to determine the instantaneous charge (q) and current (i) at a specific moment.

$$L\frac{dl}{dt}+RI+\frac{Q}{c}=E,$$

$$2\frac{d^2Q}{dt^2}+16\frac{dQ}{dt}+50Q=300, \qquad \ldots\left\{\ I=\frac{dQ}{dt}\right.$$

$$\frac{d^2Q}{dt^2}+8\frac{dQ}{dt}+25Q=150.$$

Applying LT leads to,

$$L\left[\frac{d^2Q}{dt^2}\right]+8L\left[\frac{dQ}{dt}\right]+25L[Q]=L[150],$$

$$\{s^2L[Q]-sQ(0)-Q'(0)\}+8\{sL[Q]-Q(0)\}+25L[Q]=150L,$$

$$s^2L[Q]+8sL[Q]+25L[Q]=\frac{150}{s},$$

$$(s^2+8s+25)L[Q]=\frac{150}{s},$$

$$L[Q]=\frac{150}{s(s^2+8s+25)}.$$

Taking the Inverse LP

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 11
***Article Received:**10 September 2023 **Revised:**20 October 2023 **Accepted:**30 October 2023*

$$Q = L^{-1}\left[\frac{150}{s(s^2+8s+25)}\right].$$

Considering the partial fraction

$$Q = L^{-1}\left[\frac{6}{s} - \frac{6s+48}{(s^2+8s+25)}\right],$$

$$Q = 6L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{6(s+4)}{(s+4)^2+9}\right] - L^{-1}\left[\frac{24}{(s+4)^2+9}\right].$$

With the shifting property, we obtain

$$Q = 6 - 6e^{-4t}\cos 3t - 8e^{-4t}\sin 3t,$$

And $I = \frac{dQ}{dt} = 50e^{-4t}\sin\ 3t.$

There Q is the charge and I is the current at any time $t > 0$.

Example: 2 A resistor with a resistance of 60 ohms, a capacitor with a capacitance of 1/180 farads, an inductor with an inductance of 5 henries, and a voltage supply denoted by $E(t)$ = 120 cos 6t volts are connected in series. Let us assume that at first, both the charge c and the current I are zero**.**

(a) One way to illustrate the differential equation governing the charge (t) is as follows:

$$q'' + 12q' + 36q = 24\ cos\ 6t \qquad (14)$$

(b) Given the initial conditions mentioned in Part (a), find the charge *q(t)* that satisfies the equation.

Solution:

(a) Given $L = 5$, $C = \frac{1}{180}$, $and\ \ E(t) = 120\cos\ 6t$.

Substituting in the equation

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{c}q = E(t),$

we obtain

$q'' + 12q' + 36q = 24\cos\ 6t.$

(b) The characteristic equation is

$m^2 + 12m + 36 = 0\ ;\ m_{1,2} = \frac{-12 \pm \sqrt{12^2 - 4\cdot 36}}{2} = -6.$

As a result, the homogeneous equation's overall solution is provided by:

$q_c = c_1e^{-6t} + c_2te^{-6t}.$

We try to determine a specific solution by applying the indeterminate coefficients method:

$$g(t) = 24\cos 6t\ ,$$
$$q_p(t) = A_1\cos(6t) + A_2\sin(6t),$$
$$q_p'(t) = -6A_1\sin(6t) + 6A_2\cos(6t),$$
$$q_p''(t) = -36A_1\cos(6t) - 36A_2\sin(6t).$$

This is what we get when we factorize and substitute it into the nonhomogeneous equation given in Eq. (14).

$$72A_2\cos(6t) - 72A_1\sin(6t) = 24\cos(6t)\ ,$$

where

$$A_1 = 0 \text{ and } A_2 = \frac{1}{3} \quad q_p(t) = \frac{1}{3}\sin(6t),$$

and the general solution is

$$q(t) = c_1e^{-6t} + c_2te^{-6t} + \frac{1}{3}\sin(6t),$$

$$I = \frac{dq}{dt} = -6c_1e^{-6t} - 6c_2te^{-6t} + c_2e^{-6t} + 2\cos(6t).$$

The ICs indicates $q(0) = 0\ \rightarrow\ c_1 = 0$ and $q'(0) = 0\ \rightarrow\ c_2 = -2$

Thus, the charge $q(t) = -2te^{-6t} + \frac{1}{3}\sin(6t)$.

4.2 Theory of Automatic Control

A servomechanism is an apparatus that uses mechanical, electronic, or other principles to accomplish automatic control. Suppose for the moment that a missile, $\boldsymbol{M}$, is following an enemy aircraft. In order for the missile to intercept and destroy the target, it must also modify its route by the same angle if the enemy changes direction at time t by an angle $\phi(t)$. A human might manually manipulate a steering system to make the necessary turns if they were on board the missile. But since the missile functions on its own, human control must be replaced.

In order to do this, a system that can understand data—such as a radar beam indicating the direction the missile should turn—is needed. Furthermore, a system that turns a shaft at a precise angle to provide the desired turn must exist to replace human hands. Assume for the purposes of this particular application that the radar beam indicates θ, the desired turning angle. Let the angle of the shaft at time t be represented by $\theta(t)$. It is anticipated that there will be some margin of error, indicated by *L[1]*, because these events happen quickly.

$$\text{Error } = \theta(t) - \alpha t \qquad (15)$$

i.e. The shaft must be informed of the error's existence so that a compensatory torque can be produced. The extent of the error determines the magnitude of the torque; a larger error calls for a greater torque, whereas a smaller error calls for a lesser torque.

Torque $= I\frac{d^2q}{dt^2}$.

This suggests that the torque and inaccuracy are proportionate,

$$I\frac{d^2q}{dt^2}\alpha[\theta(t)-\alpha t],$$

$$I\frac{d^2q}{dt^2}=-k[\theta(t)-\alpha t],$$

where $k > 0$.

Since torque is directly correlated with error, the relationship between positive errors and opposing torques and negative errors and supportive torques is represented by a negative sign.

This approach is predicated on a starting state with zero angular velocity and angle.

i.e. $\theta(0)=0$, and $\theta'(0)=0$

From the equation above

$$I\frac{d^2q}{dt^2}=-k\theta(t)+k\alpha t$$

$$\Rightarrow\ \frac{d^2q}{dt^2}+\frac{k}{l}\theta(t)=\frac{k}{l}\alpha t.$$

Taking the LT,

$$L\left[\frac{d^2q}{dt^2}\right]+\frac{k}{l}L[\theta(t)]=\frac{k}{I}L[\alpha t]$$

$$\Rightarrow\ s^2L[\theta]-s\theta(0)-\theta'(0)+\frac{k}{l}L[\theta]=\frac{k}{l}\alpha\frac{1}{s^2}.$$

$$\Rightarrow\ \left(s^2+\frac{k}{l}\right)L[\theta]=\frac{k}{l}\alpha\frac{1}{s^2}.$$

Using $\theta(0)=0$, and $\theta'(0)=0$

$$\Rightarrow\ L[\theta]=\frac{k}{l}\left[\frac{\alpha}{s^2\left(s^{2+}\frac{k}{I}\right)}\right].$$

Taking the Inverse LT,

$$\theta=\frac{k}{I}L^{-1}\left[\frac{\alpha}{s^2\left(s^2+\frac{k}{I}\right)}\right],$$

$$\theta=\frac{k\alpha}{I}\left[L^{-1}\left(\frac{1}{s^2}\right)*L^{-1}\left(\frac{1}{s^2+\frac{k}{I}}\right)\right].$$

By convolution theorem,

$$\theta=\frac{k\alpha}{I}\int_0^t u\frac{1}{\sqrt{k/I}}\sin\ \sqrt{\frac{k}{I}}(t-u)du$$

$$\theta=\alpha\sqrt{\frac{k}{I}}\int_0^t u\cdot\sin\ \sqrt{\frac{k}{I}}(t-u)du$$

$$\theta=a\sqrt{\frac{k}{I}}\left\{\left[u.\frac{1}{\sqrt{k/l}}\cos\ \sqrt{\frac{k}{I}}(t-u)\right]_0^t-\int_0^t\frac{1}{\sqrt{k/l}}\cos\ \sqrt{\frac{k}{I}}(t-u)du\right\}. \tag{16}$$

Integration by parts leads to

$$\theta=a\sqrt{\frac{k}{I}}\left\{\left[\frac{t}{\sqrt{k/1}}\right]+\left[\frac{I}{k}\sin\ \sqrt{\frac{k}{I}}(t-u)\right]_0^t\right\}.$$

$$\theta=a\sqrt{\frac{k}{I}}\left\{\frac{I}{\sqrt{k}}t-\frac{I}{k}sin\sqrt{\frac{k}{I}}t\right\}.$$

$$\theta(t)=at-a\sqrt{\frac{I}{k}}sin\sqrt{\frac{k}{I}}t.$$

Hence, the required turn at any time.

4.3 LT in Nuclear Physics:

Considering the nuclear physics ideas. Take a look at the first-order linear DE below.

$$\frac{dN}{dt}=-\lambda N. \tag{17}$$

This equation represents the main relationship driving radioactive decay: $N=N(t)$ represents the number of atoms in a sample of a radioactive isotope that have not decayed at time t, and λ stands for the decay constant.

This equation can be solved using the LP. We derive the following equation by rearranging the equation above.

$$\frac{dN}{dt}+\lambda N=0.$$

Taking LP on both sides,

$$sL[N]-N(0)+\lambda L[N]=0$$

$$s\bar{N}-N_0+\lambda\bar{N}=0$$

$$L[N]=\bar{N}\text{ and }N(0)=N_0$$

$$\bar{N}=\frac{N_0}{s+\lambda}. \tag{18}$$

Now, Taking Inverse LP on both sides of Eq. (16), we get

$$N(t)=N_0e^{-\lambda r}. \tag{19}$$

Which is indeed the correct form for radioactive decay.

4.4 LT in Control Engineering.

Mechanical Engineering:

The LP is a commonly used tool in mechanical engineering to solve ODEs that arise during mathematical modeling of mechanical systems. This process helps determine the transfer function of the system. The procedure for determining the transfer function with the Laplace Transform is shown in the example that follows.

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 11
***Article Received:**10 September 2023 **Revised:**20 October 2023 **Accepted:**30 October 2023*

At $t = 0$ in the scenario shown, the tank is empty. After this $\boldsymbol{t > 0}$, a constant flow rate $\boldsymbol{Q_i}$ is introduced, and the rate at which the flow leaves the tank is indicated by $\boldsymbol{Q_0}$=$\boldsymbol{CH}$. The cross-sectional area of the tank is **A.** The assignment is to find the differential equation for the head **H**'s, the time constant, and the transfer function of the system.

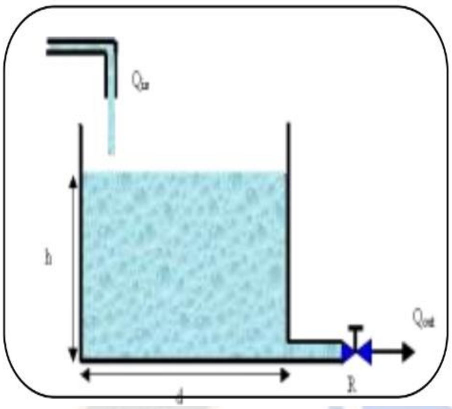

Figure 4.2: Control Engineering

Solution

Given that $Q_0 = CH$

Let, $M =$ Mass of fluid

$e =$ Density of fluid

Mass $= M =$ Volume $\times$ density

$= AH \times e$

Mass flow rate $= \dot{M} = \dfrac{dM}{dt}$

$= \dfrac{d}{dt}(AH \times e) = eA \times \dfrac{dH}{dt}$

Observe that the Mass flow rate into tank = Mass in flow rate -Mass out flow rate.

$$eA\frac{dH}{dt} = eQ_i - eQ_o$$

$$A\frac{dH}{dt} = Q_\mathrm{i} - Q_o$$

$\mathrm{A}\frac{dH}{dt} = Q_i - CH \ldots.. \{Q_o = CH$

$Q_i = \mathrm{A}\frac{dH}{dt}$+CH.

This equation represents the DE for head H

Now,

Taking LP on both sides,

$$L[Q_i] = A_s L\left[\frac{dH}{dt}\right] + C_s L[H]$$

$$Q_i(s) = A_s\{sH(s) - H(0)\} + C_s H(s)$$

$$Q_i(s) = A_s \cdot sH(s) + C_s H(s)$$

$$\{\because H(0) = 0$$

$$Q_i(s) = (sA_s + C_s)H(s)$$

$$\frac{H(s)}{Q_i(s)} = \frac{1}{sA_s + C_s}.$$

But $\mathrm{Q_0 = CH}$.

Taking LP, we obtain

$$\mathrm{Q_0(s) = C_s H(s)}$$

$$\mathrm{H(s)} = \frac{\mathrm{Q_0(s)}}{\mathrm{c_s}}$$

Which leads to

$$\frac{\mathrm{Q_0(s)}}{\mathrm{C_s Q_i(s)}} = \frac{1}{\mathrm{sA_s + C_s}}$$

$$\frac{\mathrm{Q_0(s)}}{\mathrm{Q_i(s)}} = \frac{1}{1 + \left(\frac{\mathrm{A_s}}{\mathrm{C_s}}\right)\mathrm{s}}.$$

This equation represents the transfer function of system, time constant.

5. CONCLUSION

The research presented the application of LP in different areas of physics and electrical power engineering. Apart from that, the LP is a very efficient mathematical tool for simplifying exceedingly difficult problems in the field of stability and control. Many research software's have made it feasible to directly simulate the Laplace transformable equations due to the ease with which LP may be applied in a variety of scientific applications, which has resulted in significant progress in the research field.

REFERENCES

[1] A. D. Poularikas, The Transforms and Applications Hand- book (McGraw Hill, 2000), 2nd ed.

[2] Beerends, R. J., ter Morsche, H. G., Van den Berg, J. C., & Van de Vrie, E. M. (2003). Fourier and Laplace transforms (p. 458).

[3] Bellman, R., & Roth, R. S. (1984). The laplace transform (Vol. 3). World Scientific.

[4] Davies, B., & Martin, B. (1979). Numerical inversion of the Laplace transform: a survey and comparison of methods. Journal of computational physics,33(1), 1-32.

[5] Dyke, P. P., & Dyke, P. P. (2001). An introduction to Laplace transforms and Fourier series. London: Springer.

[6] Gopal, M. (1988). Digital control engineering. New Age International.

[7] Grubbström, R. W. (1967). On the application of the Laplace transform to certain economic problems. Management Science, 13(7), 558-567.

[8] Hosono, T. (1981). Numerical inversion of Laplace transform and some applications to wave optics. Radio Science, 16(6), 1015-1019.

[9] Honig, G., & Hirdes, U. (1984). A method for the numerical inversion of Laplace transforms. Journal of Computational and Applied Mathematics, 10(1), 113-132.

[10] Kazem, S. (2013). Exact solution of some linear fractional differential equations by Laplace transform. International Journal of nonlinear science, 16(1), 3-11.

[11] Khuri, S. A. (2001). A Laplace decomposition algorithm applied to a class of nonlinear differential equations. Journal of applied mathematics, 1(4), 141-155.

[12] M.J.Roberts, Fundamentals of Signals and Systems (Mc- Graw Hill, 2006), 2nd ed.

[13] M. N. S. Charles K. Alexander, Fundamentals of Electric Circuits (McGraw Hill, 2006), 2nd ed.

[14] Oberhettinger, F., & Badii, L. (2012). Tables of Laplace transforms. Springer Science & Business Media.

[15] Ridout, M. S. (2009). Generating random numbers from a distribution specified by its Laplace transform. Statistics and Computing, 19(4), 439-450.

[16] Sheng, H., Li, Y., & Chen, Y. (2011). Application of numerical inverse Laplace transform algorithms in fractional calculus. Journal of the Franklin Institute, 348(2), 315-330.

[17] Singhal, K., Vlach, J., & Vlach, M. (1975). Numerical inversion of multidimensional Laplace transform. Proceedings of the IEEE, 63(11), 1627-1628.

[18] Spiegel, M. R. (1965). Laplace transforms (p. 249). New York: McGraw-Hill.

[19] Sudicky, E. A. (1989). The Laplace transform Galerkin technique: A time-continuous finite element theory and application to mass transport in groundwater. Water Resources Research, 25(8), 1833-1846.

[20] Watugala, G. (1993). Sumudu transform: a new integral transform to solve differential equations and control engineering problems. Integrated Education, 24(1), 35-43.

[21] Widder, D. V. (2015). Laplace transform (PMS-6). Princeton university press.

[22] Ibrahim, S. (2020). Numerical Approximation Method for Solving Differential Equations. Eurasian Journal of Science & Engineering, 6(2), 157-168, 2020.

[23] Ibrahim, S., & Isah, A. (2021). Solving System of Fractional Order Differential Equations Using Legendre Operational Matrix of Derivatives. Eurasian Journal of Science & Engineering, 7(1), 25-37, 2021.

[24] Ibrahim, S., & Isah, A. (2022). Solving Solution for Second-Order Differential Equation Using Least Square Method. Eurasian Journal of Science & Engineering, 8(1), 119-125, 2022.

[25] Ibrahim, S., & Koksal, M. E. (2021a). Commutativity of Sixth-Order Time-Varying Linear Systems. Circuits Syst Signal Process, 2021a. https://doi.org/10.1007/s00034-021-01709-6

[26] Ibrahim, S., & Koksal, M. E. (2021b). Realization of a Fourth-Order Linear Time-Varying Differential System with Nonzero Initial Conditions by Cascaded two Second-Order Commutative Pairs. Circuits Syst Signal Process. https://doi.org/10.1007/s00034-020-01617-1.

[27] Isah, A., & Ibrahim, S. (2021). Shifted Genocchi Polynomial Operational Matrix for Solving Fractional Order System. Eurasian Journal of Science & Engineering, 7(1) 25-37, 2021.

[28] Ibrahim, S., Sulaiman, T.A., Yusuf, A. et al. (2022). Families of optical soliton solutions for the nonlinear HirotaSchrodinger equation. Opt Quant Electron 54 (722). https://doi.org/10.1007/s11082-022-04149-x

[29] Ibrahim, S., Ashir, A.M., Sabawi, Y.A. et al. (2023). Realization of optical solitons from nonlinear Schr¨odinger equation using modified Sardar sub-equation technique. Opt Quant Electron 55, 617. https://doi.org/10.1007/s11082-023-04776-y

[30] Ibrahim, S. (2022a): Optical soliton solutions for the nonlinear third-order partial differential equation, Advances in Differential

Equations and Control Processes 29 (2022), 127-138. http://dx.doi.org/10.17654/0974324322037. 10

[31] Ibrahim, S. (2022b): Solitary wave solutions for the (2+1) CBS equation, Advances in Differential Equations and Control Processes 29 (2022), 117-126. http://dx.doi.org/10.17654/0974324322036.

[32] Ibrahim, S., Sulaiman, T.A., Yusuf, A. et al. (2024). Wave propagation to the doubly dispersive equation and the improved Boussinesq equation. Opt Quant Electron 56, 20.

[33] Ibrahim, S., & Baleanu, D. (2023). Classes of solitary solution for nonlinear Schrödinger equation arising in optical fibers and their stability analysis. Optical and Quantum Electronics, 55(13), 1158

[34] Rababah, A., & Ibrahim, S. (2016a). Weighted G^1-Multi-Degree Reduction of Bézier Curves. International Journal of Advanced Computer Science and Applications, 7(2), 540-545. https://thesai.org/Publications/ViewPaper?Volume=7&Issue=2&Code=ijacsa&SerialNo=70

[35] Rababah, A., & Ibrahim, S. (2016b). Weighted Degree Reduction of Bézier Curves with G^2-continuity. International Journal of Advanced and Applied Science, 3(3), 13-18.

[36] Rababah, A., & Ibrahim, S. (2018). Geometric Degree Reduction of Bézier curves, Springer Proceeding in Mathematics and Statistics, Book Chapter 8. 2018. https://www.springer.com/us/book/9789811320941

[37] Salisu, I. (2022a,). Commutativity of high-order linear time-varying systems. Advances in Differential Ð Equations and Control Processes 27(1) 73-83. Ð http://dx.doi.org/10.17654/0974324322013

[38] Salisu, I. (2022b). Discrete least square method for solving differential equations, Advances and Applications in Discrete Mathematics 30, 87-102. Ð http://dx.doi.org/10.17654/0974165822021

[39] Salisu, I. (2022c). Commutativity Associated with Euler Second-Order Differential Equation. Advances in Differential Equations and Control Processes 28(2022), 29-36. Ð http://dx.doi.org/10.17654/0974324322022

[40] Salisu, I., & Abedallah R. (2022). Decomposition of Fourth-Order Euler-Type Linear Time-Varying Differential System into Cascaded Two Second-Order Euler Commutative Pairs, Complexity, vol. 2022, . https://doi.org/10.1155/2022/3690019.