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Review on Various Forms of Lomax Distribution

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Abstract
The purpose of this brief survey is to discover the various forms of Lomax distribution developed from research articles released in recent years. This paper plainly and simply describes the most recent outcomes of different forms of lomax distribution applications. also illustrating critical aspects in the use of lomax distribution in various forms. This paper explores the significance of emphasising lomax distribution and their formulae for various forms.

Keywords: - Lomax Distribution, Exponential, Power, Gamma, Odd and Generalised.

Introduction

Lomax (1954) invented the Lomax distribution, which is also known as the Pareto type-II distribution. The Pareto type-II probability distribution is the most common type of probability distribution. It is essentially a Pareto distribution with 0 support. Because it belongs to the declining failure rate family, the Lomax distribution is regarded as an important model of lifespan models. It is mainly used for reliability modelling and life testing, applied to income and wealth distribution data, firm size, queuing problems, biological sciences, modelling the distribution of the sizes of computer files on servers and medicines. The lomax distribution is used for suggested as an alternative to the exponential distribution when the data are heavy-tailed. Now a days the lomax distribution is rapidly spread to all the fields for finding out the originality and justifying the issues faced.

We also discussed the basic concepts of Lomax distribution and their properties in this survey articles. This survey alone would make it easy to consider their review of the effects of the various forms of the Lomax distribution with their parameters and their future development in the future.

Lomax Distribution

Lomax distribution consists of two parameters which are denoted by Lomax (α, β), α is a shape parameter and β is the scale parameter 1.

The pdf and cdf of the lomax distribution is

$$f(x) = \frac{\propto}{\beta}\left[1 + \frac{x}{\beta}\right]^{-(\propto+1)} \text{ and } F(x) = 1 - \left[1 + \frac{x}{\beta}\right]^{-\propto}, x, \propto, \beta > 0$$

The variation of the lomax distribution to other distribution as follows,

(a) The Lomax distribution different of the Pareto Type I distribution shifted because its support begins at zero.

(b) The Lomax distribution or Pareto Type II distribution with location parameter $\mu = 0$ in Type-I

(c) The Lomax distribution is also beta distribution of the second kind with one of parameters is equal to1.

(d) The Lomax distribution is the Feller-Pareto distribution with location parameter $\mu = 0$, inequality parameter $\gamma = 1$, shape parameter $\gamma 1 = \alpha$ and shape parameter $\gamma 2 = 1$.

(e) The Lomax distribution is the Pareto Type IV with location parameter μ = 0 and inequality parameter γ = 1.

(f) It can be derived as a special case of a particular compound gamma distribution.

The Lomax distribution belongs to the Burr family of distributions. Although the Lomax distribution has many real-world uses, it does not allow much flexibility in modelling data. Many applications have recently demonstrated a clear requirement for the generalization of the Lomax distribution by adding one or more parameters to express the ability to fit varied data sets. Some well-known distributions are special examples of these generalized distributions.

2.1 Beta Lomax Distribution

This distribution derived at 2013 by Muhammad Rajab et.al., [2]with five parameters as shape α, scale λ, location μ, a and b are the positive parameters. The beta lomax is combination of the burr and beta family of distributions. The pdf of the beta lomax as

$$f(x)=\frac{\alpha}{\lambda\beta(a,b)}\left[1-\left\{1+\left(\frac{x-\mu}{\lambda}\right)\right\}^{-\alpha}\right]^{a-1}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-(\alpha b+1)} ;x\geq\mu \; and\,(\alpha,\lambda,a,b>0)$$

And the cdf of the beta lomax as

$$F(x)=\frac{\alpha}{\beta(a,b)}\sum_{i=0}^{a-1}(-1)^{i}\binom{a-1}{i}\frac{1}{\alpha b+i\alpha}\left[1-\left\{1+\frac{x-\mu}{\lambda}\right\}^{-(\alpha b+i\alpha)}\right]$$

2.2 Transmuted Lomax Distribution

This distribution is derived from the S.K. Ashour, and M.A. Eltehiwy in 2013 [3]. This distribution is based on the Quadratic Rank Transmutation Map the pdf of the transmuted and lomax distribution mixed together and form a new kind of the distribution is Transmuted lomax distribution.

The pdf is

$$f(x)=\frac{\alpha\theta\gamma}{(1+\gamma x)^{(\theta+1)}}\left(1+\lambda-2\lambda\left[1-(1+\gamma x)^{-\theta}\right]\right).$$

and the cdf

$$F(x)=\left[1-(1+\gamma x)^{-\theta}\right]\times\begin{pmatrix}(1+\lambda)\\-\lambda\left[1-(1+\gamma x)^{-\theta}\right]\end{pmatrix}$$

here θ is shape parameter, γ is scale parameter and $\lambda\in[-1,1]$.

2.3 Poisson Lomax Distribution

The poisson lomax distribution is derived at 2014 from Bander Al-Zahrania, Hanaa Sagorb [4]. The random variable X has the Poisson-Lomax distribution with three parameters α, β and λ.

The pdf as,

$$g(x;\alpha,\beta,\lambda)=\frac{\alpha\beta\lambda\,(1+\beta x)^{-(\alpha+1)}\,e^{-\lambda(1+\beta x)^{-\alpha}}}{(1-e^{-\lambda})},\quad x>0,\alpha,\beta,\lambda>0.$$

and the cdf as

$$\bar{G}(x;\alpha,\beta,\lambda)=\frac{1-e^{-\lambda(1+\beta x)^{-\alpha}}}{(1-e^{-\lambda})},\quad x>0,\alpha,\beta,\lambda>0.$$

2.4 Exponential Lomax Distribution

The exponential distribution was introduced by A. H. El-Bassiouny, et.,al., in 2015 [5]. In this distribution has the three parameters α, β and λ, (two shape parameters and one scale parameter).
The pdf as,

$$f(x)=\frac{\alpha\lambda}{\beta}\left(\frac{\beta}{x+\beta}\right)^{-\alpha+1}e^{-\lambda\left(\frac{\beta}{x+\beta}\right)^{-\alpha}},x\geq-\beta,\alpha,\beta,\lambda>0.$$

And the cdf as ,

$$F(x)=\int_{0}^{\frac{1}{\left(\frac{\beta}{x+\beta}\right)^{\alpha}}}\lambda e^{-\lambda x}dx,\ \ \alpha,\beta,\lambda>0,$$

$$=1-e^{-\lambda\left(\frac{\beta}{x+\beta}\right)^{-\alpha}}.$$

2.5 Gamma Lomax Distribution

The gamma lomax distribution was explained by the Gauss M. Cordeiro et., al., in 2015 [6]. The burr and gamma family of the combined to frame as Gamma Lomax Distribution. The Gamma Lomax distribution has the two parameters with special gamma function.
The pdf as,

$$f(x)=\frac{1}{\Gamma(a)}(\alpha+2\beta x)(\alpha x+\beta x^2)^{a-1}\,e^{-(\alpha x+\beta x^2)}.$$

The cdf as,

$$F(x)=\frac{\gamma(a,\alpha x+\beta x^2)}{\Gamma(a)}.$$

2.6 Weibull Lomax Distribution

The Weibull lomax distribution is derived by the M. H. Tahir, et.,al., in 2015 [7]. In combination of Burr and G family of the formation of Weibull Lomax distribution. Here additionally two shape parameters introduced so that in this distribution has four parameters.
The pdf as,

$$f(x;a,b,\alpha,\beta) = \frac{ab\,\alpha}{\beta}\left[1+\left(\frac{x}{\beta}\right)\right]^{b\,\alpha-1}\left\{1-\left[1+\left(\frac{x}{\beta}\right)\right]^{-\alpha}\right\}^{b-1} \times \exp\left\{-a\left\{\left[1+\left(\frac{x}{\beta}\right)\right]^{\alpha}-1\right\}^{b}\right\},$$

The cdf as,

$$F(x;a,b,\alpha,\beta) = 1-\exp\left\{-a\left\{\left[1+\left(\frac{x}{\beta}\right)\right]^{\alpha}-1\right\}^{b}\right\}.$$

2.7 Beta Exponential Lomax Distribution

This distribution has the five parameters derived from Mead, et., al., at 2016 [8]. The beta, exponential special functions introduced two more parameters to the lomax distribution.
The pdf as,

$$f(x;\xi) = \frac{\beta\theta\lambda}{B(a,b)}(1+\lambda x)^{-(\theta+1)}\left(1-(1+\lambda x)^{-\theta}\right)^{a\beta-1}\left[1-\left(1-(1+\lambda x)^{-\theta}\right)^{\beta}\right]^{b-1}.$$

The cdf as,

$$F(x;\zeta) = \frac{\left(1-(1+\lambda x)^{-\theta}\right)^{a\beta}}{a\,B(a,b)}\left[{}_2F_1\left(a,1-b;a+1;\left(1-(1+\lambda x)^{-\theta}\right)^{\beta}\right)\right],$$

where

$${}_2F_1(a,b;c;z) = \frac{1}{B(b,c-b)}\int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-t\,z)^{a}}\,dt,$$

Under the hypergeometric function. The parameters are (a, b, β, ɵ, λ).

2.8 Power Lomax Distribution

The lomax distribution considering with power transformation [9]. The power lomax distribution has the three parameters. It is derived by El-Houssainy A. Rady, et.,al., in 2016.
The pdf as,

$$f(x) = \alpha\beta\lambda^{\alpha}x^{\beta-1}\left(\lambda+x^{\beta}\right)^{-\alpha-1}, \quad x>0,\alpha,\beta,\lambda>0.$$

The cdf as,

$$F(x) = 1-\lambda^{\alpha}\left(x^{\beta}+\lambda\right)^{-\alpha}, \quad x>0,\alpha,\beta,\lambda>0.$$

2.9 Gampertz Lomax Distribution

Oguntunde, et.,al., derived the new generalisation of lomax distribution with gampertz family of distribution in 2017 [10]. This distribution has the three shape parameter and one scale β parameter, totally four parameters.
The pdf as,

$$f(x) = \theta\alpha\beta(1+\beta x)^{\alpha\gamma-1} e^{\frac{\theta}{\gamma}\{1-[1+\beta x]^{\alpha\gamma}\}}$$

The cdf as,

$$F(x) = 1 - e^{\frac{\theta}{\gamma}\{1-[1+\beta x]^{\alpha\gamma}\}}$$

2.10 Half Logistic Lomax Distribution

This distribution was derived from Masood Anwar and Jawaria Zahoor in 2018 [11]. The lomax distribution is mixed with logistic families and derived two parameter half logistic lomax distribution.
The pdf as,

$$f(x;\alpha,\beta) = \frac{2\alpha\beta(1+\beta x)^{-(\alpha+1)}}{\left[1+(1+\beta x)^{-\alpha}\right]^2}, \quad x > 0;\ \alpha,\beta > 0.$$

The cdf as,

$$F(x) = \frac{\left[1-(1+\beta x)^{-\alpha}\right]}{\left[1+(1+\beta x)^{-\alpha}\right]}.$$

2.11 Exponential Weibull Lomax Distribution

This distribution was introduced by Amal S. Hassan and Marwa Abd-Allah in 2018 [12]. This distribution has three parameters.
The pdf as,

$$f(x) = \frac{a\alpha\beta(G(x))^{\beta-1}g(x)}{(1-(G(x))^{\beta+1}} e^{-\alpha[\frac{G(x)}{1-G(x)}]^{\beta}} [1-exp(-\alpha\left[\frac{G(x)}{1-G(x)}\right]^{\beta}\Big)]^{a-1}$$

The cdf as,

$$F(x) = [1-exp(-\alpha[\frac{G(x)}{1-G(x)}]^{\beta})]^{a} \ ;\ x > 0\ ;\ a,\alpha,\beta > 0$$

where $a, \beta > 0$ are the two shape parameters and $\alpha > 0$ is the scale parameter.

2.12 Rayleigh Lomax Distribution

The combination of the lomax with Rayleigh family of distributions. It was explained by Kawsar Fatima, et., al., in 2018 [13]. This distribution has the three parameters.

The pdf as,

$$f(x,\beta,\lambda,\theta)=\frac{\beta\lambda}{\theta}\left(\frac{\theta}{\theta+x}\right)^{-2\lambda+1} e^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+x}\right)^{-2\lambda}}, \ x\geq-\theta \text{ and } \theta,\lambda,\beta>0.$$

The cdf as,

$$F(x)=1-e^{\frac{-\beta}{2}\left(\frac{\theta}{\theta+x}\right)^{-2\lambda}}, \ x\geq-\theta \text{ and } \theta,\lambda,\beta>0$$

2.13 Transmuted Rayleigh Lomax Distribution

Amani Alghamdi, 2018 [14] explained about the Transmuted Rayleigh Lomax distribution clearly with four parameters.

The pdf as,

$$f(x)=\frac{\alpha}{\lambda\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha-1}e^{\frac{-1}{2\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha}}[1-\beta+2\beta e^{\frac{-1}{2\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha}}].$$

The cdf as,

$$\begin{aligned}F(x)&=(1+\beta)(1-e^{\frac{-1}{2\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha}})-\beta[1-e^{\frac{-1}{2\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha}}]^2\\&=(1-e^{\frac{-1}{2\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha}})[1+\beta e^{\frac{-1}{2\sigma^2}(\frac{x+\lambda}{\lambda})^{2\alpha}}],\end{aligned}$$

where $\alpha,\lambda,\sigma>0$ and $|\beta|\leq 1$.

2.14 Odd Generalized Exponential Power Lomax Distribution

Salwa Mahmoud Assar introduced 2018 new generalization of the lomax distribution [15]. Here has four parameters.

The pdf as,

$$f(x;\alpha,\beta,\gamma,\lambda)=\frac{\alpha\beta\lambda}{\gamma}x^{\beta-1}\left(1+\frac{x^\beta}{\gamma}\right)^{\alpha-1}e^{-\lambda\left(\left(1+\frac{x^\beta}{\gamma}\right)^{\alpha}-1\right)},$$

The cdf as,

$$F(x;\alpha,\beta,\gamma,\lambda)=1-e^{-\lambda\left(\left(1+\frac{x^\beta}{\gamma}\right)^{\alpha}-1\right)}, \ x>0.$$

2.15 Inverse Power Lomax Distribution

In 2019 A.S. Hassan, et.,al., derived this new type of lomax distribution on 2019 [16]. Here has three parameters.

The pdf as,

$$f(x;\alpha,\lambda,\beta)=\frac{\alpha\beta}{\lambda}x^{-(\beta+1)}\left(1+\frac{x^{-\beta}}{\lambda}\right)^{-\alpha-1} \quad ;x,\alpha,\lambda,\beta>0$$

The cdf as,

$$F(x;\alpha,\lambda,\beta)=\left(1+\frac{x^{-\beta}}{\lambda}\right)^{-\alpha} \quad ;x,\alpha,\lambda,\beta>0$$

2.16 Weibull Generalised Lomax Distribution

In 2019,new extension of lomax distribution with three parameters is the Weibull generalised lomax distribution derived by Mona Mustafa, and et.al.,[17]

The pdf as,

$$f_{[\alpha,\theta,b]}(x)\,|_{(\alpha>0,\theta>0,b>0)}^{(x>0)}=\alpha\,\theta b\,(1+x)^{b\theta-1}\left[-1+(1+x)^{\theta b}\right]^{-1+\alpha}\,\mathrm{e}^{-\left[-1+(1+x)^{\theta b}\right]^{\alpha}},$$

The cdf as,

$$F_{[\alpha,\theta,b]}(x)\,|_{(\alpha>0,\theta>0,b>0)}^{(x>0)}=1-\mathrm{e}^{-\left[-1+(1+x)^{\theta b}\right]^{\alpha}},$$

2.17 Type II Topp Leone Power Lomax Distribution

Sanaa Al-Marzouki, et.,al., derived this distribution in 2020 [18]. In this distribution the parameters are four.

The pdf as,

$$f(x;\theta,\alpha,\beta,\lambda)=\frac{2\theta\alpha\beta}{\lambda}x^{\beta-1}\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha-1}\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]\left\{1-\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{2}\right\}^{\theta-1}$$

The cdf as,

$$F(x;\theta,\alpha,\beta,\lambda)=1-\left\{1-\left[1-\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\right]^{2}\right\}^{\theta}, \quad x>0$$

2.18 Marshall Olkin Exponential Lomax Distribution

In 2020 Nagarjuna and Vishnu both are introduced a new type of lomax with MO-G family of distribution [19]. It has four parameters.

The pdf as

$$f(x) = \frac{\frac{\theta\lambda\alpha}{\beta}\left(\frac{\beta}{x+\beta}\right)^{-\alpha+1} e^{-\lambda\left(\frac{\beta}{x+\beta}\right)^{-\alpha}}}{\left[1-\bar{\theta}e^{-\lambda\left(\frac{\beta}{x+\beta}\right)^{-\alpha}}\right]^2} \quad x > -\beta, \ (\alpha, \beta, \theta, \lambda) > 0$$

The cdf as

$$\bar{F}(x) = \frac{\theta e^{-\lambda\left(\frac{\beta}{x+\beta}\right)^{-\alpha}}}{1-\bar{\theta}e^{-\lambda\left(\frac{\beta}{x+\beta}\right)^{-\alpha}}}$$

2.19 Inverse Lomax Exponentiated G (IL-EG) Family Distribution

Falgore and Doguw 2020 derived the new distribution with lomax and inverse eponentiated G- family distribution [20]. It has additional three parameters.

The pdf as

$$j(x;\zeta) = \frac{\theta\beta\lambda m(x;\upsilon)\bar{M}^{\lambda-1}(x;\upsilon)}{M^{\lambda+1}(x;\upsilon)}\left(1+\beta\left[\frac{\bar{M}(x;\upsilon)}{M(x;\upsilon)}\right]^{\lambda}\right)^{-(\theta+1)}$$

The cdf as

$$J(x;\zeta) = \int_0^{\left[\frac{M(x;\upsilon)}{M(x;\upsilon)}\right]^{\lambda}} r(f)df = \left(1+\beta\left[\frac{\bar{M}(x;\upsilon)}{M(x;\upsilon)}\right]^{\lambda}\right)^{-\theta}; \qquad x > 0, \theta, \beta, \lambda, \upsilon > 0$$

2.20 Marshall Olkin Length Biased Lomax Distribution

In 2020 the lomax distribution mixed with MOLB distribution and form a new extend lomax distribution [21]. It has three parameter include with the denominatior function parameter.

The pdf as

$$f_{MOLBL}(x;\alpha,\beta,\gamma) = \frac{\alpha(\alpha-1)\gamma}{\beta^2}\frac{x(1+x/\beta)^{-(\alpha+1)}}{[1-(1-\gamma)(1+x/\beta)^{-\alpha}(1+\alpha x/\beta)]^2}, \quad x \geq 0,$$

The cdf as

$$F_{MOLBL}(x;\alpha,\beta,\gamma) = \frac{1-(1+x/\beta)^{-\alpha}(1+\alpha x/\beta)}{1-(1-\gamma)(1+x/\beta)^{-\alpha}(1+\alpha x/\beta)}, \quad x \geq 0,$$

2.21 New Modified Inverse Lomax Distribution

Modified logarithmic transfomed Lomax distribution has new extended lomax distribution, derived by Abdullah M. Almarashi at 2021[22]. It has three parametes.

The pdf as

$$f(x;\ \lambda,\ \alpha,\ \theta) = \frac{(\lambda-1)\alpha\theta x^{-2}\left(1+\frac{\theta}{x}\right)^{-(\alpha+1)}}{\log(\lambda)\left[\lambda-(\lambda-1)\left(1+\frac{\theta}{x}\right)^{-\alpha}\right]},\quad x>0,\ \lambda,\ \alpha,\ \theta>0,\ \lambda\neq 1,$$

The cdf as

$$F(x;\ \lambda,\ \alpha,\ \theta) = 1-\frac{\log\left[\lambda-(\lambda-1)\left(1+\frac{\theta}{x}\right)^{-\alpha}\right]}{\log(\lambda)}$$

2.22 Nadarajah-Haghighi Lomax Distribution

In 2021 Nagarjuna, and et.al., derived the NH family distribution with lomax [23]. It has two scale parameters and two shape parameters.

The pdf as

$$f_{NHLx}(x;\zeta) = \frac{ab\alpha}{\beta}\left(\frac{\beta}{x+\beta}\right)^{-\alpha+1}\left(1+b\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)^{a-1} e^{1-\left(1+b\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)^{a}},$$
$$x \geq -\beta\ \zeta = (a,b,\alpha,\beta) > 0,$$

The cdf as

$$F_{NHLx}(x;\zeta) = 1-e^{1-\left(1+b\left(\frac{\beta}{x+\beta}\right)^{-\alpha}\right)^{a}}$$

2.23 Kumaraswamy Generalized Power Lomax Distribution

Vasili B.V. Nagarjuna, et.al., 2021 described the Kumaraswamy generalized power lomax distribution with five parameters in the family Kw-G, PL distributions [24].
The cdf and pdf are

$$F_{KPL}(x;\xi) = 1-\left\{1-\left[1-\left(\frac{\lambda}{\lambda+x^{\beta}}\right)^{\alpha}\right]^{a}\right\}^{b},\quad x>0,$$

where $\xi = (\alpha,\beta,\lambda,a,b) \in (0,\infty)^5$, and

$$f_{KPL}(x;\xi) = \frac{ab\alpha\beta}{\lambda}x^{\beta-1}\left(\frac{\lambda}{\lambda+x^{\beta}}\right)^{\alpha+1}\left[1-\left(\frac{\lambda}{\lambda+x^{\beta}}\right)^{\alpha}\right]^{a-1}\left\{1-\left[1-\left(\frac{\lambda}{\lambda+x^{\beta}}\right)^{\alpha}\right]^{a}\right\}^{b-1}.$$

2.24 Tranmuted Generalized Lomax Distribution

Wael S. Abu El Azm, et.,al., 2021 explained the transmuted generalized lomax distribution with five parameters in the family geombertz family distribution [25].

The pdf as,

$$f(x;\mathbb{Z}) = \delta\vartheta\gamma(1+\gamma x)^{\delta\pi-1} e^{\vartheta/\pi\left[1-(1+\gamma x)^{\delta\pi}\right]} \left[1-\beta+2\beta e^{\vartheta/\pi\left[1-(1+\gamma x)^{\delta\pi}\right]}\right];$$

The cdf as,

$$F(x;\mathbb{Z}) = \left[1-e^{\vartheta/\pi\left[1-(1+\gamma x)^{\delta\pi}\right]}\right]\left[1+\beta e^{\vartheta/\pi\left[1-(1+\gamma x)^{\delta\pi}\right]}\right]$$

2.25 Inverse Lomax Rayleigh Distribution

Jamilu Yunusa Falgore, et.,al., 2021 developed this new extended lomax distribution [26]. This distribution depends on Rayleigh family with burr family of distributions. This distribution has three parameters.

The pdf as,

$$f_{ILR}(x;\alpha,\lambda,\sigma) = \frac{\alpha\lambda x e^{-\frac{x^2}{2\sigma^2}}}{\sigma^2(1-e^{-\frac{x^2}{2\sigma^2}})^2}\left[1+\alpha\left(\frac{e^{-\frac{x^2}{2\sigma^2}}}{1-e^{-\frac{x^2}{2\sigma^2}}}\right)\right]^{-\lambda-1}, \quad x\in\mathbb{R}$$

where $\alpha, \sigma > 0$ are scale parameters & $\lambda > 0$ is a shape parameter.

The cdf as,

$$F_{ILR}(x;\alpha,\lambda,\sigma) = \left[1+\alpha\left(\frac{e^{-\frac{x^2}{2\sigma^2}}}{1-e^{-\frac{x^2}{2\sigma^2}}}\right)\right]^{-\lambda}, \quad x\in\mathbb{R}$$

2.26 Sine Power Lomax Distribution

Vasili B. V. Nagarjuna, et.,al., 2021 explained the sine transformation in power lomax distribution [27]. This distribution has three parameters, one shape parameter and two scale parameters.

The pdf as,

$$f_{SPL}(x;\zeta) = \frac{\pi}{2}\alpha\beta\lambda x^{\beta-1}(1+\lambda x^{\beta})^{-(\alpha+1)}\sin\left(\frac{\pi}{2}(1+\lambda x^{\beta})^{-\alpha}\right), \quad x>0,$$

The cdf as,

$$F_{SPL}(x;\zeta) = \cos\left(\frac{\pi}{2}(1+\lambda x^{\beta})^{-\alpha}\right)$$

2.27 Alpha Power Lomax Distribution

Y. Murat Bulut, et.,al., 2021 introduced new distribution of alpha power lomax distribution [28]. This distribution has this parameters.
The pdf as,

$$f_{APL}(y;\alpha,\beta,\lambda) = \begin{cases} \frac{\log\alpha}{\alpha-1}\frac{\beta}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\beta+1)}\alpha^{1-\left[1+\frac{y}{\lambda}\right]^{-\beta}}, & \alpha>0, \alpha\neq 1 \\ \frac{\beta}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\beta+1)}, & \alpha=1 \end{cases}$$

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The cdf as,

$$F_{APL}(y;\alpha,\beta,\lambda)=\begin{cases}\frac{\alpha^{1-\left[1+\frac{y}{\lambda}\right]^{-\beta}}-1}{\alpha-1} & ,\alpha>0,\alpha\neq 1\\ 1-\left[1+\frac{y}{\lambda}\right]^{-\beta} & ,\alpha=1.\end{cases}$$

2.28 Maxwell Lomax Distribution

Alfred Adewole Abioduna and Aliyu Ismail Ishaq explained the new extension of lomax distribution i.e., Maxwell Lomax Distribution [29]. The burr family with maxwell family of distributions derived the three parameters.

The pdf as,

$$f(x;\lambda,\beta,\theta)=f(x)$$
$$=\frac{2\beta\theta(1+\theta x)^{-(1+\beta)}}{\lambda^3\sqrt{2\pi}\left((1+\theta x)^{-\beta}\right)^2}\left(\frac{1-(1+\theta x)^{-\beta}}{(1+\theta x)^{-\beta}}\right)^2$$
$$\exp\left(-\frac{1}{2\lambda^2}\left(\frac{1-(1+\theta x)^{-\beta}}{(1+\theta x)^{-\beta}}\right)^2\right),\ \lambda,\beta,\theta;x>0.$$

The cdf as,

$$F(x;\lambda,\beta,\theta)=F(x)$$
$$=\frac{2}{\sqrt{\pi}}\gamma\left(\frac{3}{2},\ \frac{1}{2\lambda^2}\left(\frac{1-(1+\theta x)^{-\beta}}{(1+\theta x)^{-\beta}}\right)^2\right),\lambda,\beta,\theta;$$
$$=\frac{\gamma(a,\ w)}{\Gamma(a)},$$

where $a=\frac{3}{2}$ and $w=\frac{1}{2\lambda^2}\left(\frac{1-(1+\theta x)^{-\beta}}{(1+\theta x)^{-\beta}}\right)^2$.

2.29 Sine Inverse Power Lomax Distribution

Vasilli B.V. Nagarjuna and Christophe Cheneau at 2022 [30]derived the new extension of inverse power lomax distribution by applying the sine transformation. In this distribution one shape parameter and two scale parameters.

The pdf as.

$$f_{SIPL}(x;\jmath)=\frac{\pi}{2}\frac{\alpha\beta}{\lambda}x^{-\beta-1}\left(1+\frac{x^{-\beta}}{\lambda}\right)^{-\alpha-1}\cos\left[\frac{\pi}{2}\left(1+\frac{x^{-\beta}}{\lambda}\right)^{-\alpha}\right],\quad x>0$$

The cdf as,

$$F_{SIPL}(x; \jmath) = \sin\left[\frac{\pi}{2}\left(1 + \frac{x^{-\beta}}{\lambda}\right)^{-\alpha}\right]$$

2.30 Power Odd Generalized Exponential Lomax Distribution

Hanem Mohamed, et.,a;., 2022 [31]explained the power odd generalized exponential lomax distribution with four parameters.

The pdf as,

$$f(x;\lambda,\gamma,\beta,\theta) = \frac{\lambda\gamma\beta}{\theta}\left(1+\frac{x}{\theta}\right)^{\beta-1} e^{-\lambda\left[\left(1+\frac{x}{\theta}\right)^{\beta}-1\right]} \left[1 - e^{-\lambda\left[\left(1+\frac{x}{\theta}\right)^{\beta}-1\right]}\right]^{\gamma-1}, x > 0, \lambda, \gamma, \beta > 0, \theta > 1.$$

The cdf as,

$$F(x;\lambda,\gamma,\beta,\theta) = \left[1 - e^{-\lambda\left[\left(1+\frac{x}{\theta}\right)^{\beta}-1\right]}\right]^{\gamma}.$$

2.31 Minimum Lindley Lomax Distribution

Sadaf Khan, et.,al., 2022 explained the new extended of lomax i.e., minimum Lindley lomax distribution [32]. This distribution has three parameters.

The pdf as,

$$f(x) = \frac{e^{-\theta x}}{(1+\theta)(1+\lambda x)^{\beta+1}}\left[\lambda\beta(1+\theta+\theta x) + \theta^2(1+x)(1+\lambda x)\right]$$

The cdf as,

$$F(x) = 1 - \frac{e^{-\theta x}}{(1+\lambda x)^{\beta}}\left(\frac{1+\theta+\theta x}{1+\theta}\right)$$

2.32 New Weibull Inverse Lomax Distribution

Jamilu Yunusa Falgore and Sani Ibrahim Doguwa were developed new Weibull inverse lomax distribution [33]at 2022 with two shape parameters and two scale parameters.

The pdf as,

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$$f(x;\theta)=\alpha\beta\gamma\lambda x^{-2}\left(1+\frac{\gamma}{x}\right)^{-1}\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta-1}$$
$$\times\exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}$$

The cdf as,

$$F(x;\beta,\alpha,\lambda,\gamma)=\exp\left\{-\alpha\left\{-log\left[\left(1+\frac{\gamma}{x}\right)^{-\lambda}\right]\right\}^{\beta}\right\}$$

And

$$x=\frac{\gamma}{\left[\exp\left\{-\left[-\frac{log(u)}{\alpha}\right]^{\frac{1}{\beta}}\right\}\right]^{-\frac{1}{\lambda}}-1}$$

2.33 Quasi Poisson Exponentiated Exponential Lomax Distribution

Mohamed Aboraya, et.,al., 2022 [34]explained the new extended of poisson exponentiated exponential lomax distribution used by quasi ranking method. In this distribution has three parameters.

The pdf as,

$$f_{a,b,\theta}(z)=\sum_{\tau,d=0}^{+\infty}\varepsilon_{\tau,d}h_{\tau+d+1}(z;\theta)|_{(\tau+d+1)>0},$$

where

$$h_{\tau+d+1}(z;\theta)=\theta(\tau+d+1)(1+z)^{-\theta-1}[1-(1+z)^{-\theta}]^{\tau+d}$$

$$\varepsilon_{\tau,d}=\frac{b}{[1-\exp(-a)]\tau!d!}\sum_{h,l=0}^{+\infty}a^{1+h}\frac{(-1)^{h+\tau+l}(l+1)^{\tau}\Gamma(b(h+1))\Gamma(\tau+d+2)}{h!l!(\tau+d+1)\Gamma(b(h+1)-l)\Gamma(\tau+2)}.$$

The cdf as,

$$F_{a,b,\theta}(z)=\sum_{\tau,d=0}^{+\infty}\varepsilon_{\tau,d}H_{\tau+d+1}(z;\theta)|_{(\tau+d+1)>0},$$

where $H_{\tau+d+1}(z;\theta)=[1-(1+z)^{-\theta}]^{\tau+d+1}$

2.34 Novel Extended Lomax Distribution

Aisha Fayomi, et.,al., 2023 developed a new novel extended lomax distribution with the three parameters [35].

The pdf as,

$$f_{ELo}(x;\alpha,\beta,\lambda)=\begin{cases}\alpha\beta(1+\beta x)^{-\alpha-1}[1-\lambda+\lambda\alpha\log(1+\beta x)], & x>0,\\ 0, & x\le 0,\end{cases}$$

The cdf as.

$$F_{ELo}(x;\alpha,\beta,\lambda)=\begin{cases}1-(1+\beta x)^{-\alpha}[1+\lambda\alpha\log(1+\beta x)], & x>0,\\ 0, & x\le 0,\end{cases}$$

2.35 New Lomax Extension Distribution

Recently Salem, and et.al., derived the new lomax extension distribution with special property as symmetry[36].

The pdf as

$$f_{\underline{V}}(z)=\sum_{d_1,d_2=0}^{\infty} w_{d_1,d_2}\, h_{\delta^*,\beta}(z)|_{\delta^*=\delta_2(d_1+d_2+1)},$$

$$\text{where } h_{\delta^*,\beta}(z)=\tfrac{1}{2}\xi a^*\left[1-\varphi_\beta(z)\right]^{\delta^*-1}\left(1+\tfrac{1}{2}z\right)^{-(\beta+1)}$$

$$w_{d_1,d_2}=\frac{\delta_1\delta_2}{d_1!\,\delta^*}(-1)^{d_1+d_2}\binom{-(d_1+2)}{d_2}\sum_{i=0}^{\infty}(-1)^i(i+1)^{d_1}\binom{\delta_1-1}{i}.$$

The cdf as,

$$F_{\underline{V}}(z)=\sum_{d_1,d_2=0}^{\infty} w_{d_1,d_2} H_{\delta^*,\beta}(z)|_{\delta^*=\delta_2(d_1+d_2+1)},$$

$$\text{where } H_{\delta^*,\beta}(z)=\left[1-\varphi_\beta(z)\right]^{\delta^*}$$

Conclusion

We've considered lomax distribution throughout this article. We clarified quite well during this survey, and thus, the various types of lomax distribution have developed in the previous few years and their different types of pdf and cdf . This survey paper has been compiled very clearly for the possible purpose of developing lomax distribution and its various forms for the future numerical evaluation system.

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Conflict of interest

The authors declare that they have no conflict of interest.

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