Predictor Analysis of the Non-parametric Bulk Arrival Fuzzy Queueing System

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Abstract— In general, queueing methodology is most helpful for design the system and that may achieve the described performance level. This paper, we discuss the fuzzy queueing model with fuzzy parameter. First we construct the membership function of the fuzzy queueing character where the arrival and service rates are triangular fuzzy numbers. Consider the service node as k-phase and to provide the equal service rate in all the phases. Second we shows that the method for constructing the membership function of finite capacity queueing system. A pair of nonlinear program is developed to describe the family of crisp membership functions of finite capacity through which the membership functions of the system performance measures are derived. Finally, we obtain the lower and upper bound of the system performance measure at the different possibility level of alpha. Third we analyze the optimal level of the queueing system, this work extended in [13, 14]. A numerical example is solved successfully.

Keywords*-component; Fuzzy Sets; Mixed integer nonlinear programming; k-phase Erlang Distribution; α - cut Membership function*

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I. INTRODUCTION*(HEADING 1)*

In real life problem the arriving customer need not to wait in the service stage, In this situation bulk arriving queueing model is useful for recovery the problem, such as production, telecommunication and etc., in this case service may talented in many phases.[6, 8, 16] has investigated the performance level. The basic queue characters are involved the certain probability distribution. The attractiveness is analyzing the observed data through statistical interference. The observable data's unquestionable on the queueing system actually are. It is important to utilize the data of extend possible. Many algebraic problems are connected with the simulation modeling in queueing analysis. A statistical formula can support the best use of remaining data should be taken its important of the queueing studies. The initial works on the measurements of queue was totally observed a period of time and complete information was available in the form of the arrival moments and service of each customer. In general, model of queue liable on the markov process. Clarke (1957) and Benes (1957) are assumed the processing time consider as a special distribution. They are investigated the queues parameters through statistical interfering the different models (M/M/1 & $M/M/\infty$). But, the real world situation the quantitative data's of the arrival and service rates are uncertainty. Accordingly, we apply the fuzzy set theory in queueing model. On the basis of [17] the classical queueing models are extended in fuzzy model with more applications. The fuzzy queuing models are more truthful for the classical ones. [1, 2, 7, 9, 10, 12, 15] have analyzed and proved important results on fuzzy applications using -level membership function, [4,5,13] analyze the nonlinear programming for fuzzy queues in general discipline.[11] provided the overview on the conceptual aspects for the phase service in different queueing model. Clearly, many researchers are analyzing the queueing system modeling, but, they are never being analyzed the level of queueing performance. In this paper, we extended the work [13, 14] a pair of nonlinear mixed integer program is formulated to calculate the lower and upper bounds of the α-cut. Then we analyze the consolidated solution scrutinize the statistical make steps towards to measure the system performance analysis using tree chart algorithm.

II. GENERALIZED ERLANG K-PHASE SERVICE DISTRIBUTION

In this model service time consider as an Erlang distribution. More specially, the overall rate of each service phase is k μ .Even though the service may not actually contain in k phases, Let $p_{n,i}(t)$ be the steady state probability, here "n, i" denotes customers in the system and service in k-phase. Here, we number the phases backward, so k is the first phase of service and one is the last. We can derive the steady state balance equation is: Inter arrival time: $A(t) = \lambda e^{-\lambda t}, t \geq 0$

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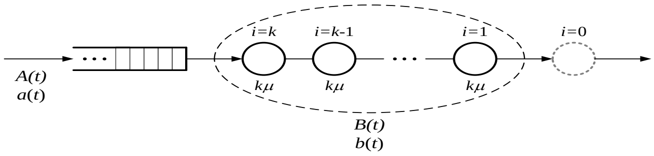
Service time:

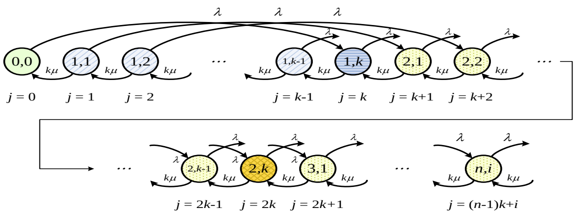
$$B(t)=\frac{k\mu(k\mu t)^{k-1}e^{-k\mu t}}{(k-1)!} \quad t\ge 0 \quad , \quad \text{here } E(x)=1/\mu \text{ and}$$

$V(x)=1/k\mu^2$.



Define the 2-dim state variable (n, i) to be the total number of customers n in the system and the customer being served is at i-stage (phase). Then $P(n)=\sum_{i=1}^{k}P(n,i)$

$$\begin{cases} i=k: \text{at the 1st phase} \\ i=1: \text{at the last phase} \\ i=0: \text{leaving the system or service completion} \end{cases}$$



$$p_{n,i}(t+\Delta t)=p_{n,i}(t)(1-\lambda\Delta t-k\mu\Delta t)+p_{n,i+1}(t)(k\mu\Delta t)+ p_{n-1,i}(t)(\lambda\Delta t) \quad n\ge 2,\ 1\le i\le k$$

----- (1)

$$p_{n,k}(t+\Delta t)=p_{n,k}(t)(1-\lambda\Delta t-k\mu\Delta t)+p_{n,k+1}(t)(k\mu\Delta t) + p_{n-1,i}(t)(\lambda\Delta t) \quad n\ge 2,\ 1\le i\le k$$

----- (2)

At steady state for n>0, $1\le i\le k$

$$-\lambda p_{0,0}+k\mu p_{1,1}=0$$

----- (3)

If n=1, then

$$\begin{cases} -(\lambda+k\mu)p_{1,i}+k\mu p_{1,i+1}=0 & ,1\le i\le k-1 \\ -(\lambda+k\mu)p_{1,k}+k\mu p_{2,1}+\lambda p_0=0 \end{cases}$$

----- (4)

if $n\ge 2$, *then*

$$\begin{cases} -(\lambda+k\mu)p_{n,i}+k\mu p_{n,i+1}+\lambda p_{n-1,i}=0 & ,1\le i\le k-1 \\ -(\lambda+k\mu)p_{n,k}+k\mu p_{n+1,1}+\lambda p_{n-1,k}=0 \end{cases}$$

----- (5)

We obtained the total phase service is:

$$G(z)=\sum_{n=1}^{\infty}\sum_{i=1}^{k}p_{n,i}z^{k(n-1)+i}+p_0=\frac{p_0(1-z)}{1-z(1+r)+rz^{k+1}}$$

$$W_q=\sum_{n=1}^{\infty}\sum_{i=1}^{k}\left[\frac{k(n-1)+i}{k\mu}\right]p_{n,i}=\frac{1}{k\mu}G'(z)\Big|_{z=1}$$

$$=\mathrm{E}(N_q)\frac{1}{\mu}+\mathrm{E}(I)\frac{1}{k\mu}$$

----- (6)

If we let $j=(n-1)k+i,\ n\ge 0,\ 1\le i\le k$ be the total number of phases for (n,i) state in the system with problem

$$p_j^{(p)}.\ p_j=0\ ,\ j>0\ ,\ p_n=\sum_{i=1}^{k}p_{n,i}=\sum_{j=(n-1)k+1}^{nk}p_j^{(p)}$$

$p_j^{(p)}$: The prob of j in the bulk-input system. Then we rewrite

$$-\lambda p_0+k\mu p_1=0$$

$$-(\lambda+k\mu)P_n+k\mu P_{n+1}+\lambda p_{n-k}=0$$

----- (7)

The Erlangian type k service model is equivalent to Bulk input model where $c_k=1,\ c_x=0,\ x\ne k$.

$$p_0=1-\frac{\lambda\mathrm{E}(X)}{k\mu}=1-\frac{k\lambda}{k\mu}=1-\frac{\lambda}{\mu}=1-\rho$$

------ (8)

Using partial fraction expansion may yield

$$G(z)=(1-\rho)\sum_{i=1}^{k}\frac{A_i}{(1-\frac{z}{z_i})} \quad \text{where} \quad A_i=\prod_{\substack{n=1\\ n\ne i}}^{k}\frac{1}{(1-\frac{z_i}{z_n})}$$

And then

$$p_j^{(P)}=(1-\rho)\sum_{i=1}^{k}A_i(z_i)^{-j} \quad ; \quad p_n=\sum_{j=(n-1)k+1}^{n-k}P_j^{(P)}$$

The performance measures as follows:

$$W_q=\mathrm{E}(T_q)=\mathrm{E}(N_q)\frac{1}{\mu}+\mathrm{E}(I)\frac{1}{k\mu}$$

$$=\sum_{n=1}^{\infty}\sum_{i=1}^{k}\left[\frac{k(n-1)+i}{k\mu}\right]p_{n,i}+0\cdot p_0$$

$$=\frac{1}{k\mu}\mathrm{G}'(z)\Big|_{z=1}$$

$$G'(z)\Big|_{z=1}=\frac{(k+1)\rho}{2(1-\rho)},$$

$$W_q=\frac{(k+1)\rho}{2k\mu(1-\rho)}=\frac{k+1}{2k}\cdot\frac{\lambda}{\mu(\mu-\lambda)} \quad \text{---- (9)}$$

$$L_q=\lambda W_q=\frac{k+1}{2k}\cdot\frac{\lambda^2}{\mu(\mu-\lambda)} \quad \text{--- (10)}$$

$$W=W_q+\frac{1}{\mu}=\frac{k+1}{2k}\cdot\frac{\lambda}{\mu(\mu-\lambda)}+\frac{1}{\mu} \quad \text{---- (11)}$$

$$L=\lambda W=\frac{k+1}{2k}\cdot\frac{\lambda^2}{\mu(\mu-\lambda)}+\frac{\lambda}{\mu} \quad \text{---- (12)}$$

$G'(z)=\dfrac{(k+1)\lambda}{2(\mu-\lambda)}$ - Denotes the average total phase in the system.

III. Fuzzy Queues with K-Phase Infinite Capacity

Consider a queueing system in k-phases single server facility in general discipline. The rate of fuzzy arrival and service rates are denoted by $\bar{\lambda}$ and $\bar{\mu}$ it's defined as $\bar{\lambda} = \{(x, \mu_{\bar{\lambda}}(x)) / x \in X\}$, $\bar{\mu} = \{(y, \mu_{\bar{\lambda}}(y)) / y \in Y\}$ where X and Y are the crisp universal sets of the arrival and service rate, $\mu_{\bar{\lambda}}(x)$ and $\mu_{\bar{\mu}}(y)$ are the corresponding membership functions. Clearly, if $\bar{\lambda}$ and $\bar{\mu}$ are fuzzy numbers then P ($\bar{\lambda}$, $\bar{\mu}$) is also fuzzy number. Let P(x, y) denote the system performance measure of interest and it's defined as $\mu_{\bar{p}(\bar{\lambda},\bar{\mu})}(z) = \sup \min \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = p(x,y)\}$ ----- (13)without loss of generality, the performance measure of interest is L_q obtained. From the derivation (sec 2) of L_q and W_q is $w_q \lambda = L_q$ and $w_q = \frac{1+(1/k)}{2} * \frac{\rho}{(1-\rho)\mu}$ where ρ is called the traffic intensity. From (i) the membership function of $\overline{W_q}$ is :

$$\mu_{\overline{w_q}}(z) = \sup \min \left\{ \mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = \left(\frac{k+1}{2k}\right)\frac{\rho}{\mu(1-\rho)} \right\} \quad \text{---- (14)}$$

$$\mu_{\overline{L_q}}(z) = \sup \min \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\mu}}(y) / z = \lambda \overline{w_q}\} \quad \text{---- (15)}$$

Now, the idea is establish the mathematical programming technique a pair of nonlinear programs is developed and the different possibility levels are calculated. So, we estimated the system performance through the statistical interference.

IV. Solution Procedure

To construct the membership function $P(\bar{\lambda}, \bar{\mu})$ is formation on the basis of derivation of α - cuts. α - cuts of $\bar{\lambda}$ and $\bar{\mu}$ is defined as follows:

$$\overline{\lambda_\alpha} = [x_\alpha^l, x_\alpha^u] = [\min\{x / \mu_\lambda(x) \ge \alpha\}, \ \max\{x / \mu_\lambda(x) \ge \alpha\}] \text{ --- (16)}$$

$$\overline{\mu_\alpha} = [y_\alpha^l, y_\alpha^u] = [\min\{x / \mu_\mu(y) \ge \alpha\}, \ \max\{y / \mu_\mu(y) \ge \alpha\}] \text{----- (17)}$$

[14] Arrival and service rates are represented as different levels of intervals. Consequently; FM/FEk/1 can be reduced to a family of crisp M/Ek/1 queues with different α -level sets $\{(\lambda_\alpha, \mu_\alpha) / 0 < \alpha \le 1\}$. Expression of the above two sets are relationship between ordinary sets and fuzzy sets [3]. The bounds fuzzy interval function of α can be obtained as $x_\alpha^l = \min(\mu_\lambda^{-1}(\alpha))$, $x_\alpha^u = \max(\mu_\lambda^{-1}(\alpha))$, $y_\alpha^l = \min(\mu_\mu^{-1}(\alpha))$ and $y_\alpha^u = \max(\mu_\mu^{-1}(\alpha))$. Clearly, as defined in [1],the membership function of $(\bar{\lambda}, \bar{\mu})$ is also parameterized by α. Consequently, we can use its α -cut to construct membership function. According to (i), $\mu_{\overline{L_q}}(z)$ and $\mu_{\overline{w_q}}(z)$ is the minimum of $\mu_{\bar{\lambda}}(x)$ and $\mu_{\bar{\mu}}(y)$. We need to either $\mu_{\bar{\lambda}}(x) = \alpha$ and $\mu_{\bar{\mu}}(y) \ge \alpha$ or $\mu_{\bar{\lambda}}(x) \ge \alpha$ and $\mu_{\bar{\mu}}(y) = \alpha$ such that $L_q(z) = \lambda W_q$ and $W_q = \frac{1+1/k}{2}\left(\frac{\rho}{\mu(1-\rho)}\right)$ to satisfy that $\mu_{L_q}(z) = \alpha$ and $\mu_{W_q}(z) = \alpha$. According the definition of (i), $y \in \mu(\alpha)$ and $x \in \lambda(\alpha)$ can be replaced by $x \in [x_\alpha^l, x_\alpha^u]$and $y \in [y_\alpha^l, y_\alpha^u]$ respectively, Thus, based on (ii &iii) to find the membership function of $\mu_{L_q}(z)$ and $\mu_{W_q}(z)$, it suffices to the $[L_{q_\alpha}^{\ l}, L_{q_\alpha}^{\ u}]$ and $[W_{q_\alpha}^{\ l}, W_{q_\alpha}^{\ u}]$ of the α -cuts of $\mu_{L_q}(z)$ and $\mu_{W_q}(z)$, which can be rewritten as:

$$L_{q_{\bar{\alpha}}}^{l}(z) = \min \left\{ \left(\frac{k+1}{2k}\right) \frac{\rho}{\mu(1-\rho)} \right\} \quad \text{---- (18)}$$

$$L_{q_{\bar{\alpha}}}^{u}(z) = \max \left\{ \left(\frac{k+1}{2k}\right) \frac{\rho}{\mu(1-\rho)} \right\} \quad \text{---- (19)}$$

s.t $x_\alpha^l \le x \le x_\alpha^u$ and $y_\alpha^l \le y \le y_\alpha^u$. There are several effective and efficient methods for solving these problems**.** Moreover, analyze how to change the optimal solution as x_α^l, x_α^u, y_α^l *and* y_α^u where $\alpha \in [0,1]$; they fall into the category of NLP [13]. If $L_{q_\alpha} = [L_{q_\alpha}^{\ l}, L_{q_\alpha}^{\ u}]$ and $W_{q_\alpha} = [w_{q_\alpha}^{\ l}, w_{q_\alpha}^{\ u}]$ are invertible with respect to α, then the left and right shape of the function is [L(z),R(z)] = $L^{-1}{}_{q_\alpha}$ and [L(z),R(z)] = $W^{-1}{}_{q_\alpha}$ can be obtained from membership function $\mu_{W_q}(z)$ and $\mu_{L_q}(z)$ constructed:

$$\mu_{L_q}(z) \,\&\, \mu_{W_q}(z) = \begin{cases} L(z) & z_1 \le z \le z_2 \\ 1 & z = z_2 \\ R(z) & z_2 \le z \le z_3 \end{cases} \quad \text{--- (20)}$$

Otherwise, if the values of L_{q_α} and W_{q_α} cannot obtained analytically, the numerical solutions for $L_{q_\alpha} = [L_{q_\alpha}^{\ l}, L_{q_\alpha}^{\ u}]$and $W_{q_\alpha} = [w_{q_\alpha}^{\ l}, w_{q_\alpha}^{\ u}]$ at different possibility level of α can be collected to approximate the shapes of L(z) and R(z).That is, the set of interval is reveals the shape of $\mu_{L_q}(z) \,\&\, \mu_{W_q}(z)$ although the exact function is not known explicitly. Other membership function of performance measures can be derived in the similar manner. The fuzziness values are converted to crisp value using Robust Ranking Technique. However, in practical point of view the management would expect without blocking the fuzzy arrival due to behavior of the service. If the system providing worst service then the level of queue size increased. Generally any management likes to avoid this kind of behavior, in this connection we estimated the queues parameters used in statistical interference.

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V. NUMERICAL EXAMPLE

Consider a centralize parallel processing system in which the arrival at different level of phases. The phase size random variable K is a Erlang distribution, which is often studied in crisp bulk arrival with the expected value of k=3; i.e., the arrival of the system in accordance with a Poisson Process, and the service times follow an Erlang distribution. Both the group arrival and service rate are triangular fuzzy numbers represented by $\lambda=[1,5,7]$ and $\mu=[9,10,11]$ per minute, respectively. The system manager wants to evaluate the performance measures of the system such as the expected number of customers in the queue and waiting in the queue and to analyze optimality level of the system.

It is clear that in this example the steady-state condition $\rho=\frac{3x}{y}<1$ is satisfied, thus the performance measures of interest can be constructed by using the approach stated in solution procedure following (18&19), two pairs of MINLP models for deriving the membership function of $\overline{L_q}$ can be formulated, and whose solutions are as follows:

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REFERENCES

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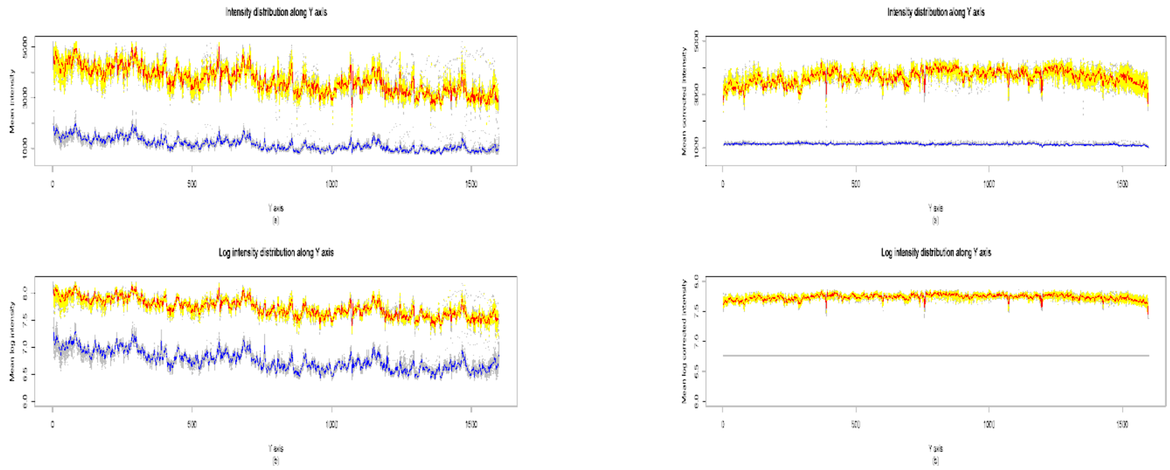

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