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Design of Turbo Trellis Coding Modulation Scheme of Rate 4/9 for Rician Fading Channel

Raj Kumar Goswami[1], Srinivasa Rao Kandula[2], S.V. Rama Rao[3], A.H. Sharief[4], Koteswararao Seelam[5], Sunil Babu Melingi[6], Swathi Nambari[7],
[1]Electronics and Communication Engineering,
Gayatri Vidya Parishad College of Engineering for Women,
Visakhapatnam, India
rajkumargoswami@gmail.com
[2]Electronics and Communication Engineering,
Dhanekula Institute of Engineering and Technology,
Vijayawada, India
Ksrinivas.ece@gmail.com
[3]Electronics and Communication Engineering,
NRI Institute of Technology,
Vijayawada, India
svr628@gmail.com
[4]shariefah@gmail.com
[5]Electronics and Communication Engineering,,
Kallam Haranatha Reddy institute of Technology
Guntur, India
drkrseelam@gmail.com
[6]sunil.babu.m@gmail.com
[7]Electronics and Communication Engineering,
Anil Neerukonda Institute of Technology and Sciences,
Visakhapatnam, India
swathiyadav23@gmail.com

Abstract— When the fading channels are encountered during data communication, errors are likely to occur at the receiving end due to multipath propagation. Researchers have been consistently striving to develop Error Correction Schemes that can effectively handle these errors and ensure error-free data reception at the receiver end. Of particular interest are the Forward Error Correction Schemes that can be implemented at the transmitter end itself. However, the implementation of error correction coding through these schemes incurs additional costs in terms of bandwidth expansion, as extra bits need to be added to facilitate error correction. Fortunately, there exists one coding scheme called Trellis Coded Modulation (TCM), which addresses this issue. TCM selects a modulation scheme based on the rate of the convolutional coding scheme. However, this coding technique has limitations in correcting the number of errors, leading to the development of Turbo Coding. This scheme utilizes two coders at the transmitter, arranged in either serial or parallel configuration, and a suitable decoder at the receiver. A design of Turbo Coding scheme has been presented in this paper, that employs convolutional coders having rate 2/3, in a serially concatenated configuration, providing an effective rate of 4/9. This turbo coding scheme is then applied to TCM scheme in order to preserve the bandwidth. Therefore, if using the convolutional coding scheme of rate 2/3, the modulation scheme is 8-QAM and in order to preserve bandwidth after coding, using the Turbo coding scheme of rate 4/9, then the modulation scheme will be 512-QAM. The simulations have been conducted in MATLAB and the error correcting capabilities of the designed scheme in comparison with convolutional coding scheme using the constituent convolutional encoder have also been compared. It has been observed that in the Rician fading channel conditions, the Turbo Trellis Coding Modulation Scheme provides approximately 5 dB gain compared to the convolutional coding scheme.

Keywords- Convolutional Code, Fading Channel, Trellis Coded Modulation, Turbo Code.

I. INTRODUCTION

Data communication, particularly through wireless mediums, is prone to errors at the receiving end due to the channels through which the data passes. While some channels introduce fewer errors, others with prominent multipath phenomena tend to produce a large number of errors [1]. Additionally, channels that use the HF medium experience error occurrences that are dependent on time, as the channels themselves are not stationary. The challenge of errors in data communication has prompted Electronics and Communication Engineers to seek solutions, leading to the invention of several error correction schemes over time. One such scheme is Block coding, where data is processed block by block, coded and then decoded correspondingly at the receiver side. However, this scheme has certain limitations, which resulted in the development of Convolutional Coding, where the data is coded continuously at a certain rate. But this scheme has its own drawbacks, such as an increase in bandwidth post coding.

To address this issue, Ungerboeck developed Trellis Coded Modulation (TCM) in 1982 [2]. TCM adjusts the modulation to match the convolutional coding scheme in order to maintain the bandwidth. For instance, if 8 QAM is used for modulation and no coding scheme is employed, three bits are modulated at a time. However, if a coding scheme with a rate of 1/2 is implemented, three bits will become six bits, effectively doubling the bandwidth. To prevent this, the modulation scheme can be switched to 64 QAM, where six bits are used for modulation, thereby preserving the bandwidth. Switching the modulation scheme from 8 QAM to 64 QAM results in the reduction of the distance amongst the points on the constellation, thereby increasing probability of errors being induced by the channel. This necessitates a compromise between the coding scheme rate and the modulation technique. This demonstrates that there is a limit to how much bandwidth can be preserved, beyond which errors will become increasingly difficult to correct. To address this limitation and improve error correction, in 1993, Turbo Coding was presented by Berrou et al [3]. Further in order to preserve the bandwidth post implementation of Turbo Coding Scheme, the same principal of TCM could be employed, which is also known as Turbo TCM coding scheme [4]. Subsequently, the effectiveness of a coding scheme is evaluated based on the Bit Error Rate (BER), that represents proportion of incorrect bits relative to the bits transmitted.

Turbo Coding scheme utilizes two encoders at the transmitter end, which are concatenated in either a series or parallel configuration along with an interleaver [5]. This interleaver randomizes the bits before they are fed as input to the second decoder.

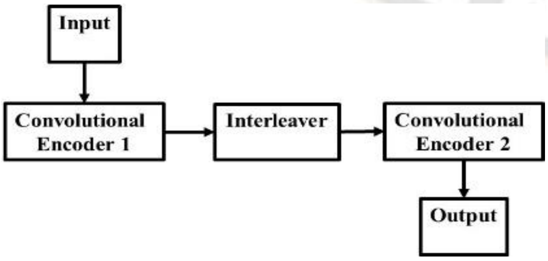

Fig. 1 SCCC Encoder

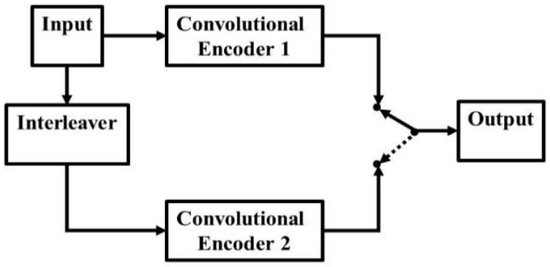

Fig. 2 PCCC Encoder

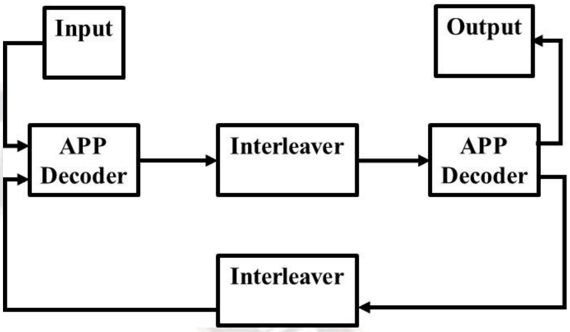

Fig. 3 SCCC Decoder

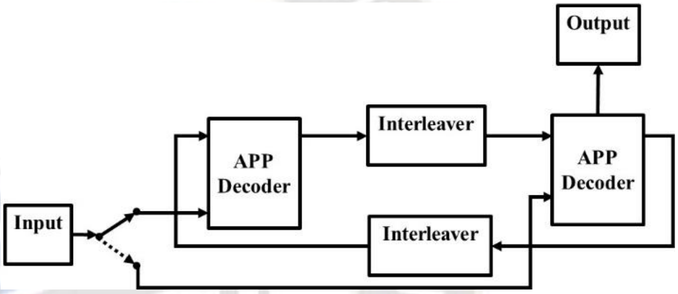

Fig. 4 PCCC Decoder

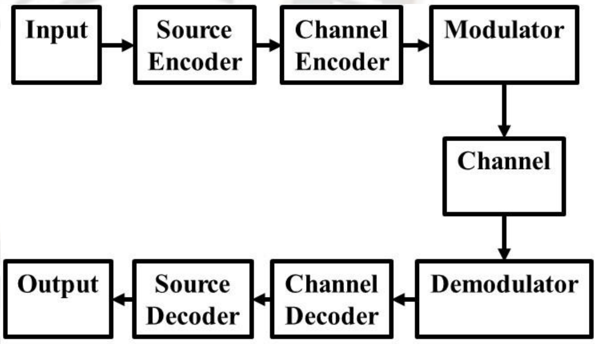

Fig. 5 Communication System block diagram

These are referred to as Serially Concatenated Convolutional Codes (SCCC) Encoder and Parallel Concatenated Convolutional Codes (PCCC) Encoder, and their configurations are shown in Figures 1 and 2, respectively.

For decoding the Turbo Codes at the receiver, APP (A Posteriori Probability) decoders are used for both SCCC and PCCC configurations, with slight changes in their respective configurations as depicted in Figure 3 and Figure 4 respectively. In both configurations, the first decoder’s output is passed through the interleaver to the second decoder, output of second

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decoder is fed back to the first decoder through the interleaver. This iterative decoding process continues until the desired error level is achieved, at which point the output of second decoder is considered the output of the Turbo decoder. The block diagram of communication system is depicted in Figure 5. Present investigation considers inclusion of a fading channel [6].

Source coding is utilized to eliminate redundant information, such as compression schemes like zip, rar, and others. Following source coding, channel coding is employed where Forward Error Correction (FEC) schemes are implemented. Once FEC is implemented, the bits are modulated and then transmitted to the channel. In this particular case, QAM has been selected as the modulation scheme, and thus the output of the Channel Encoder is organized in vector form where each vector signifies a point in the complex plane specific to the chosen QAM architecture.

After being transmitted, the received data is subject to corruption due to the channel conditions, specifically the fading channel. At the receiver, the corrupted sequence $r_l = (r_1, r_2, \ldots r_l)$ is fed to the decoder for decoding. The receiver's block diagram at the baseband level is illustrated in Figure 6.

The signal received at time i can be expressed based on the diagram of the receiver as follows

$$r_i = c_i s_i + n_i \qquad (1)$$

The equation involves n_i which represents complex Gaussian noise having zero-mean with variance $\sigma_n^2 = \frac{N_0}{2}$. c_i is channel gain of a complex Gaussian process which has variance of σ_c^2. Additionally, ci can also be represented through phasor as mentioned below

$$c_i = a_i * e^{j\phi_i} \qquad (2)$$

In which, a_i denotes amplitude and ϕ_i denotes phase.

Assuming that receiver undertakes coherent detection, compensating for the channel phase shift, equation (1) can be further simplified as

$$r_i = a s_i + n_i \qquad (3)$$

In this scenario, the occurrence of fading is depicted by a_i. When there is no direct line of sight transmission in path and

solely comprises of scattered multipath components, a_i is typically represented by a Rayleigh fading channel model. The Probability Density Function (PDF) for Rayleigh fading is expressed as:

$$P_A(a) = 2ae^{-a^2}, \qquad a \geq 0 \qquad (4)$$

In situations where there is a significant, non-fading line-of-sight path along with multipath components, the channel is simulated using Rician Fading Channel model, and its PDF is defined as:

$$P_A(a) = 2a(1+K)\, e^{-(K+a^2(1+K))} I_0\,(2a\sqrt{K(1+K)}),\ a \geq 0 \quad (5)$$

The Rician parameter K denotes the ratio between the signal received through direct path and the signal received through scattered multipaths, while $I_0(.)$ signifies the zero-order modified Bessel function of the first kind. Further the PDF should be normalized to represent the mean energy of the signal per channel symbol, E_s. In this paper, we have considered Rician Channel, and the impact of time-varying multipath is neglected, as receiver will employ equalization techniques to handle this aspect.

II. PERFORMANCE ANALYSIS

The performance analysis assumes statistically independent fading amplitudes and a memoryless channel.

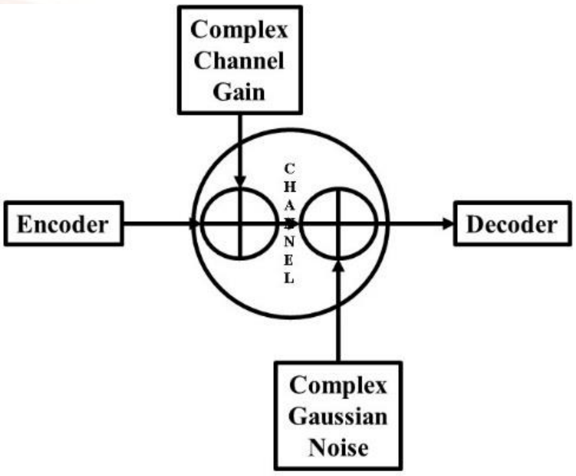

Fig. 6 Block Diagram of Baseband Receiver

The probability of an error event is essential in determining the performance of system, the lower the probability of error event, the lower the number of errors, and vice versa. The error event probability will be derived in this section. We will investigate upper bound of pairwise error probability [6]. Under the assumptions of coherent detection, ideal Channel State Information (CSI), and independent fading from symbol to symbol, the upper limit of the pairwise error probability for decoding the symbol sequence as $\hat{S}_l$ when the transmitted symbol sequence is S_l given a Rician Channel, can be represented as follows:

$$P_2\left(S_l, \hat{S}_l\right) \leq \prod_{i=1}^{l} \frac{(1+K)}{1+K+\frac{1}{4N_0}\left|(S_i-\hat{S}_i)\right|^2}\, e^{\left(-\frac{K\frac{1}{4N_0}\left|(S_i-\hat{S}_i)\right|^2}{1+K+\frac{1}{4N_0}\left|(S_i-\hat{S}_i)\right|^2}\right)} \qquad (6)$$

In scenarios where SNR is high, aforementioned equation is simplified to:

$$P_2(S_l, \hat{S}_l) \leq \prod_{i \in \eta} \frac{(1+K)e^{-K}}{\frac{1}{4N_0}\left|(S_i-\hat{S}_i)\right|^2} \qquad (7)$$

The symbol η represents the values of i where S_i and $\hat{S}_i$ are not equal. Let the number of such values be denoted by l_η. Then, equation (7) can be expressed as follows. Effective length of the error event $(S_l, \hat{S}_l)$ is also represented by l_η

$$P_2(S_l, \hat{S}_l) \le \frac{((1+K)e^{-K})^{l_\eta}}{(\frac{1}{4N_0})^{l_\eta}\, d_P^2 l_\eta} \qquad (8)$$

Here, $d_P^2(l_\eta)$ is the squared product distance of the signals when S_i and $\hat{S}_i$ are not equal, along the error event $(S_l, \hat{S}_l)$ path and is defined as

$$d_P^2(l_\eta) = \prod_{i\in\eta}\left|(S_i - \hat{S}_i)\right|^2 \qquad (9)$$

An error event occurs when the path of the received data deviates from the intended path as per the design. This concept is illustrated in Fig. 7, where the intended path is from s_1 to s_2 and up to s_l, while the actual path traced by the received symbols is $\hat{s}_1$, $\hat{s}_2$, up to $\hat{s}_l$.

If we consider all transmitted sequences and sum up the probabilities of all error events over l, l=1…∞, we can calculate an upper bound, which can be expressed as follows

$$P_e \le \sum_{l=1}^{\infty} \sum_{S_l} \sum_{\hat{S}_l \ne S_l} P(S_l) P_2(S_l, \hat{S}_l) \qquad (10)$$

In the above equation, $P(S_l)$ signifies A Priori Probability of transmitting symbol S_l. $P_2(S_l, \hat{S}_l)$ can be substituted in the equation at high SNRs using (8), and consequently, the upper bound with regards to a Rician fading channel can be expressed as:

$$P_e \le \sum_{l_\eta} \sum_{d_P^2 l_\eta} \alpha(l_\eta, d_P^2(l_\eta)) \frac{((1+K)e^{-K})^{l_\eta}}{(\frac{1}{4N_0})^{l_\eta}\, d_P^2(l_\eta)} \qquad (11)$$

Equation (11) indicates that the mean count of code sequences with effective length l_η and squared product distance $d_P^2(l_\eta)$ is denoted by $\alpha(l_\eta, d_P^2(l_\eta))$. When the SNR are high, the error event gets influenced by the lowest effective length l_η and squared product distance $d_P^2(l_\eta)$. To simplify the expression, we use L to represent the minimum effective length and $d_P^2(L)$ to represent the corresponding squared product distance. This minimum effective length is considered as the code's effective length, which allows us to approximate the error event probability as follows.

$$P_e \approx \alpha(L, d_P^2(L)) \frac{((1+K)e^{-K})^{L}}{(\frac{1}{4N_0})^{L}\, d_P^2(L)} \qquad (12)$$

In respect of Rayleigh fading channel, where K is equal to zero (12) can be further simplifies as

$$P_e \approx \frac{\alpha(L, d_P^2(L))}{(\frac{1}{4N_0})^{L}\, d_P^2(L)} \qquad (13)$$

And in respect of AWGN channel, by considering K = ∞, we can rewrite P_e as below [6]

$$P_e \approx \frac{1}{2} N(d_{free})\, erfc\left(\sqrt{\frac{d_{free}^2}{4N_0}}\right) \qquad (14)$$

where the value of d_{free} represents the code's free Euclidean distance.

In this work, focus has been on the performance analysis of a Serially Concatenated Convolutional Coding (SCCC) scheme [7] in the context of Rician Fading Channel. Specifically, the authors analyze equation (12) for the Rician parameter K, while noting that the cases where K is zero or infinity are special cases. The SCCC scheme utilizes two convolutional encoders each with rate 2/3 in a serial configuration, resulting in an effective rate of 4/9. To assess the performance of the SCCC scheme, we compare it with the convolutional coding scheme of constituent encoder. The decoding process is iterative, with the output of one decoder fed to the other and vice versa, until decoding is complete [8]. The complexity of the SCCC scheme is akin to that of a Convolutional Coding scheme, wherein APP algorithm has been implemented in the decoder [9].

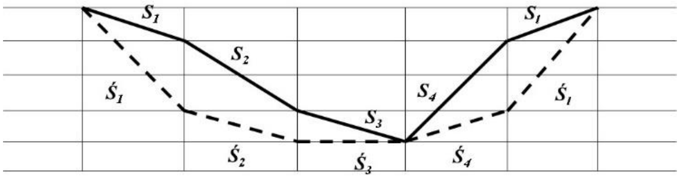

Fig. 7 Error event considering length as l.

III. DESIGN OF ENCODERS

For the encoder design, two convolutional encoders have been used in a serial configuration with an interleaver. Interleaver gets the input from first encoder, and gives output to second encoder. The encoders in this project were designed according to specific criteria. Specifically, each encoder was designed for rate of 2/3 and 8 states. TCM scheme having rate 2/3, states 8 and modulation scheme 8-PSK was developed by Ungerboeck's that had an effective length of 2 and d^2free = 4.586Es, however this was optimized for the AWGN channel [10]. Here also, the encoders have been designed using common rules however, the optimization has been performed for fading multipath channel [11].

8 x 8 matrix M represents the signals, those are linked with transitions between states of successive stages for Rate of 2/3. There are signals, those are associated with the paths going from state i to state j at stage K to stage K+1 and these are represented in trellis, by the element located in the ith row and jth column of the matrix M. Elements in the ith row denote the paths going away from state i while paths reemerging at state j are represented by the elements in the jth column. These elements indicate the signals associated with the respective paths. Two separate subsets can be created by placing alternate symbols of 8-PSK signal set into each one of them, Specifically, subset 1 includes s0, s2, s4, and s6, while subset 2 includes s1, s3, s5, and s7. For populating the other elements of matrix M by signal points, the rules mentioned below have been taken into consideration.

a. The occurrence of a signal is limited to once per row or column.

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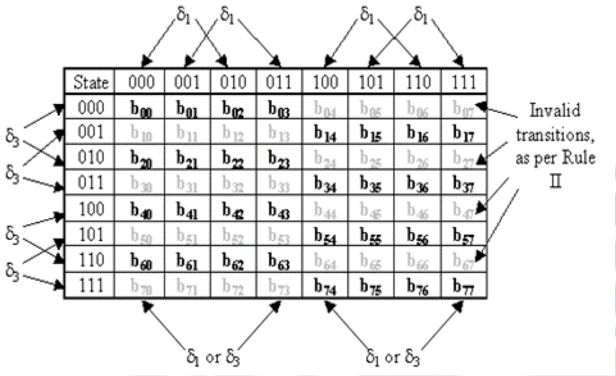
b. The number of paths emerging from a state are limited to 4 for the rate 2/3 code, whereas states present are 8. As a result, not all transitions are possible. Therefore, it has been considered that a transition path between two states can only be associated with a signal if the Least Significant Bit (LSB) of the initial state matches the Most Significant Bit (MSB) of the destination state, that is denoted as z ∈ {0, 1} [10].

c. In the given rows or columns of the matrix, there are signal pairs those are associated with them and if the distance is calculated between them then it will be observed that these are either δ_1 or δ_3, if we consider the coding rate as 2/3. The same has been illustrated in the diagram depicted in Figure 8.

Rule ‘a’ ensures that the code has a maximum effective length, while by following Rule ‘b’, the distances between every pair of paths, those are diverging from state i and every pair of paths those are converging at state j are ensured to be at least δ_1.



State	000	001	010	011	100	101	110	111
000	b_{00}	b_{01}	b_{02}	b_{03}	b_{04}	b_{05}	b_{06}	b_{07}
001	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}	b_{17}
010	b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{27}
011	b_{30}	b_{31}	b_{32}	b_{33}	b_{34}	b_{35}	b_{36}	b_{37}
100	b_{40}	b_{41}	b_{42}	b_{43}	b_{44}	b_{45}	b_{46}	b_{47}
101	b_{50}	b_{51}	b_{52}	b_{53}	b_{54}	b_{55}	b_{56}	b_{57}
110	b_{60}	b_{61}	b_{62}	b_{63}	b_{64}	b_{65}	b_{66}	b_{67}
111	b_{70}	b_{71}	b_{72}	b_{73}	b_{74}	b_{75}	b_{76}	b_{77}

Fig. 8 The diagram depicting the abovementioned rules

IV. SIMULATION AND RESULTS

The analysis has been performed in MATLAB environment and Simulink diagram is shown in Fig. 9. The simulation employs a Bernoulli Binary generator as the source of random input bits, whose output is fed to the Turbo encoder that consists of two convolutional encoders having rate 2/3 in a serially concatenated manner, producing the overall rate as 4/9. These two convolutional encoders have been separated by a random interleaver, as depicted in Fig. 10.

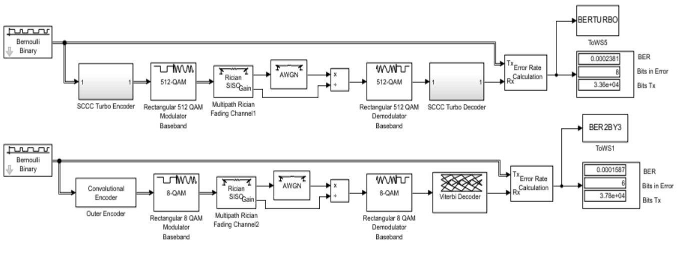


Fig. 9 Simulation Diagram

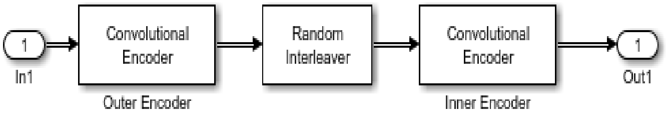


FIG. 10 Turbo Encoder diagram

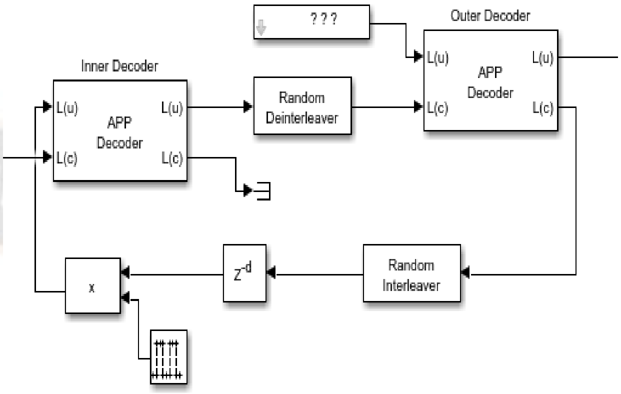


Fig. 11 Turbo Decoder Diagram

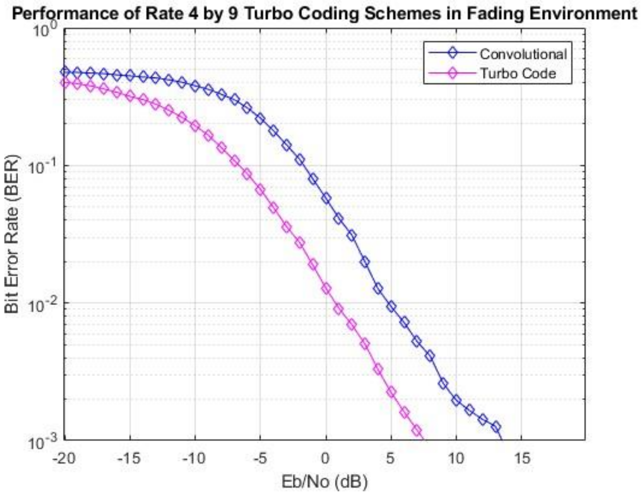


Fig. 12 Performance analysis of the proposed Turbo Coding scheme in Rician Environment

The Turbo encoder’s output has been given to 512 QAM modulator in order to preserve the band width and then the modulated data is passed through the Rician Channel in order to simulate the real time multipath propagation scenario. The received data, that has been corrupted by the channel is initially fed to the demodulator and the demodulated data is then fed to the turbo decoder, whose diagram is shown in Fig. 11, for undertaking the correction of errors those have been introduced by the channel.

For evaluating the performance of the Turbo Trellis Coding scheme, a simulation has been conducted using the Monte Carlo method. The simulation has resulted in a plot of E_b/N_0 versus Bit Error Rate (BER), which can be seen in Fig. 12.

Additionally, the BER obtained solely from the constituent convolutional encoder with a rate of 2/3 has been compared with the result obtained from the Turbo Trellis Coding scheme. Comparison has revealed that Turbo Trellis Coding scheme provides approximately 5dB gain over the Trellis Coded Modulation scheme that only utilizes the constituent convolutional coder. In other words, the Turbo Trellis Coding scheme exhibits significantly better performance than the Trellis Coded Modulation scheme in terms of error correction capability.

V. CONCLUSION

Bit Error Rate (BER) acquired from the constituent convolutional encoder operating at a rate of 2/3 was compared with the results obtained from the Turbo Trellis Coding scheme having rate 4/9. The implications of the findings are far-reaching and offer important implications for both academic as well as practical fields as the analysis indicates that the Turbo Trellis Coding scheme exhibits approximately 5dB gain over the Trellis Coded Modulation scheme that solely employs the constituent convolutional coder. In essence, the design of Turbo Trellis Coding scheme presented in this paper provides a valuable contribution to the field that demonstrates superior error correction capabilities, compared to the Trellis Coded Modulation scheme.

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