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A Novel Method of Linguistic Topsis for DMSS Technique

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Abstract**—**This paper extends the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for solving multi-attribute group decision making (MAGDM) problems under trapezoidal fuzzy linguistic variables. In situations where the information or the data is of the form of trapezoidal fuzzy linguistic numbers (TFLNs), some arithmetic aggregation operators have to be defined, namely the Trapezoid Fuzzy Linguistic Weighted Harmonic Averaging (TFLWHA)operator, Trapezoid Fuzzy Linguistic Ordered Weighted Harmonic Averaging (TFLOWHA) operator and Trapezoid Fuzzy Linguistic Hybrid Harmonic Averaging(TFLHHA) operator. A new method for determining decision maker's weights is also proposed in the paper, which is used to determine the best alternative. An extended TOPSIS model is developed to solve the MAGDM problems using a new algorithm and an illustration is given.

Keywords*-MAGDM, TOPSIS, Linguistic Weighted Operators, Decision makers weights.*

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I. INTRODUCTION

Multi-attribute group decision making (MAGDM) problems are of importance in most kinds of fields such as engineering, economics and management. It is obvious that much knowledge in the real world is fuzzy rather than precise. Imprecision comes from a variety of sources such as unquantifiable information [8], [11]. In many situations decision makers have imprecise/vague information about alternatives with respect to attributes. One of the methods which describe imprecise cases is the fuzzy set (FS) introduced by Zadeh[16]. Multi attribute group decision making (MAGDM) problems are wide spread in real life decision making situations. A MAGDM problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In order to choose a desirable solution, the decision maker often provides his/her preference information which takes the form of numerical values, such as exact values, interval number values and fuzzy numbers. However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems [14], [15]. Many authors [1,3,4,7,10,12,13] have contributed towards the field of decision making with different domain of problems.

Among the numerous approaches available for Decision Making Support Systems (DMSS), one of the most prevalent is the Technique for order preference by similarity to ideal solution (TOPSIS), which was first developed by Hwang and Yoon [5].TOPSIS is a logical decision-making approach and deals with the problem of choosing a solution from a set of candidate alternatives which are characterized in terms of some attributes. Solving a MAGDM problem involves sorting and ranking, and can be viewed as alternative methods for combining the information in a problem's decision matrix together with additional information from the decision maker to determine a final ranking or selection from among the alternatives. Besides the information contained in the decision matrix, all but the simplest MAGDM techniques require additional information from the decision matrix to arrive at a final ranking or selection. In this paper, MAGDM problems with trapezoidal fuzzy linguistic variables with the application of weight determining methods are applied.

II. TOPSIS IN DECISION MAKING SUPPORT SYSTEMS (DMSS) AND TECHNIQUES

A DSS is intended to support, rather than replace, decision maker's role in solving problems. Decision makers' capabilities are extended through using DMSS, particularly in ill-structured decision situations. In this case, a satisfied solution, instead of the optimal one, may be the goal of decision making. Solving ill-structured problems often relies on repeated interactions between the decision maker and the DMSS. Decision support systems are built upon various decision support techniques, including models, methods, algorithms and tools. A cognition-based taxonomy for decision support techniques, including six basic classes as follows [9]: Process models, Choice models , Information control techniques , Analysis and reasoning techniques, Representation aids and Human judgment amplifying/refining techniques. The Multi-criteria decision making and Multi-attribute decision making comes under the category of Choice models.

Multiple Attribute decision support systems are provided to assist decision makers with an explicit and comprehensive tool and techniques in order to evaluate alternatives in terms of different factors and importance of their weights. Some of the common Multi-Attribute Decision-Making (MADM) techniques are [2]:

- Simple Additive Weighted (SAW)
- Weighted Product Method (WPM)
- Cooperative Game Theory (CGT)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
- Elimination et Choice Translating Reality with complementary analysis (ELECTRE)
- Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)
- Analytical Hierarchy Process (AHP)

The merit of the TOPSIS method suggested in [5] is that it can deal with both quantitative and qualitative assessment in the process evaluation with little computation load. It bases upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest from the negative ideal solution. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as crisp values. In fuzzy TOPSIS, attribute values are represented by fuzzy numbers. Janic [6] stated that the TOPSIS method embraces seven steps which are as follows: (1) constructing the normalized decision matrix by using the decision making matrix; (2) constructing the weighted-normalized decision matrix; (3) determining the positive ideal and negative ideal solution; (4) calculating the separation measure of each alternative from the ideal one; (5) calculating the relative distance of each alternative to the ideal and negative ideal solution; (6) ranking the alternatives in descending order with respect to relative distance to the ideal solution;(7) identification of the preferable alternative as the closest to the ideal solution. However, in considering group decision making problems, the preferences among alternatives have to be aggregated for individual decision makers. TOPSIS logical thinking considers that the optimal decision should have the closest distance from the best alternative and the farthest distances from the worst alternative.

III. TRAPEZOID FUZZY LINGUISTIC VARIABLES

Let $S=\{s_i \setminus i=1,2,......t\}$ be a linguistic term set with odd cardinality, any label represents a possible value of the linguistic variable. Especially, and represent the lower and the upper values of the linguistic terms, respectively.

For example, a linguistic term set S could be given as follows:

$S=\{s_1=$ extremely poor, $s_2=$ very poor, $s_3=$ poor, $s_4=$ slightly poor, $s_5=$ fair, $s_6=$ slightly good, $s_7=$ good, $s_8=$ very good, $s_9=$ extremely good$\}$

Usually, in this case, and must satisfy the following additional characteristics:

1) The set S is ordered: s_i is worse than s_j, if $i<j$;
2) Maximum operator: $\max(s_i,s_j)=s_i$, if $s_i \ge s_j$;
3) Minimum operator: $\min(s_i,s_j)=s_j$, if $s_i \ge s_j$

Some calculation results, however, may not exactly match any linguistic labels in S in the calculation process. To preserve all the given information, the discrete term set Sis extended to a continuous term set $\bar{S}=\{s_i / s_0 \le s_i \le s_q, i\in[0,q]\}$, where s_i meets all the characteristics above and $q(q>t)$ is a sufficient large positive integer. If $s_i \in S$, then we call s_i the original term, otherwise, we call the virtual term. In general, the decision makers use the original linguistic terms to evaluate the alternatives, and the virtual linguistic terms can only appear in the process of the operation and ranking.

Definition:

Let $s_\alpha, s_\beta \in \bar{S}$ then we defined the distance between s_α and s_β as:

$$d(s_\alpha,s_\beta)=|\alpha-\beta| \tag{1}$$

Definition: Let $\tilde{s}=[s_\alpha,s_\beta,s_\gamma,s_\eta]\in\tilde{S}$, and α,β,γ,η the subscripts are non-decreasing numbers, and s_β,s_γ indicate the interval in which the membership value is 1, with s_β,s_γ indicating the lower and upper values of $\tilde{s}$, respectively, than is called the trapezoid fuzzy linguistic variable(TFLV), which is characterized by the following membership function

$$\mu_{\tilde{s}}(\theta)=\begin{cases} 0 & \\ \dfrac{d(s_\theta,s_\alpha)}{d(s_\beta,s_\alpha)} & s_0 \le s_\theta \le s_\alpha \\ & s_\alpha \le s_\theta \le s_\beta \\ 1 & s_\beta \le s_\theta \le s_\gamma \\ \dfrac{d(s_\theta,s_\eta)}{d(s_\gamma,s_\eta)} & s_\gamma \le s_\theta \le s_\eta \\ & s_\eta \le s_\theta \le s_q \\ 0 & \end{cases} \tag{2}$$

where $\tilde{S}$ is the set of all the trapezoid fuzzy linguistic variables. Especially, if any two of α,β,γ,η are equal, then $\tilde{s}$ is reduced to a triangular fuzzy linguistic variable; if any three of α,β,γ,η are equal, then is reduced to an uncertain linguistic variable.

IV. THE OPERATIONAL RULES AND CHARACTERISTIC OF THE TRAPEZOID FUZZY LINGUISTIC VARIABLES

Let $\tilde{s}=[s_\alpha,s_\beta,s_\gamma,s_\eta]$, $\tilde{s}_1=[s_{\alpha_1},s_{\beta_1},s_{\gamma_1},s_{\eta_1}]$ and $\tilde{s}_2=[s_{\alpha 2},s_{\beta 2},s_{\gamma 2},s_{\eta 2}]\in\tilde{S}$ be any three Trapezoid fuzzy linguistic variables, and $\lambda\in[0,1]$ and $\lambda_1\in[0,1]$, then their operational rules are defined as follows:

(1) $\tilde{s}_1 \oplus \tilde{s}_2 = \left[s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\eta_1}\right] \oplus \left[s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\eta_2}\right]$

$= [s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}, s_{\gamma_1+\gamma_2}, s_{\eta_1+\eta_2}]$;

(2) $\lambda \tilde{s} = \lambda\left[s_\alpha, s_\beta, s_\gamma, s_\eta\right]$;

$= \left[s_{\lambda\alpha}, s_{\lambda\beta}, s_{\lambda\gamma}, s_{\lambda\eta}\right]$;

(3) If $0 < \alpha \le \beta \le \gamma \le \eta$, then

$1/\tilde{s} = (\tilde{s})^{-1} = [\,1/\tilde{s}_\eta, 1/\tilde{s}_\gamma, 1/\tilde{s}_\beta, 1/\tilde{s}_\alpha\,]$

$= [s_{1/\eta}, s_{1/\gamma}, s_{1/\beta}, s_{1/\alpha}]$

In addition, the trapezoid fuzzy linguistic variables have the following characteristics:

(1) $\tilde{s}_1 \oplus \tilde{s}_2 = \tilde{s}_2 \oplus \tilde{s}_1$;

(2) $(\lambda \oplus \lambda_1)\tilde{s} = \lambda\tilde{s} \oplus \lambda_1\tilde{s}$;

(3) $\lambda(\tilde{s} \oplus s_1) = \lambda\tilde{s} \oplus \lambda s_1$

THE COMPARISON METHOD OF THE TRAPEZOID FUZZY LINGUISTIC VARIABLES

Definition:

Let $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}, s_{\gamma_1}, s_{\eta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}, s_{\gamma_2}, s_{\eta_2}]$ be two trapezoid fuzzy linguistic variables, then the possibility degree of $\tilde{s}_1 \ge \tilde{s}_2$ is defined as follows:

$$p(\tilde{s}_1 \ge \tilde{s}_2) = \min\{\max\{\frac{(\gamma_1+\eta_1)(\alpha_2+\beta_2)}{(\gamma_1+\eta_1)-(\alpha_1+\beta_1)+(\gamma_2+\eta_2)-(\alpha_2+\beta_2)}, 0\}, 1\} \quad (3)$$

Example :1

Let $\tilde{s}_1 = [s_2, s_3, s_5, s_6]$ and $\tilde{s}_2 = [s_4, s_5, s_8, s_9]$ be two trapezoid fuzzy linguistic variables, then the possibility degree of $s_1 \ge s_2$ is

$$p(\tilde{s}_1 \ge \tilde{s}_2) = \min\{\max\{\frac{(5+6)(4+5)}{(5+6)-(2+3)+(8+9)-(4+5)}, 0\}, 1\}$$

$=\min\{\max\{0.143, 0\}, 1\}$

$=0.143$

Let $\tilde{s}_i$ and $\tilde{s}_j$ be two trapezoid fuzzy linguistic variables, then the steps of the comparison method are shown as follows:

(1)Utilize the formula (3) to compare the size of $\tilde{s}_i$ and $\tilde{s}_j$, and suppose that

$p_{ij} = p(\tilde{s}_i \ge \tilde{s}_j)$, then we can contribute the possibility degree matrix $p = (p_{ij})_{n\times n}$ where $p_{ij} \ge 0$, $p_{ij} + p_{ji} = 1, p_{ij} = \frac{1}{2}, i, j = 1, 2, ..., n$ we can easily obtain the result that the matrix $P = (p_{ij})_{n\times n}$ is the complimentary judgment matrix.

(2)sum all the elements of each rows of the possibility degree matrix, and rank the orders of the trapezoid fuzzy linguistic variables based on the values p_i , where $p_i = \sum_{j=1}^{n} p_{ij} (i = 1, 2, ..., n)$. the larger the value of p_i is, the larger the trapezoid fuzzy linguistic variable $\tilde{s}_i$ is

Example :2

Let $\tilde{s}_1 = [s_2, s_3, s_5, s_6]$ and $\tilde{s}_2 = [s_4, s_5, s_8, s_9]$ be two trapezoid fuzzy linguistic variables, then we can compare the size of $\tilde{s}_1$ with $\tilde{s}_2$:

(1)The possibility degree of $\tilde{s}_1 \ge \tilde{s}_2$ is:

$$p(\tilde{s}_1 \ge \tilde{s}_2) = \min\{\max\{\frac{(5+6)-(4+5)}{(5+6)-(2+3)+(8+9)-(4+5)}, 0\}, 1\}$$

$= \min\{\max\{0.143, 0\}, 1\}$

$=0.143$

And the possibility degree of $\tilde{s}_2 \ge \tilde{s}_1$ is:

$$p(\tilde{s}_2 \ge \tilde{s}_1) = \min\{\max\{\frac{(8+9)-(2+3)}{(8+9)-(4+5)+(5+6)-(2+3)}, 0\}, 1\}$$

$= \min\{\max\{0.857, 0\}, 1\}$

$=0.857$

Then we can contribute the possibility degree matrix:

$$P = (p_{ij})_{2\times 2} = \begin{bmatrix} 0.5 & 0.143 \\ 0.857 & 0.5 \end{bmatrix}$$

(2) $p_1 = \sum_{j=1}^{2} p_{1j} = 0.5 + 0.143$

$=0.643$

$p_2 = \sum_{j=1}^{2} p_{2j} = 0.875 + 0.5$

$=1.375$

So $p_1 < p_2$

Then, we can get that: $\tilde{s}_1 < \tilde{s}_2$ ($\tilde{s}_1$ is worse than $\tilde{s}_2$).

V. Some Harmonic Operators with the Trapezoid Fuzzy Linguistic Variables

Definition:

Let $TFLWHA : \tilde{S}^n \to \tilde{S}$, if

$$TFLWHA_W(\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n) = (\sum_{j=1}^{n} \frac{w_j}{\tilde{s}_j})^{-1} \quad (4)$$

Where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables, and $\tilde{s}_j \in \tilde{S}(j = 1, 2, ..., n)$ is the trapezoid fuzzy linguistic variable. $w = (w_1, w_2, ..., w_n)$ is the weight vector, and w_i is the weight of $\tilde{s}_i$, where $w_i \ge 0, i = 1, 2, ..., n, \sum_{i=1}^{n} w_i = 1$, then TFLWHA is called the trapezoid fuzzy linguistic weighted harmonic averaging (TFLWHA)operator.

Example 3:

If $\tilde{s}_1=[s_2,s_3,s_5,s_6]$ $\tilde{s}_2=[s_4,s_5,s_8,s_9]$ $\tilde{s}_3=[s_5,s_6,s_7,s_9]$ and $\tilde{s}_4=[s_3,s_4,s_5,s_7]\in\tilde{S}$ are four trapezoid fuzzy linguistic variables, and $w=(0.3,0.2,0.1,0.4)$ is the weight vector, then

$$TFLWHA_w(\tilde{s}_1,\tilde{s}_2,\tilde{s}_3,\tilde{s}_4)=(\sum_{j=1}^{4}\frac{w_j}{\tilde{s}_j})^{-1}$$

$$=(\frac{0.3}{[s_2,s_3,s_5,s_6]}\oplus\frac{0.2}{[s_4,s_5,s_8,s_9]}\oplus\frac{0.1}{[s_5,s_6,s_7,s_9]}\oplus\frac{0.4}{[s_3,s_4,s_5,s_7]})^{-1}$$

$$=([s_{0.05},s_{0.06},s_{0.1},s_{0.15}]\oplus[s_{0.022},s_{0.025},s_{0.04},s_{0.05}]\oplus[s_{0.011},s_{0.014},s_{0.017},s_{0.02}]\oplus[s_{0.057},s_{0.08},s_{0.1},s_{0.133}])^{-1}$$

$$=[s_{0.14},s_{0.179},s_{0.257},s_{0.353}]^{-1}$$

$$=[s_{2.833},s_{3.891},s_{5.587},s_{7.143}]$$

Definition:

Let $TFLOWHA:\tilde{S}^n\to\tilde{S}$, if

$$TFLOWHA_\omega(\tilde{s}_1,\tilde{s}_2,.....,\tilde{s}_n)=(\sum_{j=1}^{n}\frac{\omega_j}{\tilde{r}_j})^{-1} \qquad (5)$$

Where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables, and $\tilde{s}_j,\tilde{r}_j\in\tilde{S}(j=1,2,.....,n)$ are the trapezoid fuzzy linguistic variables. $\tilde{r}_j$ is the j^{th} largest of $\tilde{s}_i(i=1,2,.....,n)$ and $\omega=(\omega_1,\omega_2,.......,\omega_n)$ is the position weight vector with TFLOWHA, where $\omega_j\geq 0$, j=1,2,…..,n $\sum_{j=1}^{n}\omega_j=1$, then TFLOWHA is called the trapezoid fuzzy linguistic ordered weighted harmonic averaging (TFLOWHA) operator.

The characteristic of the TFLOWHA operator is: Firstly, the order of the trapezoid fuzzy linguistic variables is ranked, then the position weights are aggregated with them, but there is no relationship between ω_j and $\tilde{s}_j$, and ω_j is only associated with the j^{th} position in the aggregation process, so $\omega=(\omega_1,\omega_2,.......,\omega_n)$ is called the position weight vector.

According to the real situation, the position weight vector $\omega=(\omega_1,\omega_2,.......,\omega_n)$ is determined. The position weight is determined by the method which proposed. The formula is shown as follows:

$$\omega_{i+1}=\frac{c^i_{n-1}}{2^{n-1}},i=0,1,......n-1 \qquad (6)$$

Example 4:

Let $\tilde{s}_1=[s_2,s_3,s_5,s_6]$ and $\tilde{s}_2=[s_4,s_5,s_8,s_9]$ be two trapezoid fuzzy linguistic variables, and we already know that $\tilde{s}_1<\tilde{s}_2$ (the calculation steps are shown in example 1) then the position weight vector is $\omega=(\frac{c^0_{2-1}}{2^{2-1}},\frac{c^0_{2-1}}{2^{2-1}})$ =(0.5,0.5)

$$TFLOWHA_\omega(\tilde{s}_1,\tilde{s}_2)=(\frac{0.5}{[s_2,s_3,s_5,s_6]}\oplus\frac{0.5}{[s_4,s_5,s_8,s_9]})^{-1}$$

$$=([s_{0.083},s_{0.1},s_{0.167},s_{0.25}]\oplus[s_{0.056},s_{0.0625},s_{0.1},s_{0.125}])^{-1}$$

$$=[s_{0.139},s_{0.1625},s_{0.267},s_{0.375}]^{-1}$$

$$=[s_{2.667},s_{3.745},s_{6.154},s_{7.217}]$$

The TFLWHA operator only focuses on the weight of the attribute value itself, but it ignores the position weight with respect to the attribute value; and the TFLOWHA operator focuses on the position weight with respect to the attribute value, but it ignores the weight of the attribute value itself. the two operators are one-sided. If the decision makers use these operators to aggregate the decision making information, some information may be lost. So, in order to avoid the disadvantage of the operators, the trapezoid fuzzy linguistic hybrid harmonic averaging (TFLHHA) operator is defined as follows:

Definition:

Let $TFLHHA:\tilde{s}^n\to\tilde{s}$ if

$$TFLHHA_{\omega,W}(\tilde{s}_1,\tilde{s}_2,....,\tilde{s}_n)=(\sum_{j=1}^{n}\frac{\omega_j}{\tilde{r}_j})^{-1} \qquad (7)$$

Where $\tilde{S}$ is the set of all trapezoid fuzzy linguistic variables, and $\tilde{s}_i,\tilde{r}_j\in\tilde{S}(i,j=1,2,......,n)$ are the trapezoid fuzzy linguistic variables. $\tilde{r}_j$ is the j^{th} largest of $\tilde{s}_i/nw_i$ (i=1,2,….,n), where $w=(w_1,w_2,....,w_n)$ is the weight vector, and w_i is the weight of $\tilde{s}_i,w_i\geq 0(i=1,2,....,n),\sum_{i=1}^{n}w_i=1$ and n is the balancing coefficient. $\omega=(\omega_1,\omega_2,....,\omega_n)$ is the position weight vector with TFLHHA, where $\omega_j\geq 0(j=1,2,....,n),\sum_{j=1}^{n}\omega_j=1$, then TFLHHA is called the trapezoid fuzzy linguistic hybrid harmonic averaging(TFLHHA) operator.

Example 5:

Let $\tilde{s}_1=[s_2,s_3,s_5,s_6]$ and $\tilde{s}_2=[s_4,s_5,s_8,s_9]$ two trapezoid fuzzy linguistic variables. We already know that the position weight vector is $\omega=(0.5,0.5)$ (the calculation steps are shown in Example 4), and the weight vector is $w=(0.3,0.7)$ given by the decision makers, then based on the method in section 2.2 calculate that

$$\tilde{r}_1=\tilde{s}_1/2w_1=[s_{3.333},s_5,s_{8.333},s_{10}] \text{ and}$$

$$\tilde{r}_2=\tilde{s}_2/2w_2=[s_{2.857},s_{3.571},s_{5.714},s_{6.429}]$$

Then,

$$TFLHHA_{\omega,W}(\tilde{s}_1,\tilde{s}_2,....,\tilde{s}_n)=(\sum_{j=1}^{n}\frac{\omega_j}{\hat{r}_j})^{-1}$$

$$=(\frac{0.5}{[s_{3.333},s_5,s_{8.333},s_{10}]}\oplus\frac{0.5}{[s_{2.857},s_{3.571},s_{5.714},s_{6.429}]})^{-1}$$

$$=$$

$$([s_{0.05},s_{0.06},s_{0.1},s_{0.15}]\oplus[s_{0.0778},s_{0.0875},s_{0.14},s_{0.175}])^{-1}$$

$$=(s_{0.1278},s_{0.1475},s_{0.24},s_{0.325})^{-1}$$

$$=(s_{3.077},s_{4.167},s_{6.780},s_{7.826})$$

Especially if $w=(1/n,1/n,....,1/n)$, then TFLHHA operator is reduced to TFLOWHA operator; if $\omega=(1/n,1/n,....,1/n)$, then TFLHHA operator is reduced to the TFLWHA operator. Obviously, TFLOWHA operator and TFLWHA operator are extended from the TFLHHA operator. The TFLHHA operator focuses on not only the importance of the weight of the trapezoid fuzzy linguistic variables itself, but also the importance of the position weight of the trapezoid fuzzy linguistic variables. So this operator is better the previous ones.

VI. ALGORITHM FOR TOPSIS BASED ON THE TRAPEZOID FUZZY LINGUISTIC VARIABLES

A multiple attribute decision making problem under the fuzzy linguistic environment is represented as follows:

Let X={x_1 , x_2 ,……. , x_n } be the set of the alternatives, and U={u_1 ,u_2 ,…….,u_m } be the set of the attributes. Let W= (w_1 ,w_2 ,………,w_m) ^{T} be the weight vector of the attributes, and W_j be the weight value of the j th attribute ,where $w_j \geq 0$ (j=1,2,…….m), $\sum_{j=1}^{m} w_j$ =1.given by the decision makers directly. Suppose that $\tilde{A}$ = $(\tilde{a}_{ij})_{n\times m}$ is the fuzzy linguistic decision matrix

$$\tilde{A}=\begin{array}{cc} \begin{matrix} u_1 & u_2 & \cdots & u_m \end{matrix} & \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nm} \end{bmatrix} & \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \end{array}$$

Where $\tilde{a}_{ij}=\left[a^{(\alpha)}{}_{ij},a^{(\beta)}{}_{ij},a^{(\gamma)}{}_{ij},a^{(\eta)}{}_{ij}\right]\in\tilde{s}$ is the attribute value which takes the form of the trapezoid fuzzy linguistic variables , given by the decision makers, for the alternative $x_i\in X$ (i=1,2,.....,n) with respect to the attribute $u_i\in U(j=1,2,......,m)$ .Let $\tilde{a}_i=\left[\tilde{a}_{i1},\tilde{a}_{i2},......\tilde{a}_{im}\right]$ be the vector of the attribute values under the alternative $x_i(i=1,2,.......,n)$

Then the decision making steps are shown as follows:

*Step : 1*Construct the weighted linguistic matrix $\tilde{A}'=(\tilde{a}'_{ij})_{n\times m}$

$$\tilde{A}'=\begin{bmatrix} \tilde{a}'_{11} & \tilde{a}'_{12} & \cdots & \tilde{a}'_{1m} \\ \tilde{a}'_{21} & \tilde{a}'_{22} & \cdots & \tilde{a}'_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}'_{n1} & \tilde{a}'_{n2} & \cdots & \tilde{a}'_{nm} \end{bmatrix}=(\tilde{a}'_{ij})_{n\times m}$$

Where $\tilde{a}'_{ij}=\tilde{a}_{ij}/nw_j$, $W=(w_1,w_2,.........w_m)$ is the weight vector of the attributes, $w_j>0(j=1,2,..........m),\sum_{j=1}^{m}w_j=1,n$ is the balancing co-efficient .

Step:2

$P(\tilde{s}_1>\tilde{s}_2)=\min\{\max\{\frac{(\gamma_1+\eta_1)-(\alpha_2+\beta_2)}{(\gamma_1+\eta_1)-(\alpha_1+\beta_1)+(\gamma_2+\eta_2)+(\alpha_2+\beta_2)},0\}1\}$ to construct the possibility degree matrixes $P_i=(P^{(i)}{}_{jk})_{n\times m}=(P^{(i)}{}_{jk}(\tilde{a}'_{ij}\geq\tilde{a}'_{ik}))_{m\times m}$ with rows of the possibility degree matrix p_i ,then get the ranking vectors $p^{(i)}=(p_1{}^{(i)},p_2{}^{(i)},........p_j{}^{(i)})(j=1,2,.......m)$ where $p^{(i)}{}_j=\sum_{k=1}^{m}p^{(i)}{}_{jk}$.

Finally, rank the orders of attribute values $\tilde{a}'_{ij}(j=1,2........m)$ with respect to the alternative x_i based on the values $p^{(i)}{}_j(j=1,2,..........m)$

Step:3

$\omega_{i+1}=\frac{c^i{}_{n-1}}{2^{n-1}},i=0,1,......n-1$ to calculate the position weight vector $\omega=\omega_1,\omega_2,........\omega_m$ of TFLHHA operator.

Step:4

$TFLHHA_{\omega,W}(\tilde{S}_1,\tilde{S}_2,......,\tilde{S}_n)=(\sum_{j=1}^{n}\frac{w_j}{\tilde{r}_j})^{-1}$ to calculate the combined attribute values,

$\tilde{z}_i=TFLHHA_{\omega,W}(\tilde{S}_1,\tilde{S}_2,..........\tilde{S}_m)=(\sum_{j=1}^{m}\frac{w_j}{\tilde{r}_j})^{-1}$ where $i=1,2,.........n$

Step:5

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$p(\tilde{s}_1 \geq \tilde{s}_2) = \min\{\max\{\frac{(\gamma_1+\eta_1)-(\alpha_2+\beta_2)}{(\gamma_1+\eta_1)-(\alpha_1+\beta_1)+(\gamma_2+\eta_2)-(\alpha_2+\beta_2)},0\}1\}$ to construct the possibility degree matrix $p=(p_{ij})_{n\times n}$ based on the combined attribute values $\tilde{z}_i$ of each alternative, then sum all the elements of each rows of the possibility degree matrix, where $p_i = \sum_{j=1}^{n} p_{ij} (i=1,2,............n)$. Rank all the combined attribute values of each alternative and choose the maximum value, which one is the best alternative based on the values p_i.

Step: 6

To transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes, normalize the matrix obtained in step-5 as follows:

$$V_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Step: 7

Calculate the weights w_j of the decision makers using a new proposed method.

Step: 8

Then calculate the weighted normalized matrix as follows:

$$R_{ij} = w_j \times V_{ij}$$

Step: 9

Identify the positive and negative ideal solutions

Step: 10

Calculate the separation measures from the ideal solutions as follows:

$$S_j^+ = \left[\sum(\mathrm{V}_{ij} - \mathrm{A}_i^+)^2\right]^{\frac{1}{2}}; \; S_j^- = \left[\sum(\mathrm{V}_{ij} - \mathrm{A}_i^-)^2\right]^{\frac{1}{2}}.$$

Step: 11

Calculate the relative closeness as follows:

$$\mathrm{C}_j = \frac{S_j^-}{S_j^+ + S_j^-}.$$

VII. ILLUSTRATIVE EXAMPLE

A decision maker intends to buy a laptop. Four types of laptops $x_i (i=1,2,3,4)$ are available. He takes into account four attributes to decide which laptop he should buy:

1) G_1 : Price,
2) G_2 : Brand name,
3) G_3 : Compatibility, and
4) G_4 : long life of battery.

The decision maker evaluates these four types of laptop $x_i (i=1,2,3,4)$ under the attributes $G_j (j=1,2,3,4)$, where the weight vector of the decision makers is unknown (to be evaluated in step 7). He uses the linguistic term set:

S={ s_1 =extremely poor, s_2 =very poor, s_3 =poor, s_4 =slightly poor, s_5 =fair, s_6 =slightly good, s_7 =good, s_8 =very good, s_9 =extremely good} and provides the linguistic decision making matrix $\tilde{A} = (\tilde{a}_{ij})_{4\times 4}$:

$$\tilde{A} = \begin{bmatrix} [s_2,s_3,s_5,s_6] & [s_4,s_5,s_8,s_9] & [s_5,s_6,s_7,s_9] & [s_3,s_4,s_5,s_7] \\ [s_3,s_5,s_6,s_7] & [s_5,s_6,s_7,s_8] & [s_4,s_5,s_8,s_9] & [s_4,s_5,s_7,s_8] \\ [s_4,s_6,s_8,s_9] & [s_4,s_5,s_6,s_7] & [s_6,s_7,s_8,s_9] & [s_3,s_4,s_5,s_6] \\ [s_5,s_6,s_7,s_9] & [s_4,s_7,s_8,s_9] & [s_3,s_5,s_6,s_7] & [s_6,s_7,s_8,s_9] \end{bmatrix}$$

Step:1

construct the weighted linguistic matrix $\tilde{A}' = (\tilde{a}'_{ij})_{4\times 4}$, where

$$\tilde{a}'_{ij} = \tilde{a}'_{ij} / nw_j \; (j=1,2,3,4)$$

$$\tilde{a}'_{11} = \frac{\tilde{a}_{11}}{4\times 0.3}$$

$$= \frac{[s_2, s_3, s_5, s_6]}{4\times 0.3}$$

$$= [s_{1.67}, s_{2.5}, s_{4.17}, s_5]$$

Similarly, calculating all the other values we get:

$$\tilde{A}' = \begin{bmatrix} [s_{1.67},s_{2.5},s_{4.17},s_5] & [s_5,s_{6.25},s_{10},s_{11.25}] & [s_{12.5},s_{15},s_{17.5},s_{22.5}] & [s_{1.875},s_{2.5},s_{3.125},s_{4.375}] \\ [s_{2.5},s_{4.17},s_5,s_{5.83}] & [s_{6.25},s_{7.5},s_{8.75},s_{10}] & [s_{10},s_{12.5},s_{20},s_{22.5}] & [s_{2.5},s_{3.125},s_{4.375},s_5] \\ [s_{3.33},s_5,s_{6.67},s_{7.5}] & [s_{3.33},s_{4.17},s_5,s_{5.83}] & [s_{15},s_{17.5},s_{20},s_{22.5}] & [s_{1.875},s_{2.5},s_{3.125},s_{3.75}] \\ [s_{4.17},s_5,s_{5.83},s_{7.5}] & [s_5,s_{8.75},s_{10},s_{11.25}] & [s_{7.5},s_{12.5},s_{15},s_{17.5}] & [s_{3.75},s_{4.375},s_5,s_{5.625}] \end{bmatrix}$$

Step: 2

$p(\tilde{s}_1 \geq \tilde{s}_2) = \min\{\max\{\frac{(\gamma_1+\eta_1)-(\alpha_2+\beta_2)}{(\gamma_1+\eta_1)-(\alpha_1+\beta_1)+(\gamma_2+\eta_2)-(\alpha_2+\beta_2)},0\}1\}$ to construct the possibility degree matrixes $p_i = (p^{(i)}{}_{jk})_{4\times 4} = (p^{(i)}{}_{jk}(\tilde{a}'_{ij} \geq \tilde{a}_{ik}))_{4\times 4}$ with respect each alternative x_i (i=1,2,3,4) and sum all the elements of each rows of the possibility degree matrix p_i , then get the ranking vectors $p^{(i)} = (p_1{}^{(i)}, p_2{}^{(i)}, ..., p_j{}^{(i)})$ (j=1,2,3,4)

Where $p^{(i)}{}_j = \sum_{k=1}^{4} p^{(i)}{}_{jk}$. finally, rank the orders of attribute values $\tilde{a}'_{ij} (j=1,2,3,4)$ with respect to the alternative x_i based on the values $p^{(i)}{}_j$ (j=1,2,3,4)

By taking,

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$$\tilde{A}'=\begin{bmatrix} R_1[s_{1.67},s_{2.5},s_{4.17},s_5] & [s_5,s_{6.25},s_{10},s_{11.25}] & [s_{12.5},s_{15},s_{17.5},s_{22.5}] & [s_{1.875},s_{2.5},s_{3.125},s_{4.375}] \\ R_2[s_{2.5},s_{4.17},s_5,s_{5.83}] & [s_{6.25},s_{7.5},s_{8.75},s_{10}] & [s_{10},s_{12.5},s_{20},s_{22.5}] & [s_{2.5},s_{3.125},s_{4.375},s_5] \\ R_3[s_{3.33},s_5,s_{6.67},s_{7.5}] & [s_{3.33},s_{4.17},s_5,s_{5.83}] & [s_{15},s_{17.5},s_{20},s_{22.5}] & [s_{1.875},s_{2.5},s_{3.125},s_{3.75}] \\ R_4[s_{4.17},s_5,s_{5.83},s_{7.5}] & [s_5,s_{8.75},s_{10},s_{11.25}] & [s_{7.5},s_{12.5},s_{15},s_{17.5}] & [s_{3.75},s_{4.375},s_5,s_{5.625}] \end{bmatrix}$$

$$R1=([s_{1.67},s_{2.5},s_{4.17},s_5][s_5,s_{6.25},s_{10},s_{11.25}][s_{12.5},s_{15},s_{17.5},s_{22.5}][s_{2.5},s_{3.125},s_{4.375},s_5])$$

$$p_1=\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix}$$

$$R_{11}=([s_{1.67},s_{2.5},s_{4.17},s_5][s_{1.67},s_{2.5},s_{4.17},s_5])$$

$$=\min\{\max\{\frac{(s_{4.17}+s_5)-(s_{1.67}+s_{2.5})}{(s_{4.17}+s_5)-(s_{1.67}+s_{2.5})+(s_{4.17}+s_5)-(s_{1.67}+s_{2.5})},0\}1\}$$

$$=0.5$$

Similarly, calculating all the other entries we get:

$$p_1=\begin{bmatrix} 0.5 & 0 & 0 & 0.59 \\ 1 & 0.5 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0.41 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(1)}{}_1=p^{(1)}{}_{11}+p^{(1)}{}_{12}+p^{(1)}{}_{13}+p^{(1)}{}_{14}$$
$$=0.5+0+0+0.59$$
$$=1.09$$

$$p^{(2)}{}_1=p^{(2)}{}_{11}+p^{(2)}{}_{12}+p^{(2)}{}_{13}+p^{(2)}{}_{14}$$
$$=1+0.5+0+1$$
$$=2.5$$

$$p^{(3)}{}_1=p^{(3)}{}_{11}+p^{(3)}{}_{12}+p^{(3)}{}_{13}+p^{(3)}{}_{14}$$
$$=1+1+0.5+1$$
$$=3.5$$

$$p^{(4)}{}_1=p^{(4)}{}_{11}+p^{(4)}{}_{12}+p^{(4)}{}_{13}+p^{(4)}{}_{14}$$
$$=0.41+0+0+0.5$$
$$=0.91$$

$$\therefore p^{(1)}=(p^{(1)}{}_1,p^{(1)}{}_2,p^{(1)}{}_3,p^{(1)}{}_4)$$
$$=(1.09,2.5,3.5,0.91)$$

$$\tilde{a}'_{13}>\tilde{a}'_{12}>\tilde{a}'_{11}>\tilde{a}'_{14}$$

Similarly,

$$R2=([s_{2.5},s_{4.17},s_5,s_{5.83}][s_{6.25},s_{7.5},s_{8.75},s_{10}][s_{10},s_{12.5},s_{20},s_{22.5}][s_{2.5},s_{3.125},s_{4.375},s_5])$$

$$p_2=\begin{bmatrix} 0.5 & 0 & 0 & 0.66 \\ 1 & 0.5 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0.34 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(2)}=(1.16,2.5,3.5,0.84)$$

$$\tilde{a}'_{23}>\tilde{a}'_{22}>\tilde{a}'_{21}>\tilde{a}'_{24}$$

$$R3=([s_{3.33},s_5,s_{6.67},s_{7.5}],[s_{3.33},s_{4.17},s_5,s_{5.83}],[s_{15},s_{17.5},s_{20},s_{22.5}],[s_{1.875},s_{2.5},s_{3.125},s_{3.75}])$$

$$p_3=\begin{bmatrix} 0.5 & 0.73 & 0 & 1 \\ 0.27 & 0.5 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(3)}=(2.23,1.77,3.5,0.5)$$

$$\tilde{a}'_{33}>\tilde{a}'_{31}>\tilde{a}'_{32}>\tilde{a}'_{34}$$

$$R4=([s_{4.17},s_5,s_{5.83},s_{7.5}][s_5,s_{8.75},s_{10},s_{11.25}][s_{7.5},s_{12.5},s_{15},s_{17.5}][s_{3.75},s_{4.375},s_5,s_{5.625}])$$

$$p_4=\begin{bmatrix} 0.5 & 0 & 0 & 0.78 \\ 1 & 0.5 & 0.0625 & 1 \\ 1 & 0.9375 & 0.5 & 1 \\ 0.22 & 0 & 0 & 0.5 \end{bmatrix}$$

$$p^{(4)}=(1.28,2.5625,3.4375,0.72)$$

$$\tilde{a}'_{43}>\tilde{a}'_{42}>\tilde{a}'_{41}>\tilde{a}'_{44}$$

Step:3

Calculate the position weight vector $\omega=(\omega_1,\omega_2,...,\omega_m)$ of TFLHHA operator by $\omega_{i+1}=\frac{c^i{}_{n-1}}{2^{n-1}}, i=0,1,...,n-1.$

$$\omega=\frac{C^0_{4-1}}{2^{4-1}},\frac{C^1_{4-1}}{2^{4-1}},\frac{C^2_{4-1}}{2^{4-1}},\frac{C^3_{4-1}}{2^{4-1}}$$

$$\omega=\frac{C^0_3}{8},\frac{C^1_3}{8},\frac{C^2_3}{8},\frac{C^3_3}{8}$$

$$\omega=(0.125,0.375,0.375,0.125)$$

Step:4

Calculate the combined attribute values by

$$TFLHHA_{\omega,W}(\tilde{S}_1,\tilde{S}_2,......\tilde{S}_n)=(\sum_{j=1}^{n}\frac{w_j}{\tilde{r}_j})^{-1}:$$

$$\tilde{z}_1=(s_{2.65},s_{3.73},s_{5.73},s_{7.02})$$

$$\tilde{z}_2=(s_{3.67},s_{5.27},s_{6.55},s_{7.55})$$

$$\tilde{z}_1=(s_{3.33},s_{4.5},s_{5.63},s_{6.53})$$

$$\tilde{z}_1=(s_{4.65},s_{6.39},s_{7.39},s_{8.87})$$

Step:5

Construct the possibility degree matrix, based on $\tilde{z}_1$ by

$$p(\tilde{s}_1\geq\tilde{s}_2)=\min\{\max\{\frac{(\gamma_1+\eta_1)-(\alpha_2+\beta_2)}{(\gamma_1+\eta_1)-(\alpha_1+\beta_1)+(\gamma_2+\eta_2)-(\alpha_2+\beta_2)},0\}1\}:$$

$$p=\begin{bmatrix} 0.5 & 0.33 & 0.46 & 0.15 \\ 0.67 & 0.5 & 0.66 & 0.29 \\ 0.54 & 0.34 & 0.5 & 0.12 \\ 0.85 & 0.71 & 0.88 & 0.5 \end{bmatrix}$$

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Let the weights of the decision makers be given by $w = (0.3, 0.2, 0.1, 0.4)$.

Step:6

To transform the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Hence the transformed matrix is given as:

$$V_{ij} = \begin{bmatrix} 0.3820 & 0.3336 & 0.3558 & 0.2463 \\ 0.5119 & 0.5055 & 0.5105 & 0.4161 \\ 0.4126 & 0.3437 & 0.3867 & 0.1970 \\ 0.6494 & 0.7178 & 0.6806 & 0.8201 \end{bmatrix}$$

Step:7

Let us suppose that there are 25 experts to give a consensus on the unknown weighting vector of the attributes. The following computational procedure is followed for calculating the unknown weights of the attributes.

Table 1: Ranks of the experts on the 4 attributes.

Expert	x_1	x_2	x_3	x_4
1	1	4	2	3
2	3	1	2	4
3	4	3	2	1
4	4	1	3	2
5	4	3	2	1
6	4	2	3	1
7	4	3	1	2
8	3	4	1	2
9	4	2	3	1
10	4	3	1	2
11	4	1	2	3
12	2	4	3	1
13	3	2	1	4
14	4	2	1	3
15	4	3	1	2
16	4	3	2	1
17	1	3	2	4
18	3	2	4	1
19	4	2	3	1
20	3	4	1	2
21	4	3	1	2
22	1	4	3	2
23	4	3	2	1
24	2	4	1	3
25	1	4	3	2
Sum of rank	***79***	***70***	***48***	***51***

Sum of rank : $\bar{t_j} = \sum_{k=1}^{r=25} t_{jk}$

$x_1 = 79;\ x_2 = 70;\ x_3 = 48\ ;\ x_4 = 51$

The average attribute rank value:

$$\bar{t_j} = \frac{\sum_{k=1}^{r=25} t_{jk}}{r}$$

$x_1 = \frac{79}{25} = 3.16\ ;\ x_2 = \frac{70}{25} = 2.8\ ;\ x_3 = \frac{48}{25} = 1.92\ ;\ x_4 = \frac{51}{25} = 2.04$

Attribute weight of value:

$$q_j = \frac{\bar{t_j}}{\sum_{j=1}^{n=4} t_j}$$

$x_1 = \frac{3.16}{10} = 0.32\ ;\ x_2 = \frac{2.8}{10} = 0.28\ ;\ x_3 = \frac{1.92}{10} = 0.192\ ;\ x_4 = \frac{2.04}{10} = 0.204$

Table 2: Calculation of $\sum_{k=1}^{r=25} \left(t_{jk} - \bar{t_j}\right)^2$.

Expert	x_1	x_2	x_3	x_4
1	4.6656	1.44	0.0064	0.9216
2	0.0256	3.24	0.0064	3.8416
3	0.7056	0.04	0.0064	1.0816
4	0.7056	3.24	1.1664	0.0016
5	0.7056	0.04	0.0064	1.0816
6	0.7056	0.64	1.1664	1.0816
7	0.7056	0.04	0.8464	0.0016
8	0.0256	1.44	0.8464	0.0016
9	0.7056	0.64	1.1664	1.0816
10	0.7056	0.04	0.8464	0.0016
11	0.7056	3.24	0.0064	0.9216
12	1.3456	1.44	1.1664	1.0816
13	0.0256	0.64	0.8464	3.8416
14	0.7056	0.64	0.8464	0.9216
15	0.7056	0.04	0.8464	0.0016
16	0.7056	0.04	0.0064	1.0816
17	4.6656	0.04	0.0064	3.8416
18	0.0256	0.64	4.3264	1.0816
19	0.7056	0.64	1.1664	1.0816
20	0.0256	1.44	0.8464	0.0016
21	0.7056	0.04	0.8464	0.0016
22	4.6656	1.44	1.1664	0.0016
23	0.7056	0.04	0.0064	1.0816
24	1.3456	1.44	0.8464	0.9216
25	4.6656	1.44	1.1664	0.0016

Calculated values:

For $x_1 = 31.36;\ x_2 = 24.00;\ x_3 = 20.14;\ x_4 = 24.96$

Dispersion of experts ranking values:

$$\sigma^2 = \frac{1}{r-1}\sum_{k=1}^{r=25}\left(t_{jk} - \overline{t_j}\right)^2$$

$x_1 = \frac{31.36}{24} = 1.31$; $x_2 = \frac{24.00}{24} = 1.00$; $x_3 = \frac{20.14}{24} = 0.84$; $x_4 = \frac{24.96}{24} = 1.04$

Variation of expert ranking value :

$$\beta_j = \frac{\sigma}{\overline{t_j}}$$

$x_1 = \frac{1.1446}{3.16} = 0.3622$; $x_2 = \frac{1.00}{2.8} = 0.3571$; $x_3 = \frac{0.9159}{1.92} = 0.47703$; $x_4 = \frac{1.0198}{2.04} = 0.4499$

Ranking sum average of expert value:

$$V = \frac{1}{n}\sum_{j=1}^{n=4}\sum_{k=1}^{r=25} t_{jk}$$

$$V = \frac{1}{4}(79 + 70 + 48 + 51) = 62$$

The total square ranking deviation:

$$S = \sum_{j=1}^{n=4}\left(\sum_{k=1}^{r=25} t_{jk} - V\right)^2$$

$$S = (79-62)^2 + (70-62)^2 + (48-62)^2 + (51-62)^2 S = 67$$

The coefficient of concordance:

$$W = \frac{12S}{r^2(n^3 - n)}$$

$W = \frac{12*670}{25^2(4^3-4)} = \frac{8400}{37500} = 0.21.$

The significance of the concordance coefficient

$$\omega_{a,y}{}^2 = \frac{12S}{rn(n+1) - \frac{1}{n}\sum_{k=1}^{r} T_k}$$

$= \frac{12*670}{25*4*5-0} = \frac{8040}{500}$ =1.60, Where $\frac{1}{n}\sum_{k=1}^{r} T_k = 0$

Compatibility of expert judgment:

$$\omega_{\alpha,v}{}^2 > \omega_{tbl}{}^2$$

$$1.60 > 13.3$$

The hypothesis about the consent of expert in ranking is not accepted. Hence we have to normalize the attribute ranks and compute the consensus values.

Table 3: Ranks of the experts on the 4 attributes& the probabilities associated with Normal Distribution.

Expert	x_1	x_2	x_3	x_4
1	1	4	2	3
2	3	1	2	4
3	4	3	2	1
4	4	1	3	2
5	4	3	2	1
6	4	2	3	1
7	4	3	1	2
8	3	4	1	2
9	4	2	3	1
10	4	3	1	2
11	4	1	2	3
12	2	4	3	1
13	3	2	1	4
14	4	2	1	3
15	4	3	1	2
16	4	3	2	1
17	1	3	2	4
18	3	2	4	1
19	4	2	3	1
20	3	4	1	2
21	4	3	1	2
22	1	4	3	2
23	4	3	2	1
24	2	4	1	3
25	1	4	3	2
6σ	***6***	***6***	***5***	***6***
σ	***1.15***	***1.00***	***0.92***	***1.01***
$\mu - 2\sigma$	***0.86***	***0.80***	***0.08***	***0.02***
$\mu + 2\sigma$	***5.67***	***4.80***	***3.76***	***4.02***

Table 4: Ranks of the experts on the 4 attributes & the probabilities associated with Normal Distribution.
(Adding the normal distribution values to the lesser expert values)

Expert	x_1	x_2	x_3	x_4
1	5.46	4	2	3
2	3	4.8	2	0.02
3	4	3	2	1
4	4	4.8	3	2
5	4	3	2	1
6	4	2	3	1
7	4	3	0.08	2
8	3	4	0.08	2
9	4	2	3	1
10	4	3	0.08	2
11	4	4	2	3
12	2	3	3	1
13	3	4.8	0.08	0.02
14	4	4	0.08	3
15	4	2	0.08	2`
16	4	2	2	1
17	5.64	3	2	0.02
18	3	3	4	1
19	4	3	3	1
20	3	2	0.08	2
21	4	2	0.08	2
22	5.64	4	3	2
23	4	3	2	1

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24	2	4	0.08	3
25	5.64	4	3	2
Sum of rank	***96.84***	***81.4***	***42.72***	***39.06***
$\boldsymbol{\sigma}$	***6***	***6***	***5***	***6***
$\boldsymbol{6\sigma}$	***1.15***	***0.80***	***0.08***	***0.02***
$\boldsymbol{\mu+2\sigma}$	***5.46***	***4.8***	***3.76***	***4.06***
$\boldsymbol{\mu-2\sigma}$	***0.86***	***0.80***	***0.08***	***0.02***

Now, sum of ranks: $\bar{t_j} = \sum_{k=1}^{r=25} t_{jk}$

$x_1 = 96.84 : x_2 = 81.4 : x_3 = 42.72 : x_4 = 39.06$

The average attribute rank value:

$$\bar{t_j} = \frac{\sum_{k=1}^{r=25} t_{jk}}{r}$$

$x_1 = \frac{96.84}{25} = 3.8736$; $x_2 = \frac{81.4}{25} = 3.24$; $x_3 = \frac{42.72}{25} = 1.708$; $x_4 = \frac{39.06}{25} = 1.5624$

Attribute weight of the expert value:

$$q_j = \frac{\bar{t_j}}{\sum_{j=1}^{=4} t_j}$$

$x_1 = \frac{3.8}{10} = 0.38$; $x_2 = \frac{3.2}{10} = 0.32$; $x_3 = \frac{1.7}{10} = 0.17$; $x_4 = \frac{1.6}{10} = 0.15$

Calculate $\sum_{k=1}^{r=25}\left(t_{jk} - \bar{t_j}\right)^2$ in the following table.

Table 5: Calculation of $\sum_{k=1}^{r=25}\left(t_{jk} - \bar{t_j}\right)^2$.

Expert	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$	$\boldsymbol{x_3}$	$\boldsymbol{x_4}$
1	2.5281	0.5776	0.0841	2.0736
2	0.7569	2.4336	0.0841	2.3716
3	0.0169	0.0576	0.0841	0.3136
4	0.0169	2.4336	1.1664	0.1936
5	0.0169	0.0576	0.0841	0.3136
6	0.0169	1.5376	1.1664	0.3136
7	0.0169	0.0576	2.6569	0.1936
8	0.7569	0.5576	2.6569	0.1936
9	0.0169	1.5376	1.6641	0.3136
10	0.0169	0.0576	2.6569	0.1936
11	0.0169	0.5776	0.0841	2.0736
12	3.4969	0.0576	1.6641	0.3136
13	0.7569	2.4336	2.6569	2.3716
14	0.0169	0.5776	2.6569	2.0736
15	0.0169	1.5376	2.6569	0.1936
16	0.0169	1.5376	0.0841	0.3136
17	2.5281	0.0576	0.0841	2.3716
18	0.7569	0.0576	5.2441	0.3136
19	0.0169	0.0576	1.6641	0.3136
20	0.7569	1.5376	2.6569	0.1936
21	0.0169	1.5376	2.6569	0.1936
22	2.5281	0.5776	1.6641	0.1936
23	0.0169	0.0576	0.0841	0.3136
24	3.4969	0.5776	2.6469	2.0736
25	2.5281	0.5776	1.6641	0.1936
Total	***21.36***	***21.088***	***41.4577***	***18.6188***

$x_1 = \frac{21.36}{24} = 0.89$; $x_2 = \frac{21.088}{24} = 0.87$; $x_3 = \frac{41.4577}{24} = 1.73$; $x_4 = \frac{18.1688}{24} = 0.76$

Variation of expert ranking values:

$$\beta_j = \frac{\sigma}{\bar{t_j}}$$

$x_1 = \frac{0.943}{3.87} = 0.244$; $x_2 = \frac{0.932}{3.24} = 0.288$; $x_3 = \frac{1.315}{1.70} = 0.77$; $x_4 = \frac{0.871}{1.56} = 0.558$

Ranking sum average of expert value:

$$V = \frac{1}{n}\sum_{j=1}^{n=4}\sum_{k=1}^{r=25} t_{jk}$$

$$V = \frac{1}{4}(96.84 + 81.4 + 42.72 + 39.06) = 0.25(260.02)$$
$$= 65$$

The total square ranking deviation :

$$S = \sum_{j=1}^{n=4}\left(\sum_{k=1}^{r=25} t_{jk} - V\right)^2$$

$$S = (96.84 - 65)^2 + (81.4 - 65)^2 + (42.7 - 65)^2 + (39.06 - 65)^2$$

$$S = 31.84^2 + 16.4^2 + (-22.28)^2 + (-25.94)^2 = 2452$$

The coefficient of concordance:

W= $\frac{12S}{r^2(n^3-n)}$;

$$W = \frac{12 * 2452}{25^2(4^3 - 4)}$$

W= 0.79

The significance of the concordance coefficient:

$$\omega_{a,y}{}^2 = \frac{12S}{rn(n+1) - \frac{1}{n}\sum_{k=1}^{r} T_k}$$

$= \frac{12*2452}{25*4*5-0} = \frac{29424}{500}$ =58.848

Compatibility of expert judgments:

$$\omega_{\alpha,v}{}^2 > \omega_{ybl}{}^2$$
$$58.84 > 13.3$$

The hypothesis about the consent of expert in ranking is accepted.

The values of the attributes weights are established as:

$$w_1 = 0.38;\ w_2 = 0.32;\ w_4 = 0.17;\ w_4 = 0.15$$

Step:8

Then calculate the weighted normalized matrix as follows:

$$R_{ij} = w_j \times V_{ij} = (0.38, 0.32, 0.17, 0.15) \times \begin{bmatrix} 0.3820 & 0.3336 & 0.3558 & 0.2463 \\ 0.5119 & 0.5055 & 0.5105 & 0.4161 \\ 0.4126 & 0.3437 & 0.3867 & 0.1970 \\ 0.6494 & 0.7178 & 0.6806 & 0.8201 \end{bmatrix}$$

$$R_{ij} = \begin{bmatrix} 0.14516 & 0.10675 & 0.06048 & 0.03695 \\ 0.19452 & 0.16171 & 0.08678 & 0.06241 \\ 0.15678 & 0.10998 & 0.06574 & 0.02955 \\ 0.24677 & 0.22969 & 0.11570 & 0.1230 \end{bmatrix}$$

Step: 9

Identifying the positive and negative ideal solutions as follows:

$$A^{+} = (0.1904, 0.1274, 0.0496, 0.3129)$$

$$A^{-} = (0.1106, 0.0658, 0.0436, 0.0698)$$

Step: 10

Calculate the separation measures from the ideal solutions as follows:

$$S_j^{+} = \left[\sum (\mathrm{V}_{ij} - \mathrm{A}_i^{+})^2\right]^{\frac{1}{2}}; \quad S_j^{-} = \left[\sum (\mathrm{V}_{ij} - \mathrm{A}_i^{-})^2\right]^{\frac{1}{2}}.$$

$$S_j^{+} = \begin{bmatrix} 0.2504 \\ 0.1429 \\ 0.2372 \\ 0.0048 \end{bmatrix} \text{ and } S_j^{-} = \begin{bmatrix} 0.0256 \\ 0.1443 \\ 0.0100 \\ 0.2867 \end{bmatrix}.$$

Step: 11

Calculate the relative closeness as follows:

$$\mathrm{C}_j = \frac{S_j^{-}}{S_j^{+} + S_j^{-}}.$$

$C_1 = 0.0927$, $C2 = 0.5024$, $C3 = 0.0404$, $C4 = 0.9835$.

Hence C_4 is the best alternative.

VIII. CONCLUSION

A novel approach of decision making is proposed with respect to MADM problems, where the data is of the form of linguistic variable. An illustrative example was presented to demonstrate and validate the effectiveness of our proposed method. This extended TOPSIS method together with the weight determining technique of the decision makers proves to be a better DMSS technique because of its exclusiveness in dealing with imprecise data in the form of linguistic variables. We shall continue to work in the extension and application of the developed method in some of the complicated domains in future.

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