Dynamics and Control of Ball and Beam System

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***Abstract*—** In this paper modeling and dynamics of Ball and Beam system have been studied and presented. Ball beam system is very useful system is widely used in control system laboratory as an experimental arrangement. Its basic principles of control is similar to control principles used in many industrial applications. So understanding control of ball and beam system makes one to understand and design control strategy for industrial application. Due to such resemblances and simplicity this system has been widely used and studied and controlled using different techniques. Here in this paper system dynamics and control of the system using PD controlled is presented.

Keywords-*ball and beam system; control; dynamics; optimal control;*

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I. INTRODUCTION

In recent age with evolution of science and technology many control problems have arisen in real physical world. Control problems can be solved by specific suitable controller. Controller design is an expression of control system imagination and creativity. The problem of designing a controller has always been a source of study for control system engineers. Some real physical problems can't be brought into laboratory to study. So to solve such problems an equivalent system is developed in laboratory. One of them is ball and beam control system. The ball and beam system is one of the most important models used in teaching control system engineering. This system is basically part of study of control system development and is widely used, because of its simplicity and safe mechanism to study dynamics of unstable system. There have been several studies on the development of automatic control system for it. The theories used to solve control of ball and beam problem is useful in many other real control applications such as in horizontal stabilization of air-plane at landing and take-off, turbulent air flow and aerospace control etc., however, bringing such type of problems in laboratory is neither easier nor safer. Thus using simpler and safer system of same control nature can be used. Ball and beam system provides such a platform. Different researchers have developed ball and beam system. A typical ball and beam system is shown in Fig.1.

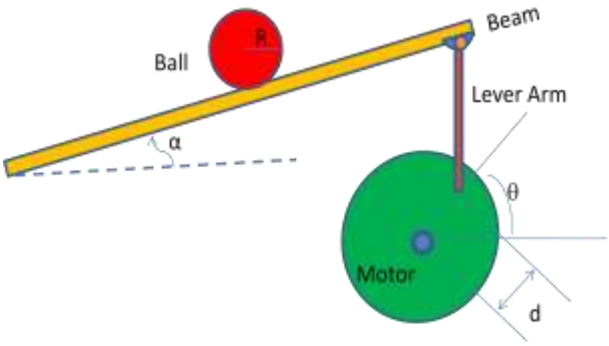

Fig.1 Ball and Beam System.

It has two degrees of freedom. In this system a ball is placed at the beam as shown in Fig.1, and lever arm attached to one end point on the beam and other end of a lever arm also attached to a rotation system or motor. When lever arm changes the position of beam from its reference position with angle α then position of ball also changes in non-linear fashion due to gravity. If the beam angle remains constant the ball will pass through the beam with changing velocity. In order to regulate motion of the ball on beam the inclination of the beam is required to be changed in such a way that can stop fall out of the ball by reversing direction of motion. Such changes in the beam angle at proper time leads to regulated motion of the ball on beam. Also by properly adjusting inclination of the beam ball can be positioned at any point on the beam. If it has to be done manually, the whole thing looks like a toy that requires concentration and manipulation in beam angle observing position of the ball on beam. However, to do the same automatically, it requires a controller that can control rotation of motor to adjust tilt of beam such that position of ball is controlled.

Many researchers have designed and studied ball and beam system. Some of them are described here for reference. Arroyo, in his research, built system ball on balancing beam using resistive wire sensor to control position of the ball using PD controller [1]. In that system design of PD controller is easier, however, the tilt angle of the beam was neither measured nor controlled which affected robustness of the system. In another work [2], Linear Quadratic Regulator (LQR) controller has been used to develop system named as ball and beam balancer in which both the position of the ball and tilt angle of the beam are measured and controlled. Quanser developed a commercial product named ball and beam module using PID controller using DC servomotor. The other system as reported in [3] which uses ultrasonic sensor to sense position of the ball and potentiometer to measure tilt angle of the beam. The ball and beam system developed by Hirsch uses ultrasonic sensor to sense position of the ball on the beam [4]. In [5], the described system has been made from acrylic and aluminum and control is similar to that of in [2]. Lieberman developed a system known as a Robotic Ball Balancing Beam. In that system resistive wire based position sensor has been used to sense position of the ball on the beam

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[6]. Ball and beam system is a simple system which is an example of unstable open loop system, however, complexity in its control, makes it very significant system, for control engineers. It has both academic and industrial applications. In the control laboratories it is used to demonstrate principles of many complex and big system which cannot be brought into laboratory setup. Ball and beam and its two dimensional version, that is ball on balancing plate, have many practical applications in real world control problems such as passenger's platform balancing in luxury cars, the control of exothermic chemical process reactions where heat increase the process, control of rocket toppling control system and aircraft vertical take–off and airplane horizontally stabilizing during landing and in turbulent airflow and moving liquid container on road etc. Ball and beam system having two degree of freedom and due to inherent nonlinearity it is a challenging control problem[7]. Ball and beam system is used for teaching, understanding control characteristic and solving difficult control problems. For many unbalanced system, present in daily life, where size and shape not suitable for study purpose in laboratory, ball and beam system can be used to learn control mechanism. Some unbalanced systems are exothermically preceding reactions, in power generation, and in aerospace. These types of real unbalanced systems are most difficult and dangerous control problems and cannot be brought in laboratory. So, ball and beam system gives an advanced role in study of this type of real unbalanced system and suitable controller design for industrial as well as academic purposes.

II. MODELING AND DYNAMICS

Mathematical modeling of any system provides ways for scientific study and understanding of the system functioning and behavior. Ball and beam system can be broken into subsystems and functioning of each subsystem provides integrated effect in which ball looks moving smoothly on the beam in accordance with tilt of the beam and vice-versa. From the mathematical analysis point of view, thus it becomes important to know contribution and co-ordination of each subsystem. From the control point of view it is essential to develop equation of motion of and transfer function of the system[8].

A. Transfer function of Ball Beam system

Transfer function of any system is defined as the ratio of Laplace transform of output and input. Using classical Newtonian balance of forces method as shown in Fig.2.We can derive transfer function and equation of ball and beam system. The equation of motion of ball on beam can be derived using Newtonian mechanics or Lagrange method. Then from these equations transfer function of the system can be obtained. Such an analysis also provides better understanding of the functioning of the system which further helps in the controller design using different methods. The mathematical modeling of the system requires relation between position of the ball and tilt angle of the beam and relation between input voltage and angle of rotation of DC motor. Thus its transfer function can be obtained as combination of transfer functions of subsystems namely (i) beam tilt to ball position and (ii) motor input voltage to its angle of rotation and tilt angle of the beam. In order to derive relation between ball position and beam angle let us consider the Fig.2 in which position of the ball at any instant of time is x, the inclination of the beam is considered along x-axis.

The motion of the ball on beam is translational and rotational. Now, let the translational acceleration of the ball is $\ddot{x}$ for which force F is given by

$$F_{tx} = m\ddot{x} = m\frac{d^2x}{dt^2}, \quad (1)$$

where $m = mass\ of\ the\ ball.$

The rotational torque T of the ball $= \mathrm{J}\frac{d\omega}{dt}$

where $\omega = angular\ velocity$

and $J = moment\ of\ inertia\ of\ the\ ball,$

and rotational force $F_{rx} = \frac{T}{R} = \frac{J}{R^2}\ddot{x}$

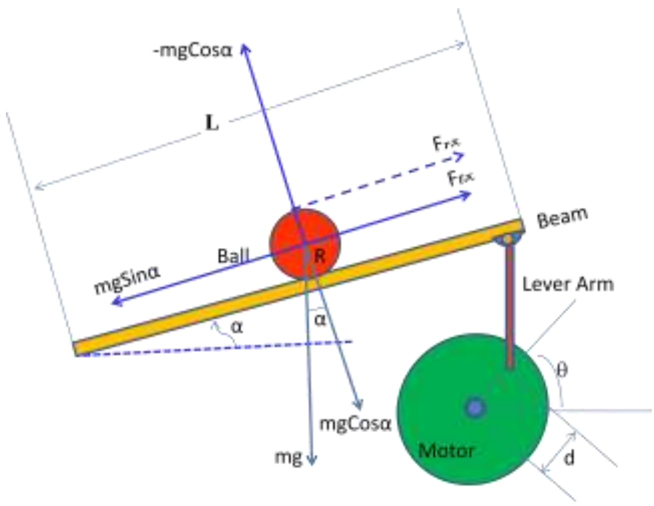

Fig.2. Ball and Beam system with Newton balances of forces

As shown in Fig.2.We have

$$F_{rx} + F_{tx} = -mgsin\alpha$$
$$\Rightarrow \left(\frac{J}{R^2} + m\right)\ddot{x} = -mgsin\alpha \quad (2)$$

Now, tilt of the beam is controlled by rotational angle θ of the gear disc connected to motor. The liver arm elevates and brings down beam as per value of θ as shown in Fig.2. In the simple way the relation between θ and α can be obtained using geometry of the lever arm section as shown in Fig.3. As is obvious from diagram of ball and beam system that the up and down motion in beam is produced due to its rotations about O_1. The said rotation of beam is produced by rotation of motor at O. The beam is connected at P to lever mechanism consisting of lever arms $SP = A$ and $OS = d$.The rotation in motor produces rotation in OS and vertical shift in arm SP which accordingly moves beam O_1P. The relation between angular displacement θ in OS and corresponding deflection in beam α is required. Let the motor arm OS reaches at T and end P of beam moves to Q such that $\angle SOT = \theta$ and $\angle PO_1Q = \alpha$. Obviously,

$$OX = d\cos\theta; TX = d\sin\theta;$$
$$XS = OS - OX = d(1 - \cos\theta)$$
$$O_1M = L\cos\alpha; QM = L\sin\alpha; MP = L(1 - \cos\alpha)$$
$$TN = A\sin\beta; QN = A\cos\beta; MN = A\cos\beta - L\sin\alpha$$
$$TX = PS - PG = MZ - MN$$

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$$\Rightarrow\ d\sin\theta = A - A\cos\beta + L\sin\alpha$$
$$\Rightarrow d\sin\theta - A(1-\cos\beta) - L\sin\alpha\ = 0$$
$$\Rightarrow d\sin\theta - A(1-\cos\beta) - L\sin\alpha\ = 0$$
$$\Rightarrow L\sin\alpha = \ d\sin\theta - A(1-\ \cos\beta)$$
$$\Rightarrow \alpha = arc\sin\left[\frac{d}{L}\sin\theta - \frac{A}{L}(1-\cos\beta)\right] \qquad (3)$$

Now

$$TN + NG = XS$$
$$\Rightarrow A\sin\beta + MP = XS$$
$$\Rightarrow A\sin\beta + (L - L\cos\alpha) = d - d\cos\theta$$
$$\Rightarrow A\ \sin\beta\ = d(1-\ \cos\theta) - L(1-\ \cos\alpha) \qquad (4)$$

The general solution for α and β is complex[10] for the given input angle θ. However, under restriction that the dependence of β on α is very weak and thus from (4) we can get

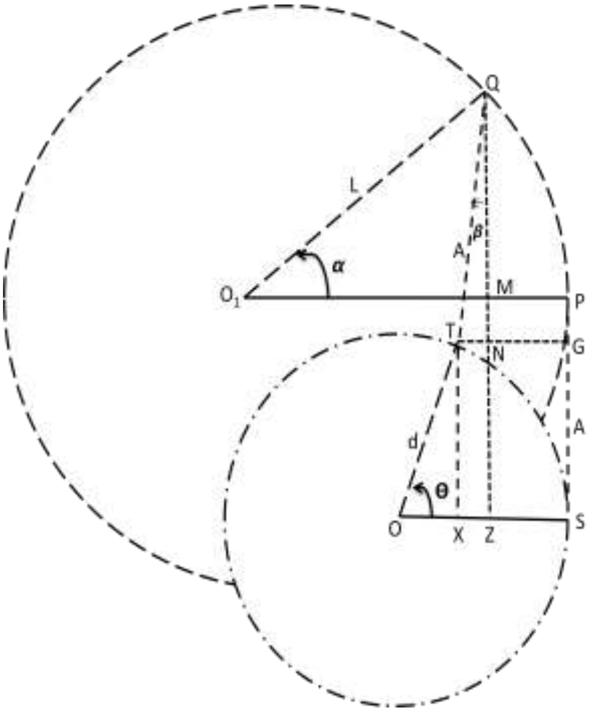

Fig.3. Relation between motor angle and beam angle

$$A\sin\beta = d(1-\cos\theta)$$
$$\Rightarrow \beta = arc\ \sin\left[\frac{d}{A}(1-\ \cos\theta)\right] \qquad (5)$$

And

$$\alpha \approx arc\ \sin\left[\frac{d}{L}\ \sin\theta\right] \qquad (6)$$
$$\alpha \approx \frac{d}{L}\theta\ , when\ \theta\ is\ small \qquad (7)$$

$$\dot{\alpha}\ =\ \frac{\frac{d}{L}\cos\theta}{\sqrt{1-\frac{d^2}{L^2}(\sin\theta)^2}}\ \dot{\theta}\quad \&\quad \dot{\beta}\ =\ \frac{\frac{d}{A}\sin\theta}{\sqrt{1-\frac{d^2}{L^2}(1-\cos\theta)^2}}\ \dot{\theta}$$

The beam angle is proportional to motor angle which in turn proportional to input voltage.
From Eq. (2) and (7), we get

$$\left(\frac{J}{R^2} + m\right)\ddot{x}\ = -mg\ \frac{d}{L}\ \theta, \qquad (\because \sin\alpha \cong \alpha)$$

Taking the Laplace transform of both sides assuming initial condition zero. We get

$$\left(\frac{J}{R^2} + m\right)s^2X(s) = -mg\ \frac{d}{L}\theta(s)$$

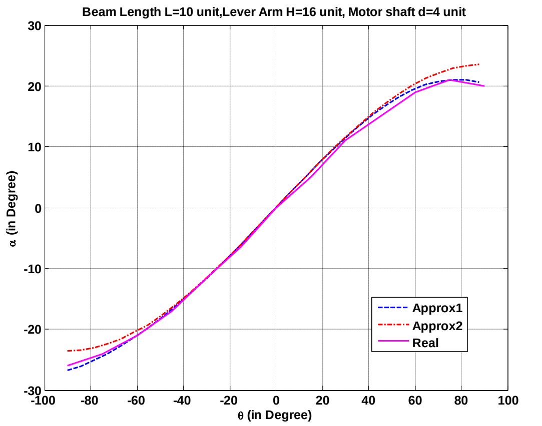

Fig.4 Relation between θ and α as per different approximated relation which shows that the relation is linear for smaller value of motor angle

$$\frac{X(s)}{\theta(s)} = G_B(s)\ =\ \frac{-mg\qquad d}{\left(\frac{J}{R^2} + m\right)\ L} \times \frac{1}{s^2} \qquad (8)$$

Obviously, transfer function is double integrator and thus provides challenging control problem. We know that the moment of inertia of ball is $J = \frac{2}{5}mR^2$. Thus the transfer function $G_B(s)$ can be finally given by

$$G_B(s) = \frac{X(S)}{\theta(S)} = \ -K\left(\frac{1}{S^2}\right) \qquad (9)$$

where constant $K = 7\frac{d}{L}$.

Since the tilt angle α of the beam is controlled by the motor angle θ and θ is controlled by the input voltage, the transfer function of motor is also essential to establish relation between θ and input voltage.The DC servomotor linked to beam pivot decides tilt of the beam which in turn controls motion of ball on the beam[9]. As discussed above the tilt angle α and angular rotation θ of the motor are related by Eq.7.The angle of rotation θ of the motor decided by the applied input voltage $E_a(s)$.The transfer function $G_m\ (s)$ of the motor is obtained as follows.

As shown in figure a DC motor is used in servo motor, we first model DC motor which may be armature controlled or field controlled. We consider here armature controlled DC motor. In the armature controlled DC motor shaft position and speed is controlled by armature current and field current is kept constant. The circuit diagram of armature controlled DC motor is shown in Fig.5.

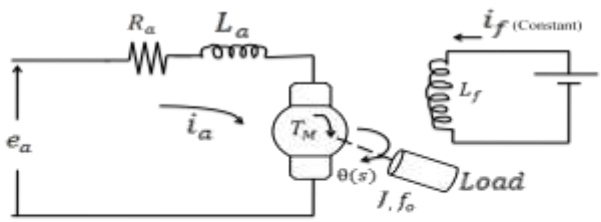

Fig.5 Armature controlled DC motor.

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Parameters shown in the Fig.5.are,

R_a = *Resistance of armature*
L_a = *Inductance of armature*
T_M = *Torque produced by motor* (N_m)
θ = *angular displacement of motor shaft*
$J = (J_m + J_L)$ =Equivalent MI of motor and Load
f_0 = *Viscous friction coefficent of motor and load acting on motor shaft*

The applied armature voltage e_a setup armature current i_a in the armature circuit the torque T_M produced by motor is proportional to airgap flux $\emptyset$ and armature current i_a and can be given by

$$T_M = k_1 \emptyset\, i_a = k_1 k_f\, i_f i_a$$
$$= k_1 k_f i_f i_a = k_1 k_f i_f i_a = k_T i_a$$

where $k_1 = constant; k_f$ = constant

k_T is known as motor constant

Application of KVL in armature circuit gives

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \qquad (10)$$

where e_b is the back emf produced by the rotation of armature in the magnetic field and is proportional to speed of the motor

$$e_b = k_b \frac{d\theta}{dt}\,, \qquad (11)$$

where k_b is the back e.m.f. constant.

The torque T_M developed by motor is used to rotate load and in fighting viscous forces.

Thus the torque equation is given by

$$J\frac{d^2\theta}{dt^2} + f_o\frac{d\theta}{dt} = T_M = k_T i_a \qquad (12)$$

Now taking Laplace transforms, assuming initial condition zero of equation (10), (11) and (12), we get

$$L_a s I_a(s) + R_a I_a(s) + E_b(s) = E_a(s)$$
$$(L_a s + R_a) I_a(s) = E_a(s) - E_b(s) \qquad (13)$$
$$E_b(s) = k_b s\, \theta(s)$$
$$(Js^2 + f_o s)\theta(s) = k_T\, I_a(s)$$

These equations can be used to represent block diagram of motor as shown in Fig.6.

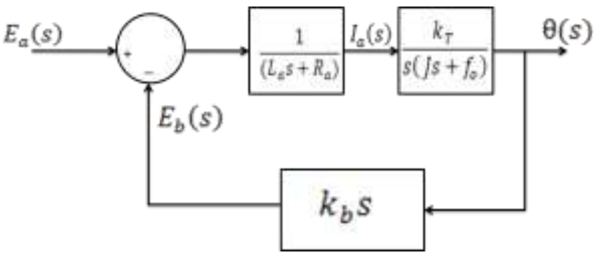

Fig.6 TF of DC motor

In Eq(12) putting $\frac{d\theta}{dt} = \omega = speed\ of\ motor$

We get

$$J\frac{d\omega}{dt} + f_o\omega = k_T i_a \qquad (14)$$

Taking Laplace transform we get

$$Js\omega(s) + f_o\omega(s) = k_T I_a(s)$$
$$(Js + f_o)\omega(s) = k_T\, I_a(s)$$
$$\therefore \omega(s) = \frac{k_T\, I_a(s)}{(Js + f_o)} \qquad (15)$$

This means position $\theta(s)$ of the shaft in above block diagram is obtained by integrating $\frac{I_a(s)k_T}{(Js+f_o)}$. Thus the block diagram of Fig.6 can be modified as shown in Fig.7.

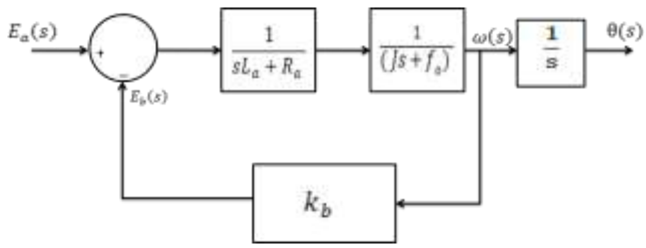

Fig.7 Modified block diagram of DC motor

It is interesting to note that that DC motor is an open loop system but the block diagram in Fig.7 shows that it keeps a built-in feedback loop for the back e.m.f. However, this feedback is taken as electric friction which is proportional to negative rate of change of position $\theta(s)$ of the motor.This feedback improves stability of the motor. The overall transfer function of the block diagram of Fig.7 is given by

$$G_m(s) = \frac{\theta(s)}{E_a(s)}$$
$$= \frac{k_T}{s^3 J L_a + (R_a J + f_o L_a)s^2 + f_o R_a s + k_T k_b s}$$

Since L_a is very small or negligible.

$$G_m(s) = \frac{k_T}{s(R_a J s + f_o R_a + k_T k_b)}$$

$$G_m(s) = \frac{k_T/R_a}{s(Js + f)}, \qquad (16)$$

where $f = f_o + \frac{k_T k_b}{R_a}$.

Appearance of term k_b in $f = f_o + \frac{k_T k_b}{R_a}$ shows that the in built feedback loop for back e.m.f. enhances viscous friction and thus known as electric friction.

$$G_m(s) = \frac{k_T/R_a f}{s\left(\frac{J}{f}s + 1\right)} = \frac{k_m}{s(\tau s + 1)}$$

(17)

where $k_m = {k_T}/{R_a f} = motor\ gain\ constant,$

$\frac{J}{f} = \tau = motor\ time\ constant.$

Further it can be shown that $k_T = k_b$ in MKS units. Also, the transfer function for the speed can be given by

$$\frac{\omega(s)}{E_a(s)} = \frac{k_m}{(\tau s + 1)} \quad (18)$$

where k_m and τ are gain and time constants depending on motor parameters such as armature resistance, back e.m.f. constant and motor load equivalent moment of inertia.

Thus the simplified block diagram of the motor is given by

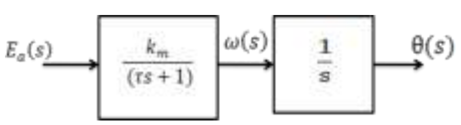

Fig.8. Simplified block diagram of DC motor

This block diagram makes it obvious that DC motor is an integrating device. In the servo system this DC motor is connected to load via gear system and a position sensor such as potentiometer is attached to the output shaft.

The overall open loop transfer function G(s) of the ball and beam system can obtained by combining motor function $G_m(s)$ and $G_B(s)$, TF of ball and beam system as shown in Fig.9.

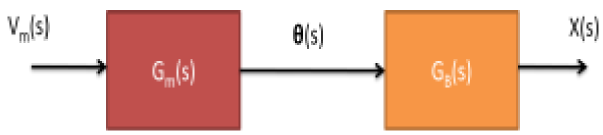

Fig.9. TF of Ball and Beam system

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System parameters used in simulation
m = 0.111 kg;R = 0.015 m;g = -9.8m/s^2;
L = 1.0h; d = 0.03m;
J = 9.99e-6 kgm^2;
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The overall transfer function G(s) of the ball and beam system is given by

$$G(s) = G_m(s).G_B(s)$$

$$= \frac{X(s)}{\theta(s)} = \frac{K_m\ K}{s^3(1+\tau s)} \quad (19)$$

III. NATURE OF TRANSFER FUNCTION

A. Beam tilt to ball positin TF $G_B(s)$

This transfer function as obtained in Eq.(7) and (8) decides position of the ball on the beam for the given tilt angle of the beam. The step response of this transfer function is shown in Fig.10 which shows that for the any given tilt of the beam the ball will roll down under gravity and go out of the beam with velocity depending upon tilt of the beam as shown in Fig.11. Thus to control and regulate motion of the ball on the beam some control mechanism is required which will change tilt of the beam in such way that will stop fall down of ball as well as regulate motion of the ball. This TF contains two poles at origin.

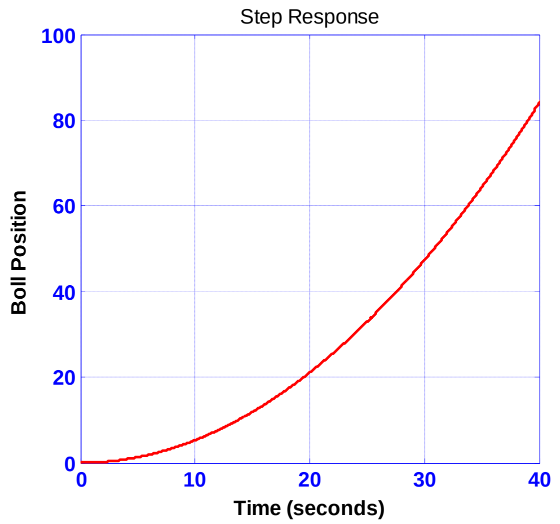

Fig.10.Step response of ball and beam system which shows that for the constant tilt angle the ball will move away from if tilt is not changed.

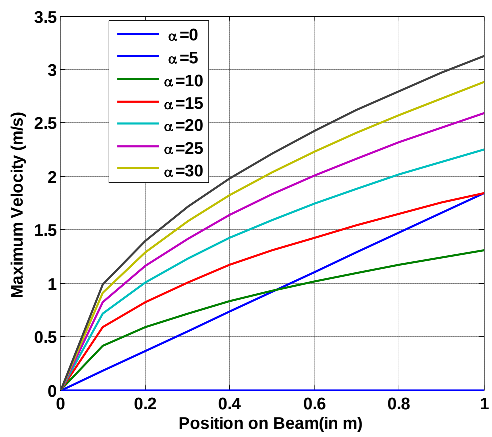

Fig.11 Velocity of ball at different positions of beam for different values of beam tilt α (in degree)

This transfer function includes relation between motor arm rotation θ and transmission of θ to beam angle α. The input voltage rotates motor and the angular displacement of motor arm is transmitted to beam rotation. Obviously this contains two parts namely (i) input voltage to motor arm rotation and motor arm to beam rotation. The relation between θ and beam angle α is also complicated and has been obtained in Eq.(3), (6) and (7). This relation is shown in Fig 12 for different d/L ratio and it is evident that for smaller range of θ, beam angle varies linearly. The value of beam tilt depends on the contact point of the lever arm with beam also. For the given setup of motor arm length and lever arm such variation in beam tilt is shown in Fig.13 as per Eq.(6). The linearity as well as range of relation reduces with increasing contact length.

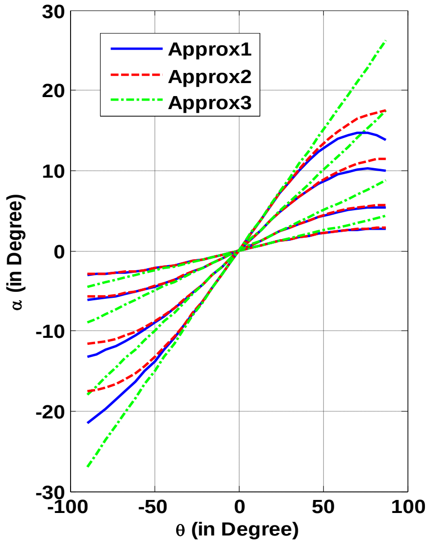

Fig.12 The relation between beam motor angle between θ and beam angle α shown by legend Approx1, Approx2 and Approx3 for Eq.(3),(6) and (7) respectively.

The dependence of β on motor angle θ for different values of motor arm *d* and lever arm *A*, as is obtained in Eq.(5) is shown in Fig.14. The practical values for the same setup obtained by making diagram as shown in Fig.3 were measured and are plotted in Fig.15 which shows agreement with Eq.(5).

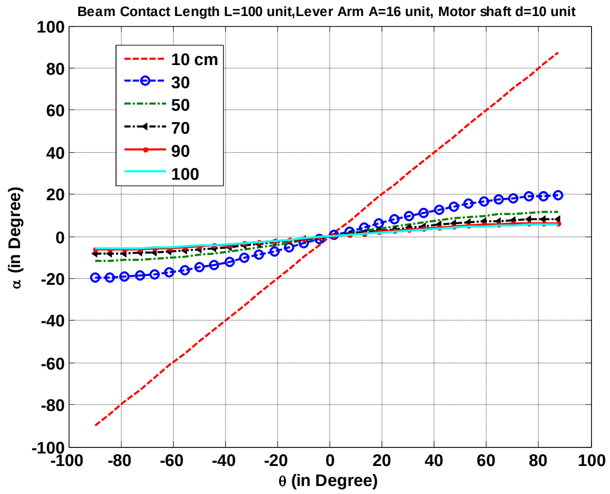

Fig.13 Variation in beam tilt α with variation in motor angle θ for different contact point of lever arm on beam.

It is obvious from these graphs that the dependence of β and motor angle θ can be ignored for ranges up to -30 t0o 30 degrees. The tilt in beam angle depends on point contact with lever arm but contact near the pivot O of the beam may introduce unwanted vibration in the beam. The ration of length of motor arm and beam length plays important role in deciding these relations. The graph shown in Fig 14 in red line presents approximation done by Eq.(7) for the relation between beam angle and motor angle. For the approximation of these angular relation in [10] authors have presented nonlinear relations taking into consideration three parameters namely beam length, lever arm length and motor arm length.

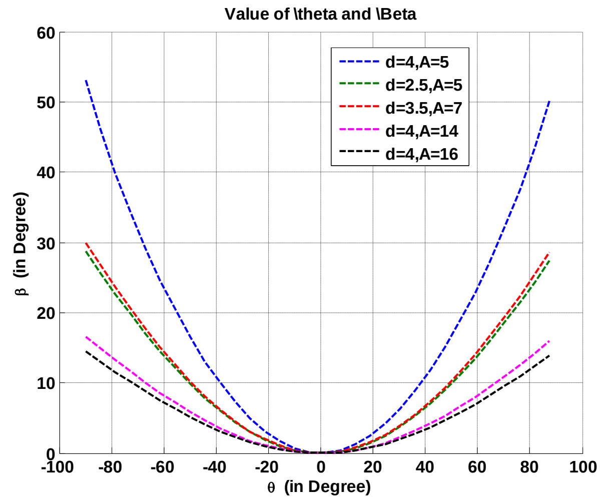

Fig14 Dependence of β on motor angle θ for different values of motor arm *d* and lever arm *A*.

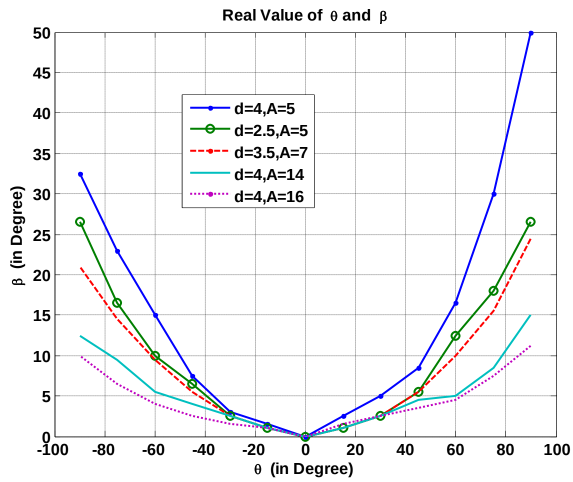

Fig.15 Relation between of β and motor angle θ measured for different values of motor arm *d* and lever arm *A*.

B. Motor transfer function $G_m(s)$

The motor transfer function as given by Eq.(17) and (18)

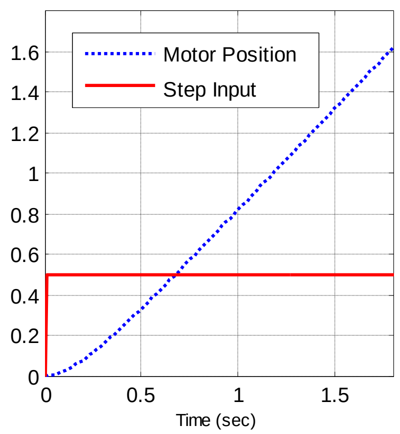

Fig.16 step response of motor (open loop)

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represents a system which requires control to direction of rotation. For the given constant input the motor shaft position is shown in Fig 16. .In order to control motor PD control can be used. The motor transfer function $G_m(s)$ represents 2nd order system and PD control will preserve its order. In the PD controller the control signal $C_1(s)$ is given by

$$C_1(s) = K_p + K_d S$$

$$\text{where } K_p = \text{Proportiooonal gain}$$

$$K_d = \text{Derivative gain}$$

These PD parameters are tuned to give optimal performance. The step response of PD controller is shown in Fig.18

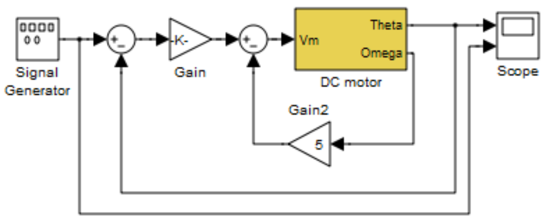

Fig.17 SIMULINK Model of DC motor control.

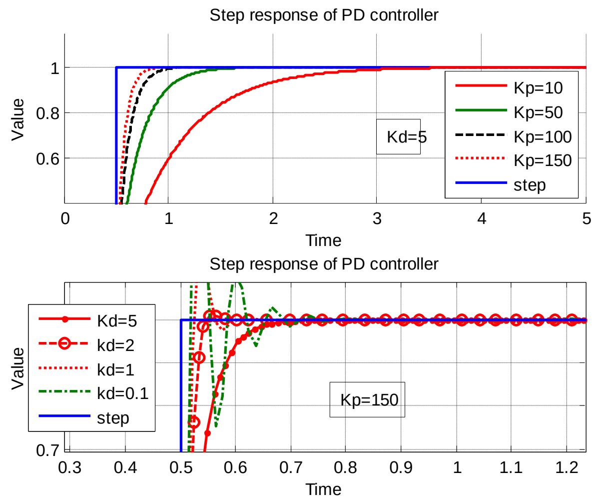

Fig.18 Step response of PD controlled DC motor.

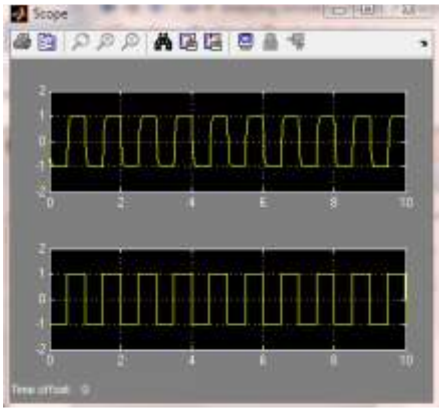

Fig.19 Tracking of input signal pattern by PD controlled motor obtained by simulation in SIMULINK.

C. Ball and beam system transfer function $G(s)$

The overall transfer function as obtained above in Eq.(19) of the ball beam system represents 4th order system. The open loop behavior of the ball and beam system, as shown in Fig. 10 indicates that with no control on the tilt of beam, the beam allows ball to move rapidly over it to fall out. The close examination of ball beam system from control point of view shows that for the controlled movement of ball on beam, beam angle needs to be changed according to position of the ball so that ball can be stopped from falling down. The beam angle α is decided by the motor angle θ through lever arms. The amount of rotation in motor is governed by the input voltage v_m supplied to motor. The functional relation for control of ball beam system is shown in Fig.20. Obviously, the equivalent block diagram contains TF of fourth order and design of the controller for the same is very difficult. However ball beam system can be controlled by two controllers one for motor control and other for the beam angle control per ball position.

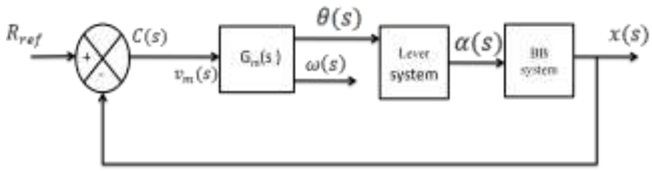

Fig.20 Control of Ball Beam System

The step response of ball and beam system using PD control is shown in Fig 21 for different values of Kp and Kd.

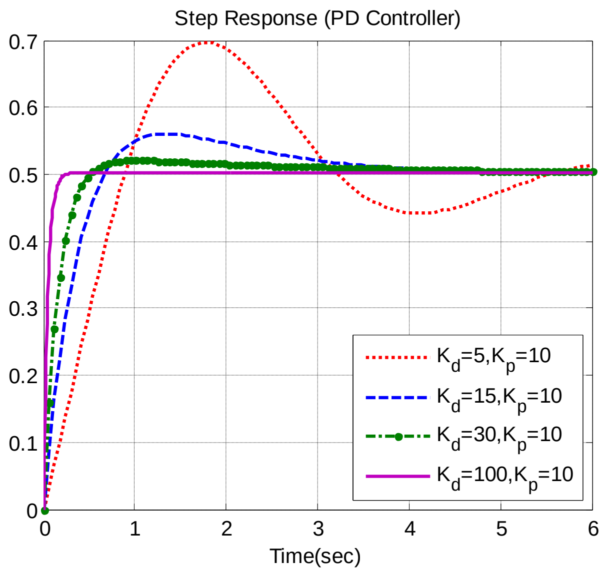

Fig.21 Step response of PD controlled Ball beam system.

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IV. Conclusions

In this paper dynamics and control of ball and beam system has been presented. The system function for different subsystems have been obtained and detailed analysis on relation between motor and beam angle has been presented with theoretical and experimental results. PD control has been used to control the system and result obtained by simulation in SIMULINK have been also presented.

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