

# Improved Direction of Arrival Estimation using Multiple Signal Classification (MUSIC) Algorithm with Decomposition and Normalization

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**Abstract**—It is important to determine the direction of arrival (DoA) of targets in various applications such as radar and sonar. Multiple Signal Classification (MUSIC), Estimation of Signal Parameters with Rotational Invariance Technique (ESPRIT), and Weighted Subspace Fitting (WSF) are subspace-based methods that can be used to improve DoA estimation. MUSIC is effective for high-resolution, uncorrelated signals, but may struggle in cases where there are two nearby targets with a low signal-to-noise ratio (SNR). The goal of this research is to improve the performance of the MUSIC algorithm for DoA estimation with low SNR signals. The proposed solution involves decomposing and normalizing the signal during transmission. Simulations were conducted to test the modified procedure with MUSIC algorithm for DoA estimation, and it was found that received signal power improved though there is noisy environment as well as system can detect more number of targets. The proposed technique of decomposition and normalization could also be applied in other areas such as WiFi communication, autonomous vehicles and biomedical signal and image processing etc.

**Keywords**- MUSIC, DoA, Decomposition, Normalization, SNR.

## I. INTRODUCTION

In the field of signal processing and communication, array signal processing is a very important task. It can be used in RADAR, SONAR and other defense applications [1]. In array signal processing, DoA estimation is the most challenging task. DoA estimation refers to, how closely spaced targets can be detected individually. DoA estimation is a direct measure of resolution. Recently, many technologies were developed to improve DOA estimation. These DoA estimation technologies involve non-coherent and coherent signals, multiple signal sources or targets, the use of different types of array structures, [2]. Multiple algorithms were also developed, such as MUSIC, ESPRIT and WSF. These algorithms are sub-spaced based and proven to be a great contribution in retrieving the information of targets from the received noisy array signal. The most efficient algorithm among all these is the MUSIC algorithm improves estimation accuracy with respect to all dimensions. Hence this work focuses only on MUSIC algorithm.

A spatial spectrum estimation algorithm is the MUSIC algorithm. The main logic behind the MUSIC algorithm is to separate the signal from noise by Eigen Value Decomposition (EVD)[3]. EVD is applied to the covariance matrix of the received signal. The signal subspace and the noise subspace are

created, by using the orthogonality property. Then spatial power spectrum of these subspaces was generated. Finally, DoA estimation is done by searching the peak of the spectrum. For MUSIC algorithm to work, there are certain assumptions. These assumptions are, the noise power is equal at all sensors and that the noise power is uncorrelated between the sensors. The MUSIC algorithm cannot work properly when more correlated signals impinge on an array[4]. The correlation between the signals increases day by day due to the multi-path environment, multi-path fading, cross-interference, and high data rates. Hence, more research is required in this area, and there is a need to improve the existing MUSIC algorithm. Many attempts have been made to modify the MUSIC algorithms to eliminate the correlation between the signals and improve DoA. This paper also focuses on one such improvement in the MUSIC algorithm, which can detect closely spaced targets in a noisy environment with great accuracy.

## II. LITERATURE REVIEW

In the literature, DoA estimation with Uniform Linear Array (ULA) geometry is carried out using algorithms like multiple signal classification (MUSIC)[5], subspace fitting[7], estimation of signal parameters using rotational invariance

techniques (ESPRIT)[8], etc. For this study, we used the MUSIC algorithm. The choice of the MUSIC algorithm was due to the high-resolution direction-finding ability of this technique, which can distinguish between closely spaced signal sources. The basic workings of the MUSIC algorithm was explained way back in the 1980's[5]. Schmidt showed that, the eigenstructure matrix which is obtained from the received signal plays a major role in DoA estimation. Schmidt also, explored the orthogonality between signal and noise subspace.

In multi-input multi-output (MIMO) arrays, for DoA estimation, the MUSIC algorithm is used and performance is evaluated by using parameters like the number of the snapshots and array elements, and the SNR values [8]. The Music algorithm was facing issues when applied to non-coherent sub-arrays, which was resolved using the weighted MUSIC (w-Music)algorithm [9]. There is a high demand to improve DoA estimation from very noisy data of received signal data. Some modified MUSIC algorithms have already been developed by considering different conditions. It is well known that in MUSIC, whole array observations are utilized to construct the noise subspace. [10] Randomly chosen sensor output to construct the noise subspace and developed a low-complexity modified MUSIC version.

If there are nearby targets in the air, then the coherency of the received signal increases. So to increase the resolution of coherent signals [11] proposed spatial smoothing before the application of MUSIC to the received signal. They found that the number of targets in the air can increase due to this. For improving DoA, conventional MUSIC is updated to the Root-MUSIC algorithm [12], where complex subspace decomposition on the entire array covariance matrix took place. Yan and his group [12] have decomposed the subspace either in the real part of the array covariance matrix or in the imaginary part of the covariance matrix for DoA estimation. Hence, the computational cost is reduced as compared to Root-MUSIC. Their group also found that instead of considering the real and imaginary part of the covariance matrix, singular value decomposition can retrieve better real-valued noise subspace[13]. Additionally, this method improved root mean square errors (RMSEs) at the Cramér-Rao Lower Bound (CRLB).

Similarly, in [14], after normalization of the covariance matrix, the author used only real values of the signal to reduce the complex computations. The Normalization reduces the resources required [14]. This methodology estimates DoA with great accuracy (low RMSE) from a very noisy environment. In the above paper, SNR is from -5db to 30db considered.

So the implementation of the MUSIC algorithm with low SNR (means below -5db) and a less number of snapshots is a great challenge. Overall, the above papers considered the decomposition and normalization of the signal at the receiver side. In this paper a decomposition and normalization of signal implemented at transmitter side and MUSIC algorithm evaluate the DoA at the receiver side. This proposed approach gives the DoA with great accuracy in noisy environment.

### III. METHODOLOGY

The block diagram of the proposed mechanism is shown in Fig. 1, after generating a signal from the signal generator, a signal matrix can be decomposed by separating the real and imaginary parts of the signal. The next step is to calculate the maximum real value. To normalize the values we divided each member of the real value matrix by the maximum value computed. The same procedure is then applied to the imaginary values of the signal matrix. These normalization increase the overall signal strength during transmission. All these normalized real and imaginary value matrices are combined together by a signal combiner. The generated output signal of a signal combiner was transmitted by a MIMO array. The overall procedure is known as decomposition and normalization. Different array steering matrices can be created depending on the different configurations of an array[15]. Given that the sensors are organized in a uniform linear structure, the signal reflected by the target travels a distance at a speed of  $c$  and arrives at the last sensor with a delayed version of the primary signal, as in the case of ULA. As a result, the received data is made up of both steering and noise matrices. To extract DoA information from noisy data, the Spatial Covariance Matrix (SCM) is introduced. The spatial covariance matrix is represented by the signal plus noise received by an array. We proposed to use the MUSIC algorithm for SCM, which is a subspace-based technique. For the MUSIC algorithm, the first step is the space spanned by its eigenvectors, which can be partitioned into two orthogonal subspaces. The first is the signal subspace, and the other is the noise subspace. This process is known as eigenvalue decomposition on the data covariance matrix. Then, by using the peak searching process or by searching all possible steering vectors, which are perpendicular to the noise subspace, DoA is calculated. The MUSIC spectrum is computed, and the largest peaks found were used to determine the DoA. This overall simulation was done in MATLAB.

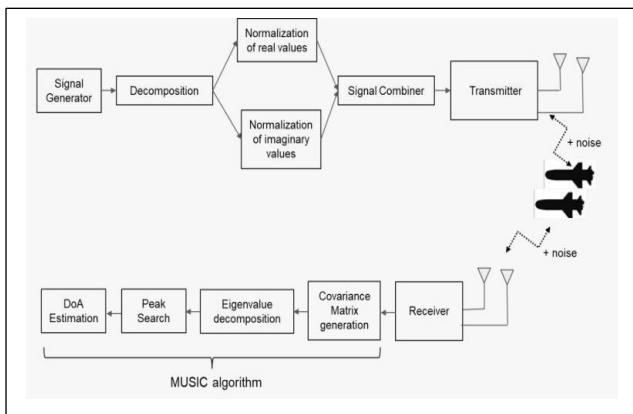


Fig.1 Block Diagram of proposed methodology showing decomposition and normalization at transmitter-side.

#### IV MATHEMATICAL MODEL

The transmitter side signal at the output of the generator, is given by equation 1.

$$S = Vj(x + iy) \dots\dots\dots(1)$$

Where,

S = Signal at the transmitter output before decomposition

Vj = Gain tuning matrix determined by SNR

(mentioned in equation 2)

x+iy= the signal with magnitude (x) and phase (y).

i =  $\sqrt{-1}$  which is an imaginary term

$$V_j = \text{diag}(\sqrt{10^{(SNR/10)}/2}) \dots\dots\dots(2)$$

Now from complex value, we have calculated real and imaginary values as follows,

The real part of equation 1 is given by,

$$S_{Re}(i) = \sqrt{(V_j \cdot x)^2 + (V_j \cdot y)^2} \dots\dots\dots(3)$$

And imaginary value of equation 1 is given by,

$$S_{im}(i) = \tan^{-1}\left(\frac{yV_j}{xV_j}\right) \dots\dots\dots(4)$$

Now consider, maximum value of real value as,

$$S_{MRe} = \max(S_{Re}(i)) \dots\dots\dots(5)$$

and maximum value of imaginary value can be written as,

$$S_{Mim} = \max(S_{im}(i)) \dots\dots\dots(6)$$

Normalized value of real numbers can be written by,

$$I_{SRe} = \frac{S_{Re}(i)}{S_{MRe}} \dots\dots\dots(7)$$

and

normalized value of imaginary number can be given by,

$$I_{Sim} = \frac{S_{im}(i)}{S_{Mim}} \dots\dots\dots(8)$$

After normalization of both real and imaginary part of signal, were combined and formed a new signal. This new signal can be denoted as Snew.

$$S_{new} = I_{sRe} + jI_{sim} \dots\dots(9)$$

Now, take ratio of S and Snew then we can say that if ratio is less than 1, then normalized signal power is better than actual power of signal. Therefore we can write,

$$\frac{S}{S_{new}} < 1 \quad \text{if and only if we consider real values only.}$$

This can proved as follows:

$$\alpha = S/S_{new} \dots\dots\dots(10)$$

Now put the values of S and Snew in equation 10,

we get,

$$\alpha = \frac{V_jx + V_jy}{I_{sRe} + jI_{sim}} \dots\dots\dots(11)$$

$$\alpha = \frac{V_jx + V_jy}{\frac{S_{Re}(i)}{S_{MRe}} + j\frac{S_{im}(i)}{S_{Mim}}}$$

$$\alpha = \frac{V_jx + jV_jy}{\frac{\sqrt{V_j^2x^2 + V_j^2y^2}S_{Mim} + jS_{MRe}\tan^{-1}\frac{V_jy}{V_jx}}{S_{MRe} \cdot S_{Mim}}}$$

$$\alpha = \frac{(x + jy)S_{Mim} \cdot S_{MRe}}{(\sqrt{x^2 + y^2})S_{Mim} + j\frac{S_{MRe}}{V_j} \cdot \tan^{-1}\frac{y}{x}}$$

After substitutions and simplification of above equation we got,

$$\alpha = \frac{x + jy}{(x^2 + y^2) + j\tan^{-1}\left(\frac{y}{x}\right)} \dots\dots\dots(12)$$

In equation 12, since Pmusic only considers real part,

then y=0, and hence, we get

$$\alpha = 1/x = S/S_{new}$$

from above term, we can say that S/Snew > 1 (assuming x > 1), which means that signal strength after decomposition and normalization get increased by keeping imaginary term zero or minimum.



V RESULTS

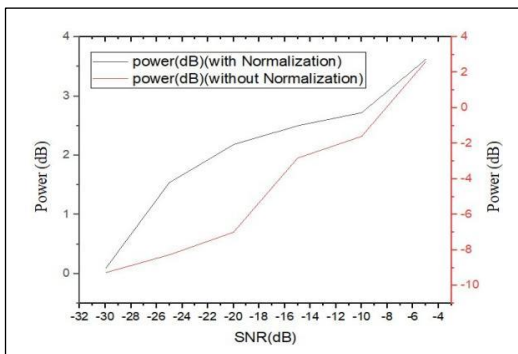


Fig 2 Effect of SNR on received power

Figure 2 shows the effect of PSNR on the proposed system in terms of the power received in dB. It can be seen that when SNR increases, the received power signal increases. The graph obtained with the proposed method (with Normalization) is found to be significantly better than the traditional one (without Normalization). For the observing nature of the graphs, two Y axis are separated and it can be seen that both graphs approximately follow the same trend line (Slope of 45 degrees). It can also be concluded from this graph is normalization increases received power and it is the frequency and SNR sensitive. The overall signal improvement was 16 % due to decomposition and normalization at the transmitter end before the MUSIC algorithm can be applied at the received side. Hence proposed modified scheme is termed as the modified MUSIC algorithm.

Figure 3 shows how SNR affects the performance of DoA estimation at two different SNRs. Fig. 3a shows the DoA estimation plot without Normalization with SNR 20dB and Fig 3b shows the output with Normalization. Fig 3c shows output without Normalization at SNR of -20 dB Fig 3d shows results with Normalization. It can be clearly seen that the results of the proposed mechanism are unaffected by the SNR value. on the other hand, the better-performing traditional algorithm fails at lower SNR of -20dB.

Figure 4 illustrates the impact of normalization on angular resolution and root-mean-square error (RMSE) in a radar system. The x-axis represents angular resolution and the y-axis represents RMSE. The graph shows two lines, one representing the results with normalization and the other without normalization.

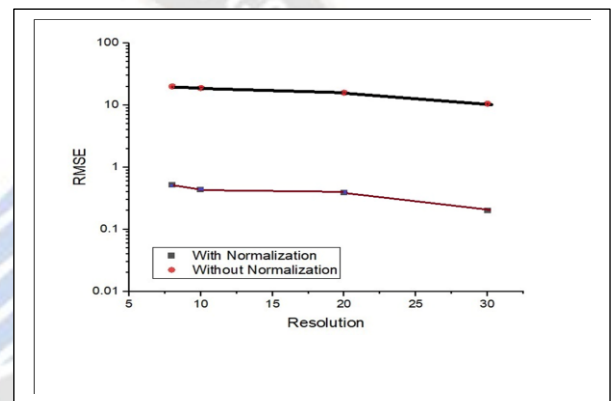


Fig.4 the relationship between angular resolution and root-mean-square error (RMSE) with and without normalization

It is clear from the graph that normalization leads to a significant reduction in RMSE, with the line representing the results with normalization being at least 10 times lower than the line representing the results without normalization, for the same signal-to-noise ratio. This indicates that normalization greatly improves the performance of the radar system in terms of its ability to accurately detect and track targets. Additionally, the graph also illustrates that normalization improves angular resolution. As the angular separation between the two targets increases, the RMSE decreases in a non-linear fashion. This suggests that as the radar system is able to distinguish between targets that are further apart, the accuracy of the system improves even more. Overall Figure 4 demonstrates the importance of normalization in improving the performance of a radar system in terms of both angular resolution and RMSE.

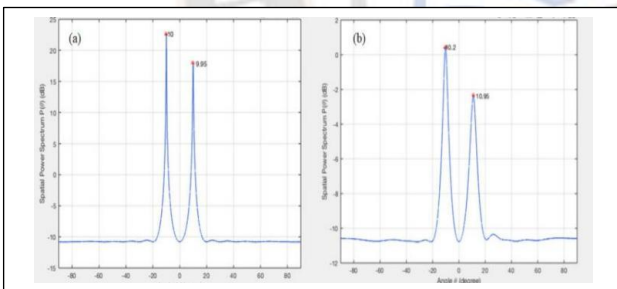


Fig.3 DoA estimation plot (Test case 20) (a) Without Normalization with SNR 20 dB (b)With Normalization with SNR 20 dB

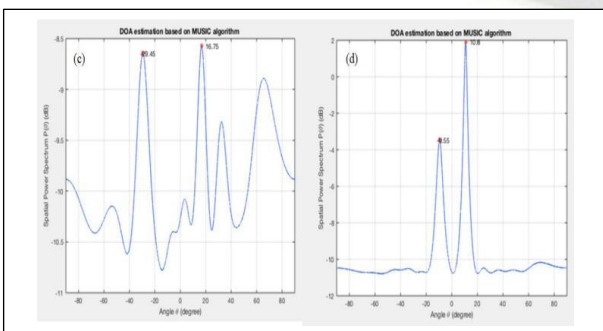


Fig.3 DoA estimation plot (Test case 20) (c) Without Normalization with SNR -20 dB (d)With Normalization with SNR -20 dB

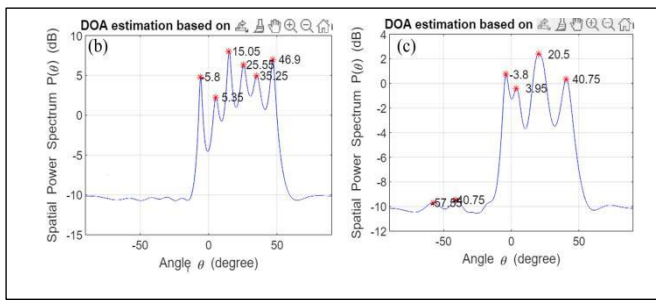


Fig.5 number of target detection with normalization and without normalization

The figure is composed of two subplots (a), (b) each showing different aspects of the effect of normalization on target detection. Subplot (a) and (b) shows the number of targets detected with normalization and without normalization respectively, with an SNR of -10 dB, using 100 snapshots. It is clear that normalization allows for the proper detection of all 6 targets which are at 5°, 10°, 15°, 25°, 35° and 45° while without normalization only 4 targets were detected. This demonstrates the superiority of normalization in detecting targets in low SNR conditions.

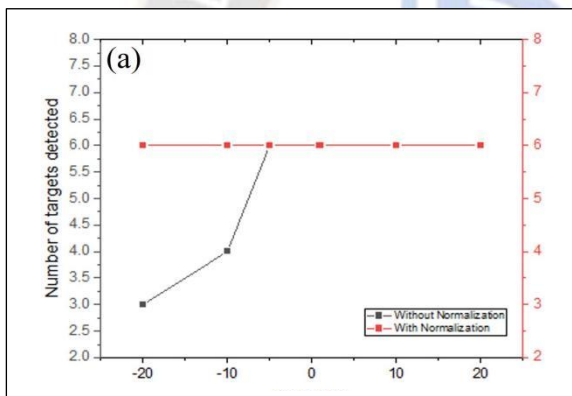


Fig.6 the relationship between signal-to-noise ratio (SNR) and the number of targets detected

In Fig.6, the relationship between signal-to-noise ratio (SNR) and the number of targets detected is shown. The plot shows two lines, one representing results with normalization and the other without normalization. It can be observed that with normalization, the number of detected targets remains unchanged regardless of the SNR value. However, without normalization, the number of detected targets decreases as the SNR drops below -10 dB. This suggests that normalization is more effective at improving resolution at lower SNR values, allowing the radar system to detect more targets even in noisy environments.

Overall, Figure 5 and Figure 6 provide a comprehensive view of the positive impact of normalization on target detection in a radar system. It illustrates the ability of normalization to improve resolution in low SNR conditions by maintaining the

number of targets detected, as well as its ability to detect all targets in a given scenario.

TABLE I

COMPARISON OF THE PROPOSED METHOD WITH THE DIFFERENT METHODS REPORTED IN THE LITERATURE WITH RESPECT TO ANGULAR RESOLUTION BETWEEN TWO TARGETS, RMSE, AND THE NUMBER OF TARGETS SIMULTANEOUSLY DETECTED BY THE SYSTEM.

METHOD	RESOLUTION IN DEGREES	RMSE	NO.OF TARGETS
Ganage [16]	5°	0.08	2
Yan [12]	2°	0.3	2
Liu [10]	8°	0.6	2
Dakilagi [17].	10°	0.3	3
Proposed Method	8°	0.2	6

Table 1 shows the comparison of the proposed method with the different methods reported in the literature with respect to angular resolution between two targets, RMSE, and the number of targets simultaneously detected by the system. It can be seen that for the resolution between the two targets, Yan et al. have achieved a maximum resolution of 2° whereas the proposed system could achieve up to 8°. The resolution of 8° was limited due to operation at low SNR of -30 dB whereas Yan et al. could go up to -10 dB. Also, we used a maximum of 12 sensor elements whereas Yan et al. can go up to 80 sensors. For RMSE again our proposed algorithm can go up to 0.2 the second best reported in the literature after Ganage et al. Their method used 200 snapshots compared to our 100. Also, their SNR was -8 dB whereas our method can work up to -30 dB. While computing the number of targets simultaneously detected we could go up to 6 targets which was the highest compared to most reported with only 2 targets.

## VI CONCLUSION

In this study, we sought to enhance the Multiple Signal Classification (MUSIC) algorithm's effectiveness for estimating direction of arrival (DoA) for signals with low signal-to-noise ratios (SNR). Radar and sonar are two common applications for subspace-based technique like MUSIC, However, MUSIC can struggle in cases where there are two nearby targets with a low SNR. To address this issue, we proposed a technique of decomposing and normalizing the signal during transmission. Through simulations, it was found that the modified procedure with MUSIC algorithm estimate DoA for closely spaced targets with good resolution. Also, due to normalization, radar can detect nearby 6 targets effectively with good signal power. The proposed technique is very useful to determine the direction of arrival of targets in various applications, like Wi-Fi, biomedical signal and image processing, and in autonomous vehicles.

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