

A Novel Approach for Quaternion Algebra Based JSEG Color Texture Segmentation

Bharat Tripathi¹, Nidhi Srivastava², Amod Kumar Tiwari³

¹Amity Institute of Information Technology, Amity University
Uttar Pradesh, Lucknow Campus, Lucknow, India
bharat.tripathi1985@gmail.com

²Amity Institute of Information Technology, Amity University
Uttar Pradesh, Lucknow Campus, Lucknow, India
nsrivastava2@lko.amity.edu

³Department Of Computer Science and Engineering
Rajkiya Engineering College Sonbhadra, India
amodtiwari@gmail.com

Abstract— In this work, a novel colour quantization approach has been applied to the JSEG colour texture segmentation using quaternion algebra. As a rule, the fundamental vectors of the colour space are derived by inverting the three RGB colour directions in the complex hyperplanes. In the proposed system, colour is represented as a quaternion because quaternion algebra provides a very intuitive means of working with homogeneous coordinates. This representation views a colour pixel as a point in the three-dimensional space. A novel quantization approach that makes use of projective geometry and level set methods has been produced as a consequence of the suggested model. The JSEG colour texture segmentation will use this technique. The new colour quantization approach utilises the binary quaternion moment preserving thresholding methodology, and is therefore a splintering clustering method. This method is used to segment the colour clusters found inside the RGB cube and the colour consistency throughout the spectrum and in the space are both considered. The results of the segmentation are compared with JSEG as well as with the most recent standard segmentation techniques. These comparisons show that the suggested quantization technique makes JSEG segmentation more robust.

Keywords: Quaternion, JSEG, Image Segmentation, Image Quantization

I. INTRODUCTION

Here we use a novel colour quantization method to JSEG colour texture segmentation. We reduced the number of dimensions of the colour space with the use of level set functions and Binary Quaternion Moment Preserving (BQMP) thresholding approach [1-3]. When using the BQMP thresholding method, the spectral bands are taken into account as a whole, such that each pixel in a given colour picture is treated as a point in a three-dimensional cube.

The suggested approach uses histogram equalisation to obtain a colour dataset with a normal distribution. The authors' concept presented in [4, 22, 25] is the basis for the suggested technique. The proposed method utilises the level set function [5, 8] and updates the level set function from the binary picture by means of the Heaviside function, as opposed to repeatedly slicing the polygon along a line and saving the local data to a file [6]. The tedious and resource-heavy polygon splitting process has been done away with, and the binary image is now being kept up-to-date through partitions in the level set routines [7]. In order to speed up the calculation, the level set functions are updated using

elementary arithmetic operations [8]. Because the suggested method views the binary image as partitions, it produces a binary tree with fewer levels and a smaller number of final clusters [9-11]. At last, a colour map is built with a limited colour scheme that helps keep everything looking straight.

This novel colour quantization approach is used in the JSEG (J measure based SEGmentation) algorithm to produce a quantized picture [31-33]. In JSEG, a nonlinear filtering approach is used to generate a quantized class map, which was then used in conjunction with a spatial segmentation algorithm [12, 33]. The spatial segmentation process included the use of a homogeneity measure, J, to generate a J-image that would reflect the interiors and bounds of the segments. We then used the J-image to run an arbitrary class region growth and region merging method to create our segments [14, 30].

The first step of the JSEG segmentation algorithm is the primary focus of the proposed segmentation framework. In this second phase, we make a few minor adjustments to strengthen the segmentation in which the best number of clusters has been determined by computing the dispersion measures within-cluster and between-cluster [13].

Recursively, the clusters with the shortest distance between them are merged into larger ones.

II. BACKGROUND AND RELATED WORK

Many scholars were drawn to JSEG since it is one of the most widely used and intuitive segmentation methods [14, 26]. JSEG used a spatial segmentation technique on a quantized classification map that is created using a nonlinear filtering approach. In spatial segmentation, a homogeneity measure, J was used to generate a J -image depicting the interiors and borders of the segments [34]. The J -image was then used as input to an arbitrary class region-growth and region-merging method [32] to carry out the segmentation. Several changes and comparisons have been made to JSEG in the previous decade due to the author's candid admission of the limitations of JSEG and suggestions for future development [15, 18, 27]. The first step of JSEG involves quantization, while the second level involves spatial segmentation. From a colour quantization outcome perspective, both [31] and [32] improved JSEG. In [9] authors created a new method called HSEG by modifying JSEG and including H measure into its spatial segmentation phase [28, 15].

In [33] authors demonstrated that H measure, like other gradient operators, is very sensitive to noise. In addition, they proposed and called a new measure J (B-JSEG) that incorporates directional operators into J measure. Since the colour quantization technique (based on vector quantization) significantly influences the final segmentation results of JSEG, while authors of [34] argued that a more robust colour quantization approach is necessary to further enhance the resilience of JSEG.

For the purposes of colour image processing, such as colour image compression, multi-class clustering of colour data, and subpixel colour-edge detection. In [28] authors have developed quaternion moment based operators using BQMP thresholding. They have also shown that the BQMP thresholding approach, a two-class classifier, produces results that are comparable to those of the best Bayes classifier for data sets with a normal distribution.

Authors in [29] developed the BSP tree, and each leaf node stands in for a polygonal area. They used the BQMP thresholding approach to convert the colour picture to a binary format, and then used the best-fit criteria to establish the partition lines inside the binary image. It was then possible to divide the area along the acquired partition lines. The BSP tree approach has the drawback of using line quantization, which lowers the system's accuracy, and requiring a large number of geometric operations and powerful calculations to find the optimal dividing line and

divide the polygon. Accuracy improves as the BSP tree becomes deeper. That is why you have to compromise one for the other: speed or precision [16, 17, 19-20].

III. PROPOSED APPROACH

Ordinarily, in colour image processing, pixel colours are treated as vectors in a physical, Euclid colour space [21-25]. It is common practise to use the three RGB components as the basic vectors of the colour space. Due to the inherent convenience of quaternion algebra for working with homogeneous coordinates, the suggested system makes use of a colour quaternion representation that views a colour pixel as a point in a 3D cube. Important to this quantization process is that the RGB cube's colour pixels be evenly distributed. However, in the RGB space of pictures of the natural world, the R , G , and B components are closely connected. Since the complete range of colours that humans can see is not supported by the RGB colour mode, the RGB colour space also has the disadvantage of being non-uniform and difficult to visualise. It is thus challenging to judge the observed differences in colours based on distances. In order to fix this issue, a histogram equalisation approach is used to the initial RGB picture.

Representation of a colour image using Quaternions

Considering the importance of local correlation, we figured out for a formula that would allow us to generate a complete quaternion matrix out of six 2D real matrices that are strongly linked and so serve as a representation of a colour picture. In order to do this, we took into account an adjoining pair of pixels situated at the coordinates (i, j) and $(i, j + 1)$, where i and j stand for the position of a row and column of pixels, respectively. Since, we are aware that each pixel in the RGB space has three values in colour space, we can conclude that the pair of pixels situated at (i, j) and $(i, j + 1)$ will have a total of six values. In order to construct a model that turns each pair of pixels into four integers, which can form a complete quaternion, we employed a fully connected feedforward auto-encoder to determine values and biases for all these six subpixels. Because each pair of neighbouring pixels in RGB colour space at coordinates (i, j) and $(i, j + 1)$ is turned into one quaternion integer, we used this model to create a quaternion matrix with the same number of rows as the original picture but having half number of columns.

Histogram Equalization

Image quality can be enhanced for human viewing by using the histogram equalisation approach, which is categorized as global contrast modification. The literature in [14] examines

more advanced histogram equalisation methods. Our proposed setup also uses a colour histogram-equalisation technique to adjust the brightness and hue of the colours in the picture. First, a nonlinear HSI colour space is created from the RGB colour space. It is assumed that the HSI colour pixel is a uniformly distributed random vector $\vec{\vartheta} = (\vartheta_H, \vartheta_S, \vartheta_I)^T$ where $\vartheta_H, \vartheta_S, \vartheta_I$ are random vectors that are modelled for hue, saturation and intensity respectively. Hue is the most important aspect of colour, and changing it leads to unintended colour artefacts [15], hence it is kept unchanged in the proposed method. Using grayscale histogram equalisation, the intensity and saturation of each

pixel are transformed into a uniformly distributed random vector so that they may be better analysed.

$$P[\vartheta \leq \vartheta_n] = \sum_{i=0}^n f(\vartheta_i) \quad \forall i = 0, 1, \dots, G - 1 \quad (1)$$

Since we are working with 8-bit pictures, the maximum number of grayscales (G) is 256. In equation 1 the function $f(\vartheta_i)$ is the probability density function of ϑ . After the histogram equalization in the HSI colour space, the picture is converted to RGB colour space that depicts uncorrelation.

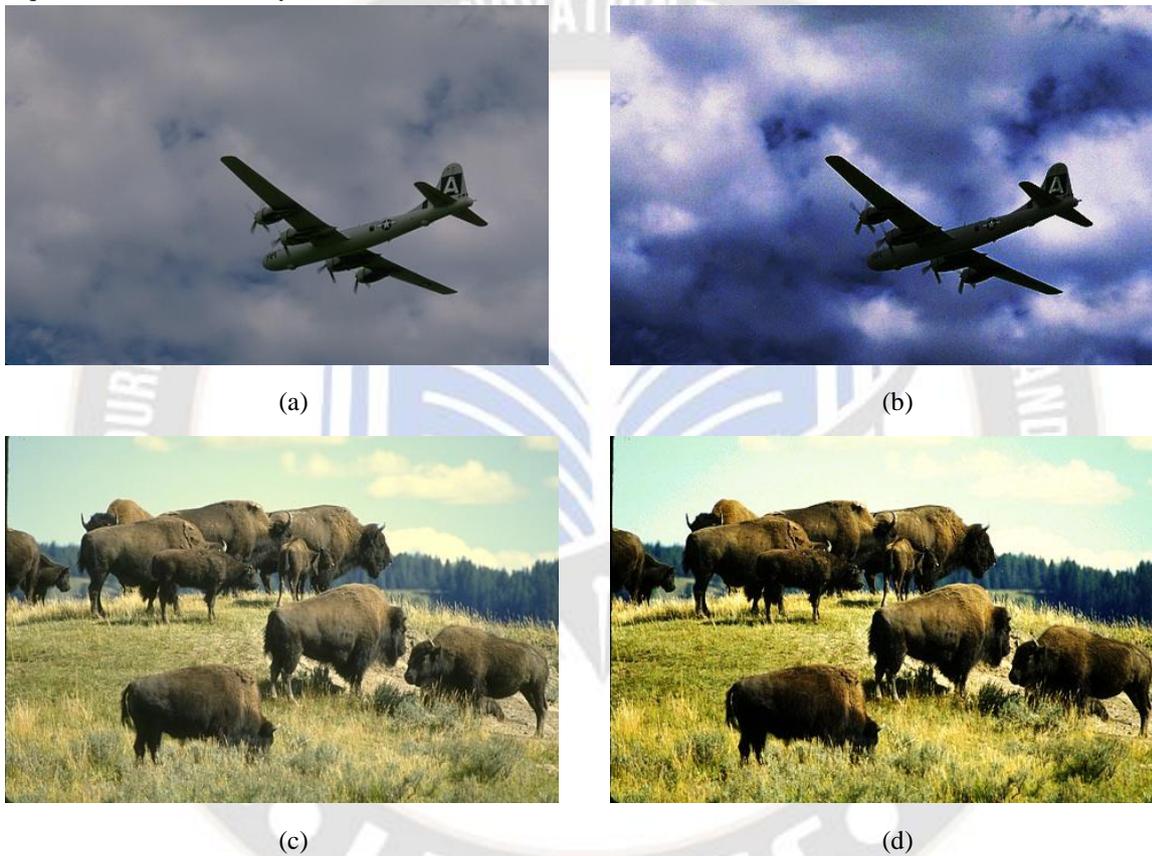


Figure 1: (a) and (c) are the original test images from the BSDS300 dataset of Berkeley where (b) and (d) are histogram equalized images.

Implementing the Multiclass Clustering Algorithm using a Level Set and the BQMP Thresholding Method

Every colour picture pixel is given a hyper-complex representation in quaternion form, as $Q = Q_0 + Q_1i + Q_2j + Q_3k$. The hyper-complex co-ordinates $i, j,$ and k operators are constrained by the rule $i^2 = j^2 = k^2 = -1$. Displaying an RGB colour pixel as a quaternion is done using the notation $Q_0 = 0, Q_1 = R, Q_2 = G$ and $Q_3 = B$.

In a quaternion-valued pixel-set, the BQMP thresholding method chooses a hyperplane as a threshold, classifying all pixels below the threshold into class 1 (C_1) and all pixels above the threshold into class 2 (C_2). Every class does this

procedure repeatedly until a target number of clusters (k) has been attained. Class variance can also be used as a termination criterion for this iterative procedure.

The suggested quantization strategy uses class variance as a cut-off condition. The provided image's attribute determines the class's threshold, which in turn determines the variance of the class. In this study, the number of colour divisions used in the quantization process is taken as the secondary stopping criterion, and the threshold for the variance of the class is set to 1/30 of the variance of the original picture. Key steps in the quantization process include developing a binary tree, formulating a level set, and putting it into action.

The sections that follow will elaborate on each of these parts.

Binary Tree Formulation

The suggested technique is based on a slicing clustering technique that results in a binary tree. The strategy of “divide and conquer” is a source of inspiration for divisive tactics. The divide-and-conquer strategy is effective for issues that can be subdivided recursively into simpler ones. Finding the underlying cause of the main issue requires solving a series of smaller, more manageable challenges and then merging their answers. The original, seed picture is used as the starting point for a divisive approach. The initial picture is then split in half. If a partition is split, the original partition's node is removed, and two new partitions, each containing their own area, are created. The process of division will continue until either M, the maximum number of leaf nodes, is achieved or the variances of all partitions remain below the threshold. Divisive algorithms are based

on straightforward ideas, but need additional implementation details to work properly. How to choose which node should be divided and how to split it is the primary concern. Before dividing something into pieces, the algorithm must make a decision. When deciding which node to split, the suggested technique uses a BFS-style ordering. Number of pixels, node variance, and threshold all play a role in determining whether or not a node may be divided. In the suggested method, the node is divided using the BQMP thresholding technique. This section introduces the main and secondary level set functions that represent the node areas and partitions, respectively. Each pixel has to be allocated a cluster membership (or node number) at each stage of the division process. For the purpose of this assignment, the parent's membership status is taken into account. This membership data is what the node membership queue is made up of, and it is what is utilised to derive the node region from the main level set function.

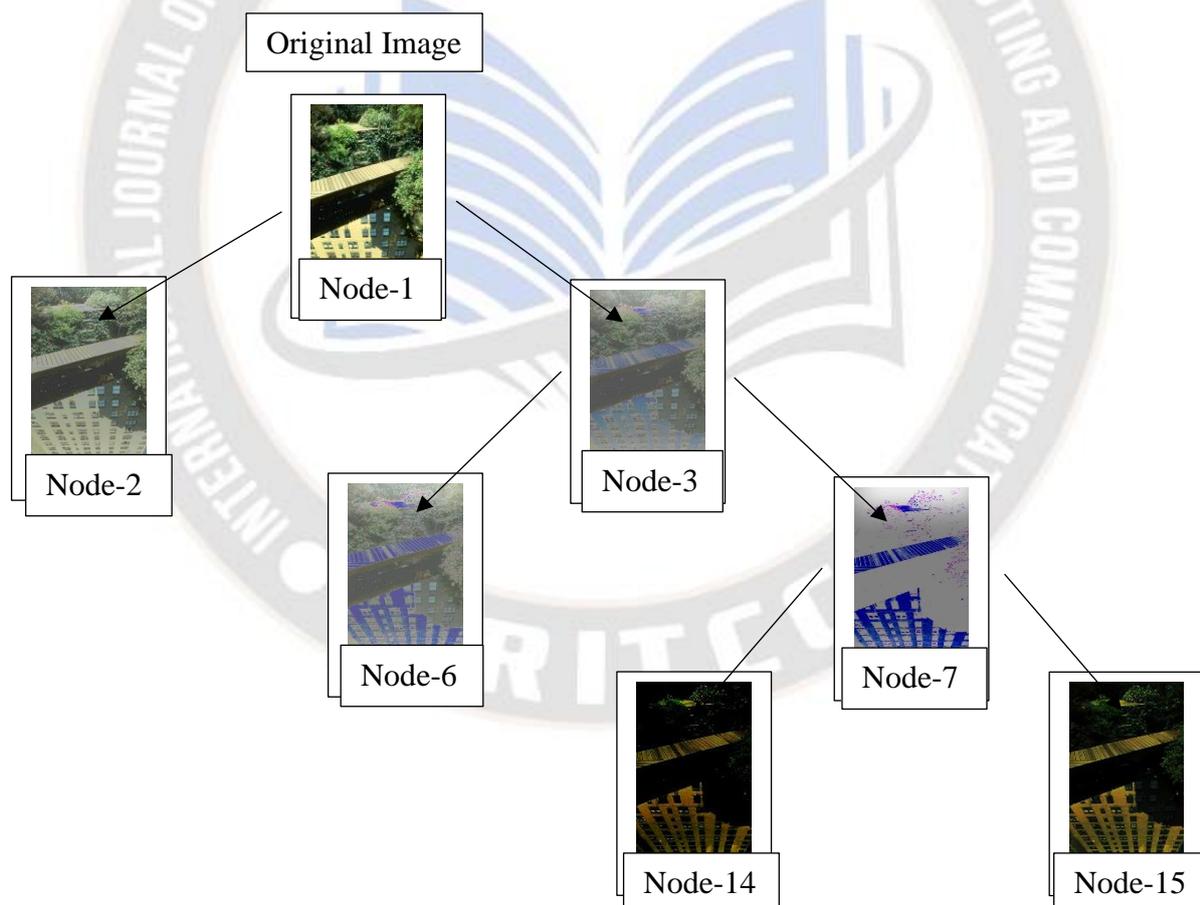


Figure 2: The suggested approach produces a hierarchical binary tree during colour quantization of a test picture for a building's shadow on a lake.

The building shadow in a lake picture obtained by the proposed approach is shown in a hierarchical binary tree structure with membership values given to each node, as shown in Figure 2. The first "node" is the original picture,

while the second and third are the offspring of that image. The left child of the nth node is always $2n$ and the right child is always $2n+1$. Each cluster is represented by a leaf

node. The suggested clustering creates clusters at nodes 2, 6, 14, and 15.

Formulation of level set

An implicit function specified in a higher dimension, called the level set function, is used to express contours between

two classes in level set techniques [14,15,16], and this function is then evolved using a Partial Differential Equation (PDE). The node areas are represented by level set functions in the proposed method.

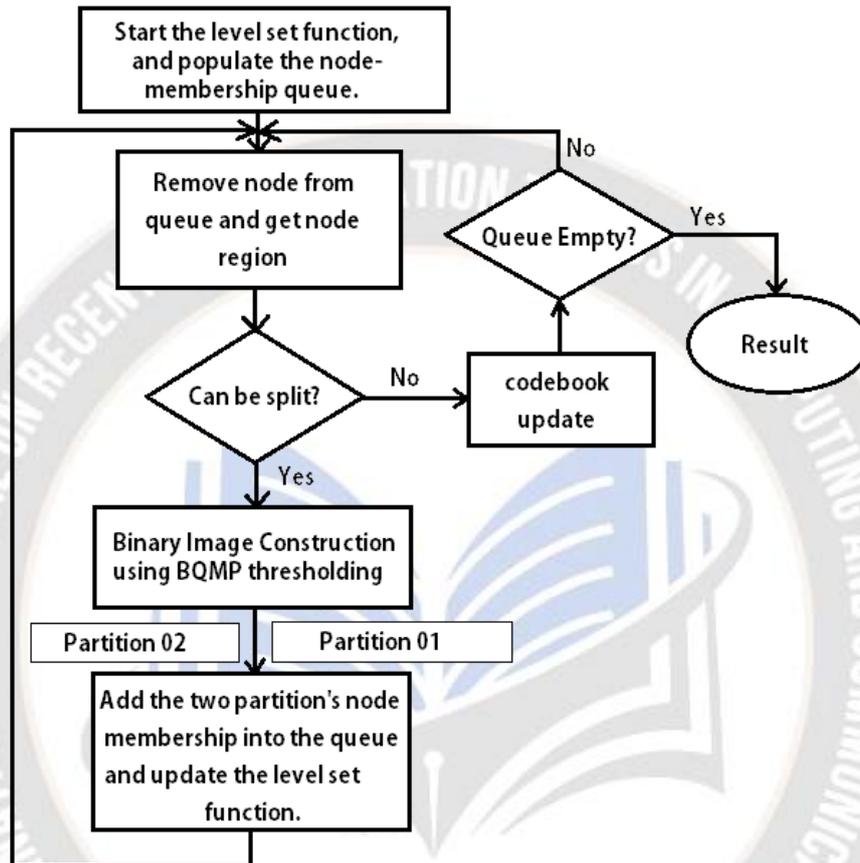


Figure 3: Diagrammatic representation of the level set formulation used in the proposed colour quantization method

The blocks used in level set formulation are shown in figure 3. Take into account an $m \times n$ pixel picture whose pixel's locations are depicted by (x, y) . This level set formulation employs two major level set functions, α and β , of size $m \times n$, and three supplementary level set functions, μ_1 , μ_2 and μ_3 also of size $m \times n$. Pixel association values are stored in a numeric level set function called δ . If the association values are completed, the Ω function of the level set reflects this. The node area N and the two node divisions are defined by the secondary binary level set functions μ_1 , μ_2 , and μ_3 . Take the case of the N node area, for which the secondary level set functions required to construct the node region and its partitions are specified by the corresponding equations (2, 3, and 4).

$$\mu_1(x, y) = \begin{cases} 1 & \text{if } \mu_1(x, y) \in \text{node}N \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_2(x, y) = \begin{cases} 1 & \text{if } Q(x, y) \in C_1 \text{ and } \mu_1(x, y) = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_3(x, y) = \begin{cases} 1 & \text{if } Q(x, y) \in C_2 \text{ and } \mu_1(x, y) = 1 \\ 0 & \text{otherwise} \end{cases}$$

Where, C_1 and C_2 are the two classes we get after for the pixel $Q(x,y)$ after BQMP thresholding process.

All primary level set functions are preserved during the quantization process. A short-term solution, supplementary level set functions are employed to refresh the main level set functions. Node area extraction, BQMP thresholding, partitions representation, and level set function updates are important features of every level set system. BQMP

partitions are represented by μ_2 and μ_3 , which are extracted from the secondary level set function μ_1 using the same notation. The node area μ_1 is fed through the BQMP thresholding method, and the resulting partitions μ_2 and μ_3 are produced. Listed below is a description of the level set formulation used to extract node regions having membership value as N and portray them as partitions. Equation 5 provides the formula for determining the node area.

$$\mu_1(x, y) = \sim H_\epsilon((\alpha(x, y) - (N + 1)U)\beta(x, y))$$

Where, U is a equivalent unit matrix size to that of μ_1 and H_ϵ is a Heaviside function approximated by smoothing function which can be depicted as in equation 6.

$$H_\epsilon(\beta) = 1 + \frac{2}{\pi} \arctan\left(\frac{\beta}{\epsilon}\right)$$

When, $\epsilon=1$ then equation 6 reduces to equation 7

$$H(\beta(x, y)) = \begin{cases} 0 & \beta(x, y) < 0 \\ 1 & \beta(x, y) \geq 0 \end{cases}$$

The Heaviside operator returns a binary representation, where 1 represents a positive or zero input parameter and 0 represents a negative one. If it is determined that the present node may be divided into two, the main level set functions are revised accordingly. The result of the quantization process has been shown in figure 4 and we can see that the quality has not been degraded even after decreasing colour information several times.



Raw Image (20659 Colours) Quaternion (13 Colours)



Raw Image (45329 Colours) Quaternion (43 Colours)



Raw Image (33657 Colours)



Quaternion (25 Colours)



Raw Image (22118 Colours)



Quaternion (13 Colours)



Raw Image (39431 Colours)



Quaternion (16 Colours)

Figure 4: The proposed Quaternion based image quantization approach has drastically reduced the number of colours to represent the raw images from the Berkeley image dataset.

Quaternion based JSEG Segmentation

Due to spatially variable lighting, which modifies both the intensity and chrominance components of the colour picture, the developers of the JSEG method have noted that their algorithm has an over-segmentation issue. To address these issues, the suggested segmentation framework proposes a two-stage process: first, the introduction of histogram equalisation and a quaternion-based divisive quantization approach; and second, the determination of the appropriate number of classes. Since it employs a quaternion-based quantization approach, the proposed full segmentation framework in this section is called Q-JSEG. Figure 5 shows a flowchart of such an inclusion in the original JSEG algorithm.

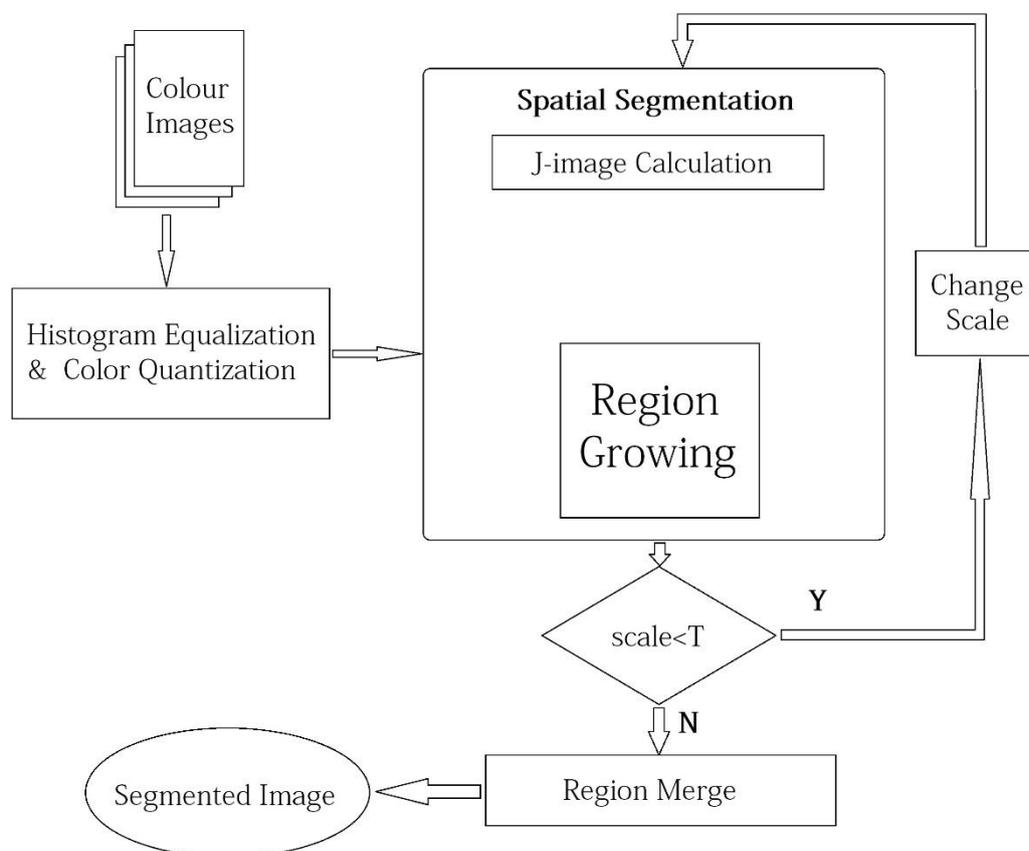


Figure 5: A flow diagram of Proposed Q-JSEG

Algorithm used in the proposed quantization technique is an iterative process makes use of the queue data structure. A binary tree as depicted in figure 2 is representing the segments of the provided colour picture is the result of this process. The algorithm for the proposed approach can be

broken down into main parts as algorithm 1 and figure 5. The algorithm 1 deals with the quaternion based image quantization approach and the figure 5 involves integrating it into the JSEG colour segmentation approach.

Algorithm 1: Quaternion based image quantization approach

Input:

Output:

Start Store Image and Initialize Primary Level set functions and Membership Queue

Store Database of Image and Primary Level set function

1: **for** every image in the database **do**

2: *read membership value from the queue*

3: **if** *queue_empty* **do**

4: *Stop*

5: **Else**

6: *extract node region τ_1*

```
7:      calculate number of pixels ( $P$ )
8:      calculate total number of pixels ( $T_P$ )
9:      if ( $P > T_P$ ) do
10:         calculate Squared Error ( $SE$ )
11:         if ( $SE > T$ ) do
12:            split the node region using BQMP thresholding
13:            calculate  $\tau_2$  and  $\tau_3$ 
14:            update levelset function  $\mu_1$ 
15:            compute membership values of 2 partitions
16:            goto step 2
17:         Else
18:            update levelset function and calculate average color
19:            goto step 2
20:         else
21:            goto step 2
22:     end for
```

IV. RESULTS AND DISCUSSION

The photos in the Berkeley collection are taken in the wild and as such have irregular textures, blurry edges, and poor contrast. The longest side of the colour photos in this collection has been scaled to 192 pixels for optimal viewing experience. Q-JSEG and JSEG were tested in order to qualitatively compare their segmentation accuracy on this dataset. For the suggested Q-JSEG method to work, many crucial parameters, including the number of quantization divisions, the region growth scales, and the threshold set to the number of pixels in the area merging process, must be defined. This study quantizes the picture to around 16 colours by building a binary tree with 5 layers (15 divisions). In this experiment, we selected window sizes (scales) of 9x9, 5x5, and 3x3 due to the large (128x192 or 192x128) size of the images captured. Fifty pixels are required as the bare minimum for an area. The suggested system's main benefit is that the number of clusters is determined automatically based on the excellence factor. In Figure 6, we see how the segmentation results of the

proposed Q-JSEG algorithm is compared to those of JSEG and FCR. The segmented results of FCR clearly show that it produces two boundary lines. Due to the fact that the FCR method employed a window of size 7x7 during the segmentation process, the thickness of the border line is around 7 pixels. Instead, it makes use of the benefits that come with using a variety of colour spaces and colour histograms.

It can be seen in figure 6 that the number of clusters that resulted from the suggested segmentation is lower, and that the segmentation itself provides better relevant findings in comparison to JSEG and FCR. The Q-JSEG boundary line is a single line, and the amount of boundary error has been drastically cut down. Figure 6 shows the segmentation results for the photos that have a lighted sky in the first, second, and fourth rows, respectively. Because the suggested structure makes use of an algorithm for sky correction, the sky pixels that have (spatially variable) various colour compositions have been grouped together into a single class.



Figure 6: A visual Comparison of results obtained from the compared three algorithms

Table 1: Performance in tabular format of the compared three algorithms with Q-JSEG

| Algorithm | Mean Performance Index | Standard Performance Index |
|-----------|------------------------|----------------------------|
| FCR | 0.78 | 0.081 |
| Q-JSEG | 0.82 | 0.13 |
| CTex | 0.79 | 0.091 |
| JSEG | 0.76 | 0.11 |

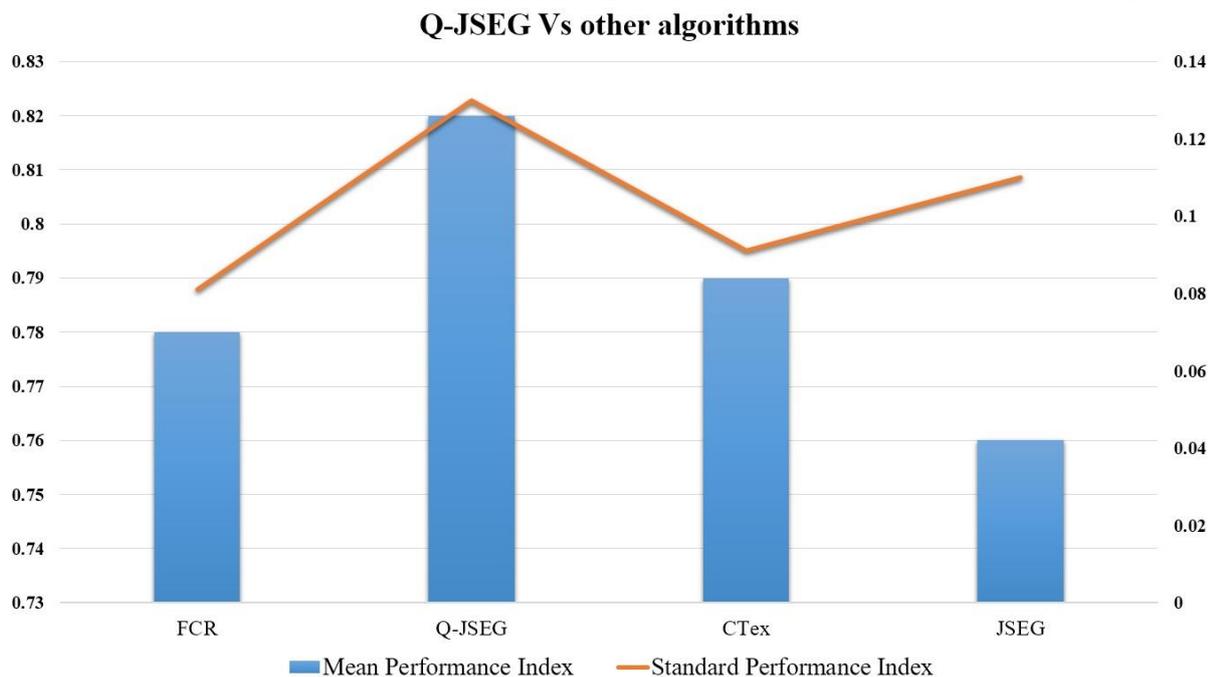


Figure 7: A graphical representation of the proposed Quaternion based JSEG image segmentation over other state-of-the-art algorithms.

V. CONCLUSION AND FUTURE WORK

In order to better segment JSEG colour textures, a novel quantization approach based on projective geometry and level set techniques has been created. The new colour quantization methodology is a partitioning clustering method that divides the RGB cube's colour clusters according to their spectral and spatial homogeneity. To increase the contrast between colours and normalise the RGB cube, histogram equalisation was used. In the suggested technique, each node of the binary tree stands in for a division. The internal nodes are purely logical and get no dedicated physical memory. The quantized colour data is only present in the leaf nodes. The level-set functions keep track of data about nodes and partitions. This makes the suggested divisive clustering technique more efficient in terms of both memory space and processing time. To further mitigate the excessive segmentation issue, the suggested quantization approach may be refined to take into account the angular characteristic of the hyper plane used in the colour division procedure. Better algorithm performance may be achieved by using advanced histogram equalisation methods. The following chapter provides a quantitative analysis of all the algorithms provided in this thesis and a comparison of the findings with the current state of algorithms.

REFERENCES

[1] Voight, John. Quaternion algebras. Springer Nature, 2021.

- [2] Zhang, Shuai, Yi Tay, Lina Yao, and Qi Liu. "Quaternion knowledge graph embeddings." *Advances in neural information processing systems* 32 (2019).
- [3] Parcollet, Titouan, Mohamed Morchid, and Georges Linarès. "A survey of quaternion neural networks." *Artificial Intelligence Review* 53, no. 4 (2020): 2957-2982.
- [4] Zhu, Xuanyu, Yi Xu, Hongteng Xu, and Changjian Chen. "Quaternion convolutional neural networks." In *Proceedings of the European Conference on Computer Vision (ECCV)*, pp. 631-647. 2018.
- [5] Liu, Yang, Yanling Zheng, Jianquan Lu, Jinde Cao, and Leszek Rutkowski. "Constrained quaternion-variable convex optimization: A quaternion-valued recurrent neural network approach." *IEEE transactions on neural networks and learning systems* 31, no. 3 (2019): 1022-1035.
- [6] Chen, Yongyong, Xiaolin Xiao, and Yicong Zhou. "Low-rank quaternion approximation for color image processing." *IEEE Transactions on Image Processing* 29 (2019): 1426-1439.
- [7] Ngan, Roan Thi, Mumtaz Ali, Dan E. Tamir, Naphtali D. Rishe, and Abraham Kandel. "Representing complex intuitionistic fuzzy set by quaternion numbers and applications to decision making." *Applied Soft Computing* 87 (2020): 105961.
- [8] Qi, Liqun, Ziyang Luo, Qing-Wen Wang, and Xinzheng Zhang. "Quaternion matrix optimization: Motivation and analysis." *Journal of Optimization Theory and Applications* 193, no. 1 (2022): 621-648.
- [9] Liu, Long-Sheng, Qing-Wen Wang, Jiang-Feng Chen, and Yu-Zhu Xie. "An exact solution to a quaternion matrix equation with an application." *Symmetry* 14, no. 2 (2022): 375.

- [10] Neshat, Mehdi, Meysam Majidi Nezhad, Seyedali Mirjalili, Giuseppe Piras, and Davide Astiaso Garcia. "Quaternion convolutional long short-term memory neural model with an adaptive decomposition method for wind speed forecasting: North aegean islands case studies." *Energy Conversion and Management* 259 (2022): 115590.
- [11] Xie, Mengyan, Qing-Wen Wang, and Yang Zhang. "The Minimum-Norm Least Squares Solutions to Quaternion Tensor Systems." *Symmetry* 14, no. 7 (2022): 1460.
- [12] Zhang, Ziyue, Xiaofeng Wei, Shuzhan Wang, Chong Lin, and Jian Chen. "Fixed-time pinning common synchronization and adaptive synchronization for delayed quaternion-valued neural networks." *IEEE Transactions on Neural Networks and Learning Systems* (2022).
- [13] Mehany, Mahmoud Saad, and Qing-Wen Wang. "Three symmetrical systems of coupled Sylvester-like quaternion matrix equations." *Symmetry* 14, no. 3 (2022): 550.
- [14] Brignone, Christian, Gioia Mancini, Eleonora Grassucci, Aurelio Uncini, and Danilo Comminiello. "Efficient sound event localization and detection in the quaternion domain." *IEEE Transactions on Circuits and Systems II: Express Briefs* 69, no. 5 (2022): 2453-2457.
- [15] Shang, Yuanyuan, Yuchen Pan, Xiao Jiang, Zhong Shao, Guodong Guo, Tie Liu, and Hui Ding. "LQGDNet: A Local Quaternion and Global Deep Network for Facial Depression Recognition." *IEEE Transactions on Affective Computing* (2021).
- [16] Yan, Hongyun, Yuanhua Qiao, Lijuan Duan, and Jun Miao. "Novel methods to global Mittag-Leffler stability of delayed fractional-order quaternion-valued neural networks." *Neural Networks* 142 (2021): 500-508.
- [17] Sun, Yehan, Qingtang Su, Huanying Wang, and Gang Wang. "A blind dual color images watermarking based on quaternion singular value decomposition." *Multimedia Tools and Applications* 81, no. 5 (2022): 6091-6113.
- [18] Wu, Tingting, Zhihui Mao, Zeyu Li, Yonghua Zeng, and Tiejong Zeng. "Efficient Color Image Segmentation via Quaternion-based $\mathbb{H} \otimes L_1/L_2$ Regularization." *Journal of Scientific Computing* 93, no. 1 (2022): 1-26.
- [19] Huang, Chaoyan, Yingying Fang, Tingting Wu, Tiejong Zeng, and Yonghua Zeng. "Quaternion Screened Poisson Equation for Low-Light Image Enhancement." *IEEE Signal Processing Letters* 29 (2022): 1417-1421.
- [20] Li, Jiaxue, and Yicong Zhou. "Automatic Color Image Stitching Using Quaternion Rank-1 Alignment." In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 19720-19729. 2022.
- [21] Bi, Xiuli, Chao Shuai, Bo Liu, Bin Xiao, Weisheng Li, and Xinbo Gao. "Privacy-Preserving Color Image Feature Extraction by Quaternion Discrete Orthogonal Moments." *IEEE Transactions on Information Forensics and Security* 17 (2022): 1655-1668.
- [22] Li, Qi, Xingyuan Wang, Bin Ma, Xiaoyu Wang, Chunpeng Wang, Zhiqiu Xia, and Yunqing Shi. "Image steganography based on style transfer and quaternion exponent moments." *Applied Soft Computing* 110 (2021): 107618.
- [23] Liu, Xilin, Yongfei Wu, Hao Zhang, Jiasong Wu, and Liming Zhang. "Quaternion discrete fractional Krawtchouk transform and its application in color image encryption and watermarking." *Signal Processing* 189 (2021): 108275.
- [24] Sun, Yehan, Qingtang Su, Huanying Wang, and Gang Wang. "A blind dual color images watermarking based on quaternion singular value decomposition." *Multimedia Tools and Applications* 81, no. 5 (2022): 6091-6113.
- [25] Xia, Zhiqiu, Xingyuan Wang, Chunpeng Wang, Bin Ma, Hao Zhang, and Qi Li. "Novel quaternion polar complex exponential transform and its application in color image zero-watermarking." *Digital Signal Processing* 116 (2021): 103130.
- [26] Kumar, Gangavarapu Venkata Satya, and Pillutla Gopala Krishna Mohan. "Enhanced Content-Based Image Retrieval Using Information Oriented Angle-Based Local Tri-Directional Weber Patterns." *International Journal of Image and Graphics* 21, no. 04 (2021): 2150046.
- [27] Bianconi, Francesco, Antonio Fernández, and Raúl E. Sánchez-Yáñez. "Special Issue Texture and Color in Image Analysis." *Applied Sciences* 11, no. 9 (2021): 3801.
- [28] Patil, Kiran H., and M. Nirupama Bhat. "Survey of Color Feature Extraction Schemes in Content-Based Picture Recovery System." In *Computational Vision and Bio-Inspired Computing*, pp. 719-732. Springer, Singapore, 2021.
- [29] Tripathy, Debi Prasad, and K. Guru Raghavendra Reddy. "Separation of gangue from limestone using GLCM, LBP, LTP and Tamura." *Journal of Mines, Metals and Fuels* (2022): 26-33.
- [30] Patel, Bhagwandas, Kuldeep Yadav, and Debashis Ghosh. "Current Trend and Methodologies of Content-Based Image Retrieval: Survey." In *Proceedings of Second International Conference on Smart Energy and Communication*, pp. 647-665. Springer, Singapore, 2021.
- [31] WT, CHEMBIAN. "Colour Image Retrieval Based on Adaptive Jseg Image Segmentation and Statistical Distance Measure." Available at SSRN 4159538.
- [32] Tsaryk, V., and V. Hnatushenko. "Research of methods of distribution of graphic objects on websites for assessment of interface quality." *System technologies* 3, no. 140 (2022): 143-154.
- [33] Wang, Yong-Gang, Jie Yang, and Yu-Chou Chang. "Color-texture image segmentation by integrating directional operators into JSEG method." *Pattern Recognition Letters* 27, no. 16 (2006): 1983-1990.
- [34] Wang, Chao, Ai-Ye Shi, Xin Wang, Fang-ming Wu, Feng-Chen Huang, and Li-Zhong Xu. "A novel multi-scale segmentation algorithm for high resolution remote sensing images based on wavelet transform and improved JSEG algorithm." *Optik* 125, no. 19 (2014): 5588-5595.
- [35] Wang, Zuyuan, and Ruedi Boesch. "Automatic forest boundary delineation in aerial images." In *2006 IEEE International Symposium on Geoscience and Remote Sensing*, pp. 2605-2608. IEEE, 2006.