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Reconstruction of an Image Based on 13/19 Triplet Half-Band Wavelet Filter Bank and Orthogonal Matching Pursuit

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Abstract
Compressive Sensing Scheme for image reconstruction presented in this paper is depending on a combination of Orthogonal Matching Search and a 13/19 triplet half band filter bank (THFB) which is resulting from 1/2-band polynomial. Here, the consideration is made for 13/19 triplet half band wavelet filter sets. The half-band polynomial is applied which is generalized and used to receive the required frequency response. The image reconstruction is done later based on this. The designed triplet wavelet filters give a sparse image which is used for the input image. Gaussian probability density function and the Orthogonal Matching Pursuit (OMP) are presented for reconstructing the image. The results and observations demonstrate that the compressive sensing by using OMP and designed wavelet filters offers good result for performance as compared to the existing wavelet filters.

Keywords: Half-Band Wavelet, Image Reconstruction, Orthogonal Matching Pursuit (OMP), Compressive Sensing etc.

I. INTRODUCTION

Compressive Sensing is a very innovative and evolving area which gained significant interest of signal processing research area. It is called as Nyquist Sampling Theorem. In this theorem, the sampling rate should be double than the larger the signal-frequency. This theorem is commonly used to read the image. It rises the sampling frequency [1]. More efficient approximation can be obtained due to it. This technique is not recommended when the signal is high. To address the problems, Donoho [2] presented the theory of compressive sensing where there is a use of a random linear projection to obtain effective direct compressible signals representation. The signal is sparse and compressible in [3]. The Compressive Sensing (CS) combines compression and data acquisition operations. That signal is called as *p-sparse* if the coefficients consist of *p* non-zero values only in some transform domain [4]. Many other important application areas are present where signal or image sparse reconstruction may be required for time sequences [5]. The sparse reconstruction is studied as mentioned in [6]. The latest contribution on compressive sensing brings the constraints for actual reconstruction [7,8,9,10]. The compressive sensing could be used efficiently on sparse signals [9]. So, to obtain the sparse representation, DCT or DWT could be applied first to the signals. The compressive sensing based on wavelet becomes successful due to time-frequency localization and multi-resolution analysis feature. The literature survey tells that more CS algorithms use orthogonal wavelet (Daubechies wavelet), 9/7 biorthogonal wavelet and Haar transform. Still the problem remains in image processing based on the wavelets choice. The wavelet filters design and findings of their characteristics are not mentioned in the CS literature. It is also known very well that the wavelet systems performance is very much dependent on the wavelet's choice. Hence, in this paper, a compressive sensing technique by using recent wavelet class is done from 3-step lifting scheme and examines their properties for CS.

This paper is arranged as follows. 2nd section explains the survey of the literature work and triplet half-band wavelet filter-bank design. Section 3 describes a method of reconstructing the image based on presented orthogonal matching. In Section 4, the implementation results are presented for evaluating the presented scheme. Section 5 describes about the conclusion.

II. Review of one dimensional bi-orthogonal wavelet filters

In image processing, bi-orthogonal wavelet is selected than the orthogonal wavelets. It has a linear phase characteristic. Daubechies Wavelet Transform proposes orthogonal wavelets construction. But, these wavelets don't offer linear-phase and symmetric characteristics which

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needed for removing some disturbances in the images. Hence, symmetric filters with linear phase gained by diluting the situation of orthogonality, is referred as bi-orthogonal wavelets. Many bi-orthogonal FBs which have more number of 0's at $z=-1$ so that they can attain proper regularity. LHBP filters don't contain any degree of freedom and hence, direct control is not present over frequency response of the filters. So, for obtaining some independent attributes, the authors in [11] preferred filter factorization of half band to generate two channel bi-orthogonal wavelet. The approach of factorization advances the frequency response. The frequency response enhancement of the two filters depends on polynomial factorization. Lifting structure is also very good scheme for designing FBs. The authors in [12] presented a half-band class filter-bank described by two kernels, is depending upon lifting approach. It finds some limitations for its frequency response control. To resolve this limitation, the paper [13] produced a category of triplet half-band filter bank by using 3 kernels having feature rich structure, structural PR and simple design. In the proposed research paper [13], two techniques are presented to design 2-channel one dimensional bi-orthogonal FBs which are depending upon the half-band filters. The authors in [14] presented a new method to design a category of THFB. The authors, in [15], employed a THFB with Semi Definite Programming (SDP). The authors, in [16], proposed SDP method. The authors in [17] addressed some issues for designing THFB and proposed a novel category of THFB wavelets which are depending on half band polynomial. The authors of [18-20] have used Fourier transform based descriptors as representative features in image processing.

2.1. Basic Background

Fig. 1 explains about BWFB which is two channel. Quadrature mirroring of LPF is used for analysis of HPF and synthesis. Hence, the cancellation is obtained as:

$$H_1(z)=z^{-1}\times G_0(-z),\ G_1(z)=z\times H_0(-z)$$

……………………. (1)

The *scaling* and *wavelet* functions mentioned in the following equations:

$$\phi(t)=\frac{2}{H_0(\omega)\,|\,\omega=0}\sum_n h_0(n)\phi(2t-n)$$

…………………….. (2)

$$\psi(t)=\frac{2}{G_0(\omega)\,|\,\omega=0}\sum_n h_1(n)\phi(2t-n)$$

……………………… (2')

Where, LP and HP filter coefficients are $h_0(n)$ and $h_1(n)$.

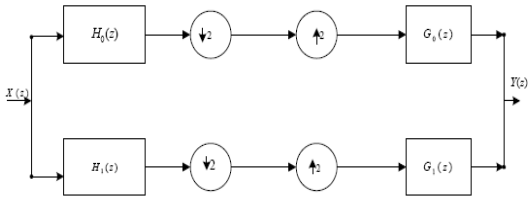

Fig. 1. 2-channel filter bank

2.2 Triplet Half-band Filter Bank (THFB)

LP filters of a THFB category are analyzed and synthesized and they consist of 3 half-band filters and mentioned in paper [13] are as shown below:

$$H_0(z)=\frac{1+p}{2}+\frac{1}{2}t_1(z)-\frac{p}{2}t_0(z)t_1(z)$$ ………(3)

$$G_0(z)=\frac{1+pt_0(z)}{1+p}+\frac{1-p}{1+p}t_2(z)\left[\frac{1+p}{2}-\frac{1}{2}t_1(z)(1+pt_0(z))\right]$$

……………………………………….(4)

Where, $t_0(z), t_1(z)$ and $t_2(z)$ are the half-band filters and that are approximating as 0 in stop band and 1 in pass band. HPFs are analyzed and synthesized in equation (1). The *p* parameter which is called as a degree of freedom, are more flexible to choose the magnitude at $\omega=0.5\pi$.

2.3. Review on the design of novel class of THFB

A proper symmetric HBP $P(z)$ of 6[th] order is assumed in [17] as follows:

$$p(z)=ar_0+ar_2z^{-2}+z^{-3}+ar_2z^{-4}+ar_0z^{-6}$$

……………………………………….. (5)

This $p(z)$ is used to produce $p_1(z), p_2(z)$ and $p_3(z)$ using extraction of no. of 0's at z=-1 by using synthetic-division approach to get 3 HBPs in the form of $ar_0(z)$. These are calculated as follows:

$$p_1(z)=ar_0+(-ar_0+0.5)z^{-2}+z^{-3}+(-ar_0+0.5)z^{-4}+ar_0z^{-6},$$

$$p_2(z)=(1+z^{-1})\times(ar_0+(-ar_0)z^{-1}+0.5z^{-2}+0.5z^{-3}+(ar_0)z^{-4}+ar_0$$
$$p_3(z)=(1+z^{-1})^2\times(ar_0+(-2ar_0)z^{-1}+(0.5+2ar_0)z^{-2}+(-2ar_0)z^{-3}+$$

……….. (6)

It is noticed that 3 HBPs are of the argument ar_0. The optimistic value of ar_0 =-0.062499 is applied. So, the energy of the kernels reduced. The kernels THFB $t_0(z), t_1(z), t_2(z)$ are gained from (6) as below:

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$$t_0(z) = ((1 + z^{-1})^0 . R_1(z)) - 1,$$

$$t_1(z) = ((1 + z^{-1})^1 . R_2(z)) - 1,$$

$$t_2(z) = ((1 + z^{-1})^2 . R_3(z)) - 1.$$

………….. (7)

There is double degree of freedom. It provides the expected frequency response for the reconstruction of image.

The responses of the frequency of HPF and LPF are distinguished with the available FBs as presented in Fig. 2. It provides proper frequency response.

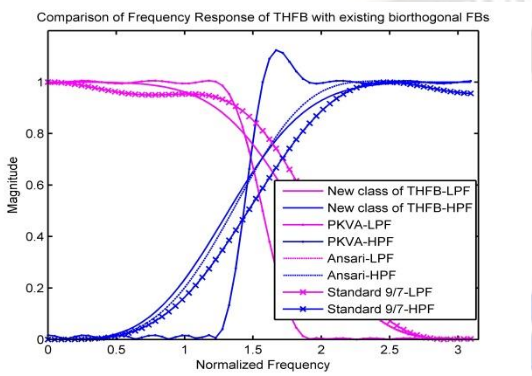

Fig. 2. Differentiation of magnitude responses of the FB pair.

In next part, the characteristics of the obtained wavelet filters are presented that can be important.

2.4. *Features of the 13/19 wavelet filters Desirable for CS [17]*

Some features are as below:

1) Linear Phase (Symmetry):

These types of wavelets fulfil the characteristics of linear phase. This characteristics is very important. It avoids the distortions at boundary.

2) Near-Orthogonality:

These types of wavelets provide near-orthogonality that is significant. There is improvement in quality for reconstructing the output image [21-22].

3) Frequency Selectivity:

It is very helpful for reconstruction the image with better quality. This is possible by measuring the energy or error of the ripple [21-22].

4) Regularity:

It is essential to know that the regularity in the design. The output wavelet has greater power of approximation to decompose. It is very consistent in the reconstruction [21-22]. This calculation is mostly needed for the reconstruction of image.

5) Time-Frequency Localization:

These filters which are designed for providing better time frequency resolution.

III. Proposed Image Reconstruction depending on compressive sensing

A purpose of this approach for getting a sparse image. DWT, DFT, and DCT is incorporated. Here, DWT is implemented to get the sparse image. it has most important feature of multi-resolution analysis. The authors presented matching pursuit (MP) based algorithms. It contains orthogonal matching (OM), basis pursuit (BP) and sparsity adaptive matching pursuit, etc. For the reconstruction of the image, OMP has been used [23].

3.1 Orthogonal Matching Pursuit for Image Reconstruction

The CS encoding price and the decoding depends on the measurement matrix type. Consider *S* as a random *n*-sparse image and $\{P_1, P_2, \ldots, P_N\}$ are N-dimentional vectors. The matrix which is named M for measurement is received using $M = N \times d$. *N*-measurement of the signal is gathered in N-dimensional data vector $V = MS$, where $\{\psi_1, \psi_2, \ldots \psi_d\}$ are columns of matrix M[23]. The vector $V = MS$ is created from M, because the signal S has *n*-nonzero components. The motive is to identify which specific column of M participate in the data vector $V = MS$ for recognizing the actual signal S[23]. The presented approach for reconstruction of image using the approach of OMP which is based the sparse image. The algorithm is as below:

1. Read an image.
2. Develop 13/19 THFB based wavelet filters from the generalized half-band polynomial to resolve some problems with the existing wavelets.
3. Operate on the image with the wavelets to get the sparse image.
4. Produce the measurement matrix by applying Gaussian Probability Density function. It is not dependent as well as equally distributed of size *Nxd*, where *d* is the no. of rows of the original image.
5. Initialization of the residual is done as $r_0 = y$.
6. For every iterative loop execution, do
7. Calculate the position of the column i_t of $M\psi$ such that,

$$i_t = \text{argmax}_i \left| < r_{t-1}, (M\psi)_i > \right|$$

8. Do the augmentation of the column position and the matrix of $M\psi$.

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9. Find the solution of the Least Square Problem for getting the new residual as follows:

$$r_t = y - P_t y$$

10. Do increment of t and go to step 7, if t<N.

IV. EXPERIMENTAL RESULTS

The presented CS approach's performance is checked over two 8-bit standard images of size 256x256 for distinct values of *N*. The PSNR is measured to obtain a thoughtful accurate analysis. The PSNR values are convenient, so they are used to identify image quality generally.

Firstly in the results, there is a direct use of the CS theory for both of the Cameraman and Lena images i.e. there is a direct application of random measurement to the tested images. It is noted that the PSNR values are smaller and the image reconstruction properly is not possible. The reconstructed output image is displayed in Fig. 3 (a). For improving the quality of reconstructed image and the PSNR values, 13/19 THFB wavelet filters

are designed and the PSNR values for different values of N (measurement matrix) obtained. The experimental results are shown in Table 2 and Table 3.

Table 2: PSNR values (dB) of the proposed CS method

CS Methods	**Lena Image**			
	N=150, d=256	N=190, d=256	N=200, d=256	N=210, d=256
Direct method (image + OMP)	9.12	10.23	11.21	11.98
Standard 9/7 wavelet filter +OMP	24.092	25.53	25.98	26.25
Proposed method (11/9 wavelet filters +OMP)	**24.82**	**25.75**	**26.094**	**27.98**
Proposed method (THFB wavelet filters +OMP)	**28.3554**	**32.1906**	**32.9274**	**33.5478**

Table 3: PSNR values (dB) of proposed CS method

CS Methods	**Cameraman Image**			
	N=150, d=256	N=190, d=256	N=200, d=256	N=210, d=256
Direct method (image + OMP)	5.16	7.58	8.01	8.59
Standard 9/7 wavelet filter +OMP	23.123	25.47	26.32	27.12
Proposed method (11/9 wavelet filters +OMP)	**24.08**	**26.08**	**26.66**	**27.96**
Proposed method (THFB wavelet filters +OMP)	**25.0953**	**29.6310**	**30.8416**	**31.8020**

From Table 2 and Table 3, it is found that DWT with the designed 13/19 filter and OMP performs better than the existing standard cdf-9/7 wavelet filters. Even Fig. 3 (a-c) presents the reconstructed image of Lena using the presented and existing approach.

Thus, the designed 13/19 THFB wavelet filters are applied for the compressive sensing.

Fig. 3 (a) Original Image (M= 162)

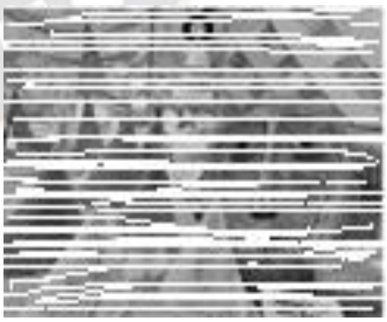

Fig. 3 (b) Reconstruction using Direct (without DWT) (M=162)

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Fig. 3 (c) Reconstruction using cdf-9/7 wavelet filters (M=162)

Fig. 3 (d) Reconstruction using designed 13/19 wavelet filters (M=162)

Fig. 2 (a) Original Image for M=162; (b) Image Reconstruction using direct measurement for M=162 without DWT (c) Image Reconstruction using cdf-9/7 wavelet filters used for M=162 (d) Image reconstruction using designed 13/19 wavelet filters for M=162

V. CONCLUSION

The proposed algorithm of compressive sensing is based on a novel category of wavelets and OMP. A wavelet class is designed which is called as 13/19 THFB wavelet filters. The half-band polynomial is considered here and examined its features. For compressive sensing, they are required. The results is measured with the existing wavelet cdf-9/7. The developed 13/19 wavelet filters performs efficiently.

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