Predetermination of Performance Parameters of 3phase Induction Motor using Numerical Technique Tools

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Abstract:

The Circle Diagram was used in past for the performance evaluation of 3-phase induction motor under different load conditions. But, now it is not preferred due to availability of programs for detailed analysis. The per-phase equivalent circuit is also not that suitable to estimate the dynamic performance. In this paper, a method is suggested to determine the performance using dq-model. The objective of this paper is to describe modelling in terms of dq variables. The analysis under steady state condition is done using numerical technique tools. The numerical tools such as Gauss-Seidel and Successive Relaxation methods are used and the results are compared with that obtained from the experiment conducted in the laboratory. The comparison of results confirms the validity as well as the accuracy of the proposed methods, as yet another approach to predetermine the performance.

Keywords- iteration, convergence, flux, linkage, performance.

INTRODUCTION

The squirrel cage type 3- ϕ induction motors are used in industries and have been favoured because of low cost, ruggedness and variable speed options [1]. Though the dynamic model equations are complex but are highly useful in understanding and analyzing the interaction of rotor flux and mmf waves. There are different models available and used based on the reference frame chosen such as stationary, rotor axis, synchronously rotating reference frames. The dq (synchronously rotating) reference frame provides a direct means to obtain the voltage equations to estimate the performance. The dq-model also describes the basic concept of transient modeling and internal fault diagnoses [2, 3]. In dqmodel, to establish the required mmf in the air gap the two orthogonal windings are used. Consider two orthogonal windings, one oriented with d-axis and another with q-axis. Because of orthogonal displacement the dq-axis windings are mutually as well as magnetically not coupled. Thus, the current

in dq-windings produce same mmf distribution similar to that of the three phase windings [4].

Over the several years numerical methods are used for providing approximate solutions associated with systems or equations iteratively with the advantage of having high accuracy. The methods used for numerical analysis are listed as direct and iterative. In direct method, a procedure is to be followed with number of steps involved to obtain the solution. An approximate value is assumed and is made to approach towards the accurate solution in iterative method. The merit of iterative method is less memory requirement in computational device. The Gauss-Seidel method is more accurate than other methods and converges quickly. The Successive Relaxation method is employed when the coefficient matrix is symmetric and has a 'property A' and it is a generalization of Gauss-Seidel method [5].

II 3-PHASE INDUCTION MOTOR MATHEMATICAL MODELING

A set of mathematical equations can be used to represent a $3-\varphi$ induction motor. Consider an arbitrary reference frame, which rotates at a speed of ω_d . Here θ_{da} is the arbitrary angle of dq-winding in regard to stator a-axis. Similarly θ_{dA} is the arbitrary angle of dq-winding set with reference to rotor A-axis is revealed from figure 1.

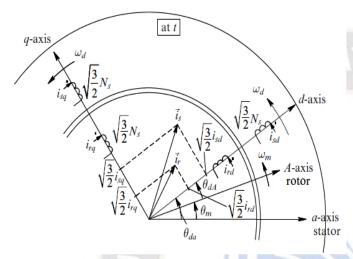


Fig. 1 Stator and rotor dq-windings of 3-phase induction motor

The following notations are used throughout this paper. The rotor dq-winding exhibits a leakage inductance L_{lr} and resistance R_r . Similarly, R_s represent resistance and leakage inductance is represented as L_{ls} of stator dq-winding. The dq-windings of stator and rotor have $\sqrt{(3/2)*N_s}$ number of turns. Here, N_s depict the number of turns in phase winding of 3-phase induction motor. Because of orthogonal property, there will be magnetic mutual uncoupling between dq-windings. The mutual inductance is zero, because of uncoupling. Thus, L_m represents the magnetizing inductance of dq-winding. Here, P represents total number of poles and p represents derivative operator [6].

The q and d-axis stator current components are represented as i_{sq} and i_{sd} respectively. Similarly i_{rd} and i_{rq} represents dq-axis components of rotor current. The flux linking with either d or q winding is due to current in other winding and that in the same winding. Thus the stator flux linkages in terms of current space vectors are given in equation 1 and 2 [7].

$$\varphi_{sd} = L_m i_{rd} + L_s i_{sd} \tag{1}$$

$$\varphi_{sa} = L_m i_{ra} + L_s i_{sa} \tag{2}$$

Similarly, rotor flux linkages in terms of current space vectors are given in equation 3 and 4.

$$\varphi_{rd} = L_m i_{sd} + L_r i_{rd} \tag{3}$$

$$\varphi_{rq} = L_m \cdot i_{sq} + L_r \cdot i_{rq} \tag{4}$$

Where, L_r and L_s exemplify the coefficient of rotor and stator self inductances respectively as mentioned in equation 5.

$$L_s = L_m + L_{ls}, L_r = L_m + L_{lr}$$
⁽⁵⁾

Consider an orthogonal set $\alpha\beta$ -winding fixed on the stator. Consider the α -axis in phase with stator winding a- axis. The stator winding voltage is equal to develop the electromotive force to establish the stator flux linkage and to sustain the potential drop in stator resistance. The $\alpha\beta$ -winding voltage equations are stated in 6 and 7.

$$\mathbf{V}_{\mathbf{s}\alpha} = \mathbf{R}_{\mathbf{s}} \cdot \mathbf{i}_{\mathbf{s}\alpha} + \mathbf{p}\boldsymbol{\varphi}_{\mathbf{s}\alpha} \tag{6}$$

$$\mathbf{V}_{\mathbf{s}\boldsymbol{\beta}} = \mathbf{R}_{\mathbf{s}} \cdot \mathbf{i}_{\mathbf{s}\boldsymbol{\beta}} + \mathbf{p}\boldsymbol{\varphi}_{\mathbf{s}\boldsymbol{\beta}} \tag{7}$$

The α -axis voltage space vectors are related to the d-axis as given in equation 8.

$$\mathbf{V}_{(\mathbf{s}_{\alpha}\alpha\beta)} = \mathbf{V}_{(\mathbf{s}_{\alpha}dq)} \,\mathbf{e}^{\mathbf{j}\,\boldsymbol{\ell}\mathbf{d}\,\mathbf{a}} \tag{8}$$

Where, $\omega_d = p\theta_{da}$ is the dq-winding speed expressed in electrical radians/ second with regard to stator a-axis speed in air gap. The real component and fictitious component of the dq-winding voltage expressions are given in 9-12.

$$\mathbf{V}_{sd} = \mathbf{R}_s \cdot \mathbf{i}_{sd} - \boldsymbol{\omega}_d \cdot \boldsymbol{\varphi}_{sq} + \mathbf{p} \boldsymbol{\varphi}_{sd}$$
(9)

$$\mathbf{V}_{sq} = \mathbf{R}_{s} \mathbf{i}_{sq} + \omega_{d} \boldsymbol{\varphi}_{sd} + \mathbf{p} \boldsymbol{\varphi}_{sq}$$
(10)

$$\mathbf{V}_{rd} = \mathbf{R}_{r} \cdot \mathbf{i}_{rd} - \omega_{dA} \cdot \varphi_{rq} + \mathbf{p}\varphi_{rd}$$
(11)

$$\mathbf{V}_{\mathbf{r}q} = \mathbf{R}_{\mathbf{r}}.\mathbf{i}_{\mathbf{r}q} + \omega_{\mathbf{d}A}.\varphi_{\mathbf{r}d} + \mathbf{p}\varphi_{\mathbf{r}q}$$
(12)

where, $p\theta_{dA} = \omega_{dA}$ is the dq-winding speed expressed in electrical radians/second with regard to the speed of rotor A-axis. Further the voltage expressions in terms of inductances given in 13-16.

$$V_{sd} = L_{m} \cdot p(i_{sd} + i_{rd}) + R_{s} \cdot i_{sd} + L_{ls} \cdot p \cdot i_{sd} - \omega_{d} \cdot \varphi_{sq}$$
(13)

$$V_{sq} = L_{m}.p(i_{sq} + i_{rq}) + \omega_{d}.\varphi_{sd} + R_{s}.i_{sq} + L_{ls}.pi_{sq}$$
(14)

$$V_{rd} = L_m \cdot p(i_{sd} + i_{rd}) + R_r \cdot i_{rd} + L_{1r} \cdot p i_{rd} - \omega_{dA} \cdot \varphi_{rq}$$
(15)

$$V_{rq} = L_m \cdot p(i_{sq} + i_{rq}) + \omega_{dA} \cdot \varphi_{rd} + R_r \cdot i_{rq} + L_{lr} \cdot p i_{rq}$$
(16)

Let $V_{rd} = V_{rq} = 0$, because in squirrel cage motor the rotor conductor are shorted. The above equations explain the 3-phase induction motor dynamic model. The instantaneous reactive power and real power input in terms of dq variables are given in equation 17 and equation 18 [8].

$$Q_{in} = (3/2).(-V_{sd}.i_{sq} + V_{sq}.i_{sd})$$
(17)

$$P_{in} = (3/2).(V_{sd}.i_{sd} + V_{sq}.i_{sq})$$
(18)

The output power is given by equation 19.

$$P_{out} = (3/2).\omega_m.(\varphi_{rq}i_{rd} - \varphi_{rd}i_{rq})$$
(1)

The developed torque obtained is given by equation 20.

$$T_{em} = (3/4).P.(\varphi_{rq}.i_{rd} - \varphi_{rd}.i_{rq})$$
(20)

9)

III NUMERICAL TECHNIQUE TOOLS

The Successive Relaxation and Gauss-Seidel methods are available to estimate the flux linkages of 3-phase induction motor and to illustrate the steady state behaviour. Consider a system of n algebraic equations with n unknowns as given in equation 21.

 $a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$

Here a_{ij} represents the known coefficients, b_i are the known values and x_i are the unknowns to be determined. Where, i,j=1(1)n. In the matrix notation, the equation 21 can be written as equation 22 [9].

$$Ax = b \tag{22}$$

The solution using linear iterative method for the system of linear equations is given in equation 23.

 $X^{(k+1)} = H.X^{(k)} + c, k = 0, 1, 2, \dots$ (23)

Here, c is a column vector and H is the iteration matrix that depends on matrix A. The approximation for x at the kth iteration is $x^{(k)}$ and the approximation at the (k+1)th iterations is $x^{(k+1)}$. When k tends to infinity, $x^{(k)}$ converges to the exact solution as given in equation 24.

$$x = A^{-1}b \tag{24}$$

(25)

Now the equation 23 can be written as given in equation 25.

$$A^{-1}b = HA^{-1}b + c$$

3.1 Gauss-Seidel method: It is a most commonly used numerical method and it uses iterative schemes to solve a set of algebraic equations. Here $x_i^{(k+1)}$ substitutes $x_i^{(k)}$ instantaneously and the iteration is carried out till the desired degree of accuracy is reached. Hence, this method is also referred as successive displacement method. The Gauss-Seidel method can be written as shown in equation 26.

$$\begin{aligned} x_1^{(k+1)} &= (-1/a_{11}).(a_{12}x_2^{(k)} + a_{13}x_3^{(k)} + ... + a_{1n}x_n^{(k)}) + (b_1/a_{11}) \\ x_2^{(k+1)} &= (-1/a_{22}).(a_{21}x_1^{(k+1)} + a_{23}x_3^{(k)} + ... + a_{2n}x_n^{(k)}) + (b_2/a_{22}) \\ \dots \\ x_n^{(k+1)} &= (-1/a_{nn}).(a_{n1}x_1^{(k+1)} + a_{n2}x_2^{(k+1)} + ... + a_{n,n-1}x_{n-1}^{(k+1)}) + (b_n/a_{nn}) \end{aligned}$$

(26)

This is rearranged and represented in the matrix form as given in equation 27.

$$(D+L).x^{(k+1)} = -U.x^{(k)} + b$$
(27)

Using Gauss-Seidel method solution to equation 27 is obtained iteratively. The above equation is solved for x in left hand side by using earlier value of x found on right hand side.

3.2 Successive Relaxation method: It is used to solve the system of equations. We define an auxiliary vector \hat{x} as given in equation 28.

$$\hat{x}^{(k+1)} = -D^{-1}.L.x^{(k+1)} - D^{-1}.U.x^{(k)} + D^{-1}.b$$
(28)

The final solution is written as equation 29.

$$x^{(k+1)} = (1-w).x^{(k)} + w.\hat{x}^{(k+1)}$$
(29)

Substituting equation 28 in equation 29 and upon simplification, we get equation 30.

$$x^{(k+1)} = (D + w.L)^{-1} [(1 - w).D - w.U] .x^{(k)} + w.(D + w.L)^{-1} .b$$
(30)

When w=1, equation 30 becomes the Gauss-Seidel method. The quantity w is referred as relaxation parameter and $x^{(k+1)}$ is a weighted mean of $\hat{x}^{(k+1)}$ and $x^{(k)}$. The weights are non-negative for 0<w<1. If w is greater than one, then this method is referred as an over relaxation method. If w is less than 1 then it is an under relaxation method. The reason of using relaxation method is to increase the speed of iteration by minimizing the largest residual to zero.

IV PERFORMANCE EVALUATION OF 3-Φ INDUCTION MOTOR

The behaviour of stator and rotor flux linkages, torque and efficiency characteristics are obtained for different load conditions. Further power factor at various load conditions is evaluated. An induction motor with the following name plate details/specifications is used for experimentation and simulation. Power: 2.2 kilo Watts / 3HP, Voltage: 415 V, Number of phases: 3, Frequency: 50 cycles/sec., Number of poles: 4, Rated Current: 4.6 A, Rated Speed: 1400 revolutions per minute.

No-load and blocked rotor test are conducted to obtain perphase circuit parameters of the 3-phase induction motor. The obtained values are: R_s =9.3 Ω , R_r =13.1 Ω , X_s = X_r =13.94 Ω (at 50 cycles/sec.), X_m =371.13 Ω (at 50 cycles/sec.), and corresponding L_s = L_r =1.2244H, L_m =1.18H, synchronous speed (ω_s)=157 rad/sec. The following results are obtained and curves are drawn using both Gauss-Seidel and Successive-Relaxation methods. The results are compared with results obtained from the experimental results on 3-phase induction motor. In the experimental method shown in figure 2 a DC shunt machine is connected to a 3-phase induction motor.

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Fig. 2 Experimental setup: 3-phase induction motor coupled with DC shunt machine.

The following are the specifications of DC shunt machine: Power=3 kilo Watts, Voltage=220 Volts, Rated Full-Load Current=15.9 A, Rated Speed=1500 revolutions per minute, Excitation Voltage=220 Volts, Excitation Current=1.02 A, Efficiency=80%.

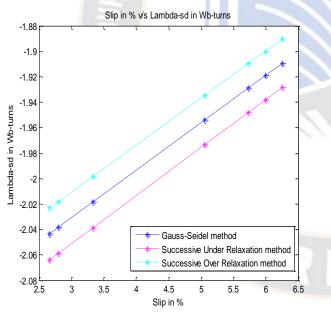


Fig. 3 Slip in % versus stator flux linkages ϕ_{sd} in weberturns.

It is depicted from figure 3 that φ_{sd} increases with increase in slip. It is because i) stator self inductance is assertive ii) as slip increases i_{sd} increases and i_{rd} decreases. Hence, as slip increases φ_{sd} increases. The value of φ_{sd} obtained at 6.26% slip is -1.9093 Wb-turns using Gauss-Seidel method and it is -1.8903 Wb-turns and -1.9285Wb-turns using Successive Over-Relaxation method and Successive Under -Relaxation method respectively.

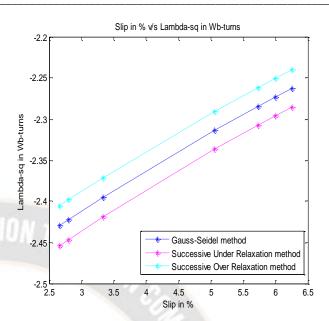


Fig. 4 Slip in % versus stator flux linkages ϕ_{sq} in weberturns.

Similarly as slip increases i_{sq} decreases and i_{rq} increases. Hence ϕ_{sq} increases with slip as shown in figure 4.

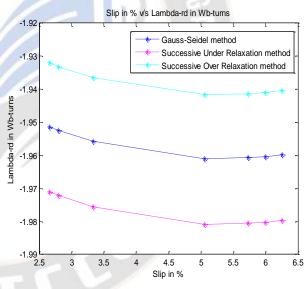
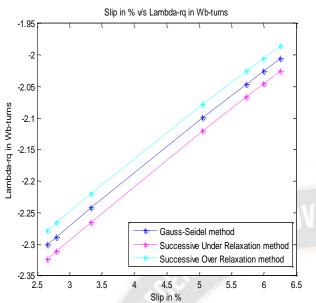
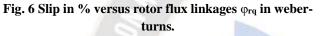


Fig. 5 Slip in % versus rotor flux linkages φ_{rd} in weberturns.

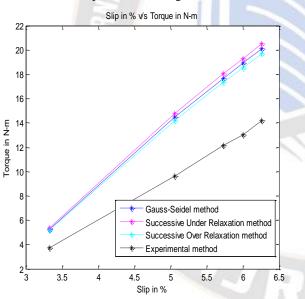
It is depicted from figure 5 that φ_{rd} decreases w.r.t. increase in slip. It is because i) the influence of rotor self inductance ii) as slip increases i_{rd} decreases and i_{sd} increases. Hence, as slip increases φ_{rd} decreases. The value of φ_{rd} obtained at 6.26% slip is -1.9598 Wb-turns using Gauss-Seidel method and it is-1.9404 Wb-turns and -1.9796 Wb-turns using Successive Over-Relaxation method and Successive Under-Relaxation method respectively.

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Similarly as slip increases i_{rq} increases and i_{sq} decreases. Hence ϕ_{rq} increases with slip as shown in figure 6.





As shown in figure 7, torque increases with increase in slip. It is because of decrease in φ_{rd} and increase in φ_{rq} . Hence the difference increases and torque increases with slip. This is given in equation 20. The value of torque obtained at 6.26% slip is 20.08 N-m using Gauss-Seidel method. The value of torque at full load obtained using Successive Under-Relaxation and Over-Relaxation method are 20.49 N-m and 19.69 N-m respectively. The torque obtained at 6.26% slip by experimental method is 14.20 N-m. The difference is assumed to be associated with meters accuracy, reading accuracy and dynamics of machine in real time condition.

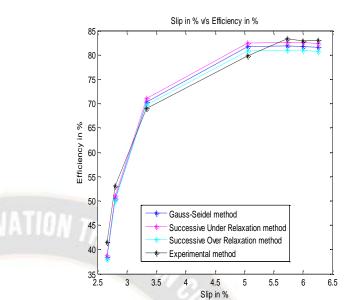
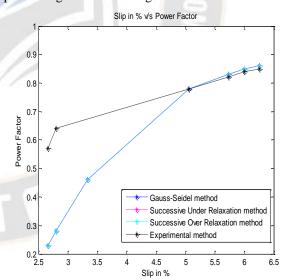
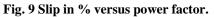


Fig. 8 Slip in % versus efficiency in %.

The efficiency obtained at 6.26% slip is 81.53% using Gauss-Seidel method. The efficiency at 6.26% slip using Successive Under-Relaxation and Over-Relaxation method are 82.35% and 80.72% respectively. The efficiency obtained from experimental test at 6.26% slip is 82.97%. The obtained efficiency by Gauss-Seidel and Relaxation methods are comparable with the experimental method and are within the acceptable range as shown in figure 8.

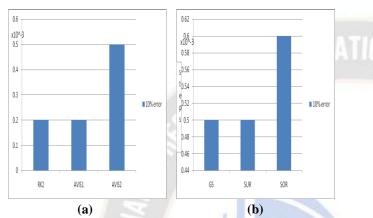


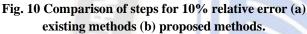


The power factor obtained at 6.26% slip is 0.86 using Gauss-Seidel method and both Relaxation methods. The obtained value of power factor from the experimental method at 6.26% slip is 0.85. The power factor obtained by both Gauss-Seidel method and relaxation methods are analogous with the experimental method. The power factor variation with slip is shown in figure 9. The machine, when running at $3/4^{\text{th}}$ to full load exhibit good power factor.

4.1 Comparison of the results with the published work:

The Gauss-Seidel method and both Relaxation methods require calculation of first-order derivative. The number of calculations increases with the order of the equations also increases the simulation time. The Gauss-Seidel method and both Relaxation methods took almost same number of steps and comparable with existing methods. The step size that provides 10% relative error for different methods is summarized as shown in figure 10 [10].





RK2: Second-order Runge–Kutta,AVIS2: Average Voltage at the Integration Step

The results obtained from no–load, locked rotor and dc resistance test are as follows. The change in efficiency observed is 14.6% from no load to full load. The highest power factor obtained is 0.686 [11]. The above results are compared with the results obtained from both Relaxation and Gauss-Seidel methods. The change in efficiency is 15% to 20% from no load to full load. The highest value of power factor obtained is 0.86.

V CONCLUSION

The dynamic model is developed and the mathematical model is presented for a $3-\varphi$ induction motor. The numerical technique tools are used to study the behaviour. It is observed that there is a close matching between the results obtained from the Successive Relaxation methods, Gauss-Seidel method and the experimental method. Thus proves the validity of numerical methods for analysing the steady state behaviour. The use of numerical methods, reduce the computational overheads while analysing the steady-state behaviour. It is evident that as slip increases the power factor of the motor increases. Thus, the input current increases and addresses also power quality related issues.

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