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Lukasiewicz Fuzzy Implication Operator on Pythagorean Fuzzy Tautological Matrices

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Abstract— In this paper, introduced Pythagorean fuzzy tautologial matrices and Pythagorean fuzzy cotautological matrices and some properties of Lukasiwicz implication operator over Pythagorean fuzzy tautologial matrices and Pythagorean fuzzy cotautological matrices are discussed. Also discussed the relation between implication with Lukasiewicz disjunction and conjunction operations of PFCMs and PFCTMs.

Keywords- Intuitionistic Fuzzy Matrix, Pythagorean Fuzzy Set, Pythagorean Fuzzy Matrix, Disjunction, Conjunction, Implication.

I. INTRODUCTION

Pal et.al[3, 10] introduced Intuitionistic fuzzy matrices also in[2], they discussed new operations on intuitionistic fuzzy matrices and investigated their algebraic properties. Atanassov and Tcvetkov[1] defined on Lukasiewicz's intuitionistic fuzzy disjunction and conjunction operations and investigated their algebraic properties. Muthuraji and Sriram[6] introduced two operators conjunction and disjunction from Lukasiewic'z type over intuitionistic fuzzy matrix(IFM) and investigated their algebraic properties. Also in [6], they proved the set of all IFMs is a commutative monoid under these operations.

Yager[15] introduced Pythagorean fuzzy set(PFS) characterized by a membership degree and a non membership degree satisfying the condition that the square sum of its membership degree and non membership degree is equal to or less than 1, has much stronger ability than intuitionistic fuzzy set to model such uncertain information in multi-criteria decision making(MCDM) problems.

Zhang and Xu[16] defined some novel operational laws of PFS and discuss its desirable properties. The motivation of introducing PFSs is that in the real-life decision process, the sum of the support degree and the against degree to which an alternative satisfying a criterion provided by the decision maker may be bigger than 1 but their square sum is equal to or less than 1.

Silambarasan and Sriram[11] introduced Pythagorean fuzzy matrix(PFM) and its algebraic operations. Venkatesan and Sriram[12] they defined multiplicative operations of Pythagorean fuzzy matrices and studied some of the basic properties of these operations with other predefined operators. Muthuraji[7, 8] studied some properties of operations conjunction, disjunction and implication from Lukasiewicz's type over Intuitionistic fuzzy matrices. Venkatesn and sriram[13, 14] extend these operations to PFMs and studied some of the basic properties of these operations with other predefined operators.

Pal and Khan[4] introduced intuitionistic fuzzy tautological matrices and its algebraic operations. Murugadas and Lalitha[5] they defined intuitionistic fuzzy cotautological matrices and its algebraic operations.

Muthuraji and Anitha[9] they discused the Lukasiewicz imlication on intuitionistic fuzzy tautological matrices discussed its desirable properties. I extend these operations to PFMs and studied some of the basic properties of these operations with other predefined operators.

The remainder of this paper is organized as follows. In Section 2, the basic definitions of PFM are given. In Section 3, introduced Pythagorean fuzzy tautologial matrices and Pythagorean fuzzy cotautological matrices and some properties of Lukasiwicz implication operator over Pythagorean fuzzy tautologial matrices and Pythagorean fuzzy cotautological matrices are discussed.

II. PRELIMINARIES

In this section, some basic definitions which are essential for the development of this dissertation are discussed.

Definition 2.1. [3] A intuitionistic fuzzy matrix(IFM) is a matrix of pairs $A = (\langle a_{ij}, a'_{ij}\rangle)$ of non negative real numbers $a_{ij}, a'_{ij} \in [0,1]$ satisfying the condition $0 \leq a_{ij} + a'_{ij} \leq 1$, for all i, j.

Definition 2.2. [2] For any two IFMs A and B of the same size, then

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$$A \rightarrow_1 B = \left(\left\langle max(a'_{ij}, b_{ij}), min(a_{ij}, b'_{ij})\right\rangle\right).$$

Definition 2.3. [15] Let a set X be a universe of discourse A Pythagorean fuzzy set(PFS) P is an object having the form, $P = \left(\left\langle x, P(\mu_p(x), \nu_p(x)) \mid (x \in X)\right\rangle\right)$, where the function $\mu_p: X \rightarrow [0,1]$ and $\nu_p: X \rightarrow [0,1]$ defines the degree of membership and degree of non-membership of the element $x \in X$ to P, respectively, and for every $x \in X$, it holds that $(\mu_p(x))^2 + (\nu_p(x))^2 \leq 1$.

Definition 2.4. [11] A Pythagorean fuzzy matrix(PFM) is a matrix of pairs $A = \left(\left\langle a_{ij}, a'_{ij}\right\rangle\right)$ of a non negative real numbers $a_{ij}, a'_{ij} \in [0,1]$ satisfying the condition $a_{ij}^2 + {a'_{ij}}^2 \leq 1$, for all i, j.
The numbers a_{ij} and a'_{ij} denote the degree of membership and the degree of non-membership of the ij^{th} element in A respectively.

Definition 2.5. [13] For any two PFMs $A, B \in \mathbb{P}_{mn}$, we have
(i)$A^C = \left(\left\langle a'_{ij}, a_{ij}\right\rangle\right)$ (The complement of A),
(ii)$A^T = \left(\left\langle a_{ji}, a'_{ji}\right\rangle\right)$ (The transpose of A),
(iii)$A \leq B$ if and only if $a_{ij} \leq b_{ij}$ and $a'_{ij} \geq b'_{ij}$,
(iv) $A \geq B$ if and only if $a_{ij} \geq b_{ij}$ and $a'_{ij} \leq b'_{ij}$,
(iv)$A \wedge B = \left(\left\langle min(a_{ij}, b_{ij}), max(a'_{ij}, b'_{ij})\right\rangle\right)$,
(v)$A \vee B = \left(\left\langle max(a_{ij}, b_{ij}), min(a'_{ij}, b'_{ij})\right\rangle\right)$,
(vi) $\Box A = \left(\left\langle a_{ij}, \sqrt{1 - a_{ij}^2}\right\rangle\right)$,
(vii) $\diamond A = \left(\left\langle \sqrt{1 - {a'_{ij}}^2}, a'_{ij}\right\rangle\right)$,
(viii) The $m \times n$ zero PFM O is an PFM all of whose entries are $\langle 0,1\rangle$,
The $m \times n$ universal PFM J is an PFM all of whose entries are $\langle 1,0\rangle$.

Definition 2.6. [13] For any two PFMs $A, B \in \mathbb{P}_{mn}$, we have
(i) $A \vee_L B =$
$$\left(\left\langle \sqrt{min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right),$$
(ii) $A \wedge_L B =$
$$\left(\left\langle \sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)}\right\rangle\right).$$

Definition 2.7. [14] For any two PFMs $A, B \in \mathbb{P}_{mn}$, we have
(i) $A \rightarrow_L B =$
$$\left(\left\langle \sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)}, \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right),$$

III. MAIN RESULTS

In this section, define Pythagorean Fuzzy Tautological Matrix (PFTM) and Pythagorean Fuzzy Cotautological Matrix (PFCTM) and some properties are conjunction, disjunction and implication are discussed over PFTM and have shown that some expressions involving all the above said operators always an PFCTM.

Definition 3.1. An Pythogorean fuzzy matrix of order m × n is called Pythogorean fuzzy tautological matrix(PFTM) if and only if $a_{ij} \geq a'_{ij}$ for all i, j.

Definition 3.2. An Pythogorean fuzzy matrix of order m × n is called Pythogorean fuzzy cotautological matrix(PFCTM) if and only if $a_{ij} \leq a'_{ij}$ for all i, j.

Property 3.1. For any two PFTMs $A, B \in \mathrm{P}_{mn}$, we have
(i) $A \vee_L B$ is an PFTM.
(ii) $A \wedge_L B$ is an PFM.
(iii) $A \rightarrow_L B$ is also an PFTM.

Proof:
From Definition 2.6,
$A \vee_L B$
$$= \left(\left\langle \sqrt{min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right).$$
Since A and B are PFTMs.
From Definition 2.5, $a_{ij} \geq a'_{ij}$ and $b_{ij} \geq b'_{ij}$ then
$$\sqrt{min(1, a_{ij}^2 + b_{ij}^2)} \geq \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}$$
Hence $A \vee_L B$ is an PFTM.

(ii) $A \wedge_L B =$
$$\left(\left\langle \sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)}\right\rangle\right).$$
Since $a_{ij} \geq a'_{ij}$,
$\sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)} \geq \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)}$ or
$\sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)} \leq \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)}$.
Hence $A \wedge_L B$ is an PFM.

(iii) $A \rightarrow_L B =$
$$\left(\left\langle \sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)}, \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right).$$
Since $b_{ij} \geq b'_{ij}$,
$\sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)} \geq \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)}$.
Hence $A \rightarrow_L B$ is an PFTM.

Property 3.2. For any two PFCTMs $A, B \in \mathrm{P}_{mn}$, we have
(i) $A \vee_L B$ is an PFM.
(ii) $A \wedge_L B$ is an PFCTM.
(iii) $A \rightarrow_L B$ is also an PFTM.

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Proof:
From Definition 2.6,

$$A \vee_L B = \left(\left\langle \sqrt{min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)} \right\rangle\right).$$

Since A and B are PFCTMs.
From Definition 2.5, $a_{ij} \leq a'_{ij}$ and $b_{ij} \leq b'_{ij}$ then

$\sqrt{min(1, a_{ij}^2 + b_{ij}^2)} \leq \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}$ or

$\sqrt{min(1, a_{ij}^2 + b_{ij}^2)} \geq \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}$.

Hence $A \vee_L B$ is an PFM.

(ii) $A \wedge_L B =$

$$\left(\left\langle \sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)} \right\rangle\right).$$

Since $a_{ij} \leq a'_{ij}$,

$\sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)} \leq \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)}$.

Hence $A \wedge_L B$ is an PFCTM.

(iii) $A \rightarrow_L B =$

$$\left(\left\langle \sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)}, \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)} \right\rangle\right).$$

Since $b_{ij} \leq b'_{ij}$,

$\sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)} \leq \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)}$.

Hence $A \rightarrow_L B$ is an PFCTM.

Property 3.3. For any A be an PFTM and B be an PFCTM $A, B \in \mathrm{P}_{mn}$, we have
(i) $A \vee_L B$ is an PFTM.
(ii) $A \wedge_L B$ is an PFCTM.
(iii) $A \rightarrow_L B$ is also an PFM.

Proof:
(i) From Definition 2.6,

$$A \vee_L B = \left(\left\langle \sqrt{min(1, a_{ij}^2 + b_{ij}^2)}, \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)} \right\rangle\right).$$

From Definition 2.6, $a_{ij} \geq a'_{ij}$ and $b_{ij} \geq b'_{ij}$ then

$\sqrt{min(1, a_{ij}^2 + b_{ij}^2)} \geq \sqrt{max(0, {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}$.

Hence $A \vee_L B$ is an PFTM.

(ii) $A \wedge_L B =$

$$\left(\left\langle \sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)}, \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)} \right\rangle\right).$$

Since $a_{ij} \leq a'_{ij}$,

$\sqrt{max(0, a_{ij}^2 + b_{ij}^2 - 1)} \leq \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2)}$.

Hence $A \wedge_L B$ is an PFCTM.

(iii) $A \rightarrow_L B =$

$$\left(\left\langle \sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)}, \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)} \right\rangle\right).$$

Since $a_{ij} \geq a'_{ij}$ and $b_{ij} \leq b'_{ij}$,

$\sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)} \geq \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)}$ or

$\sqrt{min(1, {a'_{ij}}^2 + b_{ij}^2)} \leq \sqrt{max(0, a_{ij}^2 + {b'_{ij}}^2 - 1)}$.

Hence $A \rightarrow_L B$ is an PFM.

Corollary 3.1: Suppose A is an PFCTM and B is an PFTM then $A \rightarrow_L B$ is an PFTM.

Property 3.4. For any two PFMs $A, B \in \mathrm{P}_{mn}$, then the given statements are PFTMs.
(i) $A \rightarrow_L A$.
(ii) $A \rightarrow_L (B \rightarrow_L A)$.

Proof:
(i) $A \rightarrow_L A =$

$$\left(\left\langle \sqrt{min(1, {a'_{ij}}^2 + a_{ij}^2)}, \sqrt{max(0, a_{ij}^2 + {a'_{ij}}^2 - 1)} \right\rangle\right).$$

Now it is clear

$\sqrt{min(1, {a'_{ij}}^2 + a_{ij}^2)} \geq \sqrt{max(0, a_{ij}^2 + {a'_{ij}}^2 - 1)}$.

Hence $A \rightarrow_L A$ is an PFTM.

(ii) $B \rightarrow_L A =$

$$\left(\left\langle \sqrt{min(1, {b'_{ij}}^2 + a_{ij}^2)}, \sqrt{max(0, b_{ij}^2 + {a'_{ij}}^2 - 1)} \right\rangle\right).$$

$$A \rightarrow_L (B \rightarrow_L A) =$$

$$\left(\left\langle \sqrt{min(1, {a'_{ij}}^2 + {b'_{ij}}^2 + a_{ij}^2)}, \sqrt{max(0, a_{ij}^2 + b_{ij}^2 + {a'_{ij}}^2 - 1 - 1)} \right\rangle\right).$$

Since $\sqrt{min(1, ({a'_{ij}}^2 + a_{ij}^2) + {b'_{ij}}^2)} \geq \sqrt{max(0, (a_{ij}^2 + {a'_{ij}}^2) + b_{ij}^2 - 1 - 1)}$ for all i,j.

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$$\sqrt{min(1, \quad a'^2_{ij} + b'^2_{ij} + a^2_{ij})} \geq \sqrt{max(0, a^2_{ij} + b^2_{ij} + a'^2_{ij} - 1 - 1)}$$

Suppose $(B \rightarrow_L A) = (1,0)$ for some i,j then
$A \rightarrow_L (B \rightarrow_L A)$ is also (1,0).
Hence $A \rightarrow_L (B \rightarrow_L A)$ is an PFTM.

Property 3.5. For any two PFMs $A, B \in P_{mn}$, then the given statements are PFTMs.
(i) $A \rightarrow_L (A \vee_L B)$.
(ii) $B \rightarrow_L (A \vee_L B)$.
(iii) $A \rightarrow_L (B \rightarrow_L (A \vee_L B))$.

Proof:
(i)
$$A \vee_L B = \left(\left\langle \sqrt{min(1, a^2_{ij} + b^2_{ij})}, \sqrt{max(0, a'^2_{ij} + b'^2_{ij} - 1)} \right\rangle\right)$$
$$A \rightarrow_L (A \vee_L B) = A \rightarrow_L \left(\left\langle \sqrt{min(1, a^2_{ij} + b^2_{ij})}, \sqrt{max(0, a'^2_{ij} + b'^2_{ij} - 1)} \right\rangle\right)$$

If $\sqrt{min(1, a^2_{ij} + b^2_{ij})}, \sqrt{max(0, a'^2_{ij} + b'^2_{ij} - 1)} = (1,0)$
then $A \rightarrow_L (A \vee_L B) = (1,0)$.
Otherwise
$$A \rightarrow_L (A \vee_L B) = \left(\left\langle \sqrt{min(1, a'^2_{ij} + a^2_{ij} + b^2_{ij})}, \sqrt{max(0, a^2_{ij} + a'^2_{ij} + b'^2_{ij} - 1 - 1)} \right\rangle\right).$$

It is clear that $\sqrt{min(1, a'^2_{ij} + a^2_{ij} + b^2_{ij})} \geq \sqrt{max(0, a^2_{ij} + a'^2_{ij} + b'^2_{ij} - 1 - 1)}$.
Hence $A \rightarrow_L (A \vee_L B)$ is an PFTM.

(ii) The proof of (ii) is similar to (i).

(iii) From (ii) $B \rightarrow_L (A \vee_L B)$ is an PFTM.
From Properties 1,2 and Corollory 3.1 it is evident that
$A \rightarrow_L (B \rightarrow_L (A \vee_L B))$ is an PFTM.

Property 3.6. For any two PFMs $A, B \in P_{mn}$, then the given statements are PFTMs.
$(i)(A \wedge_L B) \rightarrow_L A$.
(ii) $(A \wedge_L B) \rightarrow_L B$.

Proof:
(i) $A \wedge_L B$
$$= \left(\left\langle \sqrt{max(0, a^2_{ij} + b^2_{ij} - 1)}, \sqrt{min(1, a'^2_{ij} + b'^2_{ij})} \right\rangle\right).$$
$$(A \wedge_L B) \rightarrow_L A = \left(\left\langle \sqrt{max(0, a^2_{ij} + b^2_{ij} - 1)}, \sqrt{min(1, a'^2_{ij} + b'^2_{ij})} \right\rangle\right) \rightarrow_L A$$
If $A \wedge_L B = (1,0)$ then$(A \wedge_L B) \rightarrow_L A = (1,0)$.
Otherwise
$$(A \wedge_L B) \rightarrow_L A = \left(\left\langle \sqrt{min(1, a'^2_{ij} + b'^2_{ij} + a^2_{ij})}, \sqrt{max(0, a^2_{ij} + b^2_{ij} + a'^2_{ij} - 1 - 1)} \right\rangle\right)$$

Since $\sqrt{min(1, a'^2_{ij} + b'^2_{ij} + a^2_{ij})} \geq \sqrt{max(0, a^2_{ij} + b^2_{ij} + a'^2_{ij} - 2)}$.
Hence $(A \wedge_L B) \rightarrow_L A$ is an PFTM.

(ii) The proof of (ii) is similar to (i).

Property 3.7. If $A \geq B$ then $A \rightarrow_L B$ is an PFTM.

Proof:
$A \leq B$ if and only if $a_{ij} \leq b_{ij}$ and $a'_{ij} \geq b'_{ij}$.
Since $a'_{ij} \geq b'_{ij}$
$$\sqrt{a'^2_{ij} + b^2_{ij}} \geq \sqrt{b'^2_{ij} + a^2_{ij} - 1}.$$
Hence $A \rightarrow_L B$ is an PFTM.

Property 3.8. For any two PFMs $A, B \in P_{mn}$, then the given statements are PFTMs.
$(i)(A \wedge_L (A \rightarrow_L B)) \rightarrow_L B$.
(ii) $((A \rightarrow_L B) \wedge_L B^C) \rightarrow_L A^C$.

Proof:
$$A \rightarrow_L B = \left(\left\langle \sqrt{min(1, \quad a'^2_{ij} + b^2_{ij})}, \sqrt{max(0, a^2_{ij} + b'^2_{ij} - 1)} \right\rangle\right).$$
Case (i) If $A \rightarrow_L B = (1,0)$ then
$A \wedge_L (A \rightarrow_L B) = (\langle a_{ij}, a'_{ij} \rangle) \wedge_L (\langle 0,1 \rangle) = (\langle a_{ij}, a'_{ij} \rangle) = A$.
$(A \wedge_L (A \rightarrow_L B)) \rightarrow_L B = (A \rightarrow_L B) = (\langle 0,1 \rangle)$.

Case (ii) If $A \rightarrow_L B = \sqrt{a'^2_{ij} + b^2_{ij}}, \sqrt{b'^2_{ij} + a^2_{ij} - 1}$.

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$$A \wedge_L (A \rightarrow_L B)$$
$$= \left(\langle a_{ij}, a'_{ij}\rangle\right) \wedge_L \left(\left\langle \sqrt{{a'_{ij}}^2 + b_{ij}^2}\,, \sqrt{{b'_{ij}}^2 + a_{ij}^2 - 1}\right\rangle\right)$$
$$= \left(\left\langle \sqrt{max(\,0, \quad a_{ij}^2 + {a'_{ij}}^2 + b_{ij}^2 - 1)},\right.\right.$$
$$\left.\left.\sqrt{min(\,1, a_{ij}^2 + {a'_{ij}}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right)$$

Subcase (ii.a) If $A \wedge_L (A \rightarrow_L B) = (\langle 0,1\rangle)$ then
$\left(A \wedge_L (A \rightarrow_L B)\right) \rightarrow_L B = (\langle 0,1\rangle) = \left(\langle b_{ij}, b'_{ij}\rangle\right) = (\langle 0,1\rangle).$

$$\left(\left\langle \sqrt{min(\,1, \quad {a'_{ij}}^2 + a_{ij}^2 + {b'_{ij}}^2 - 1 + b_{ij}^2)},\right.\right.$$
$$\left.\left.\sqrt{max(\,0, a_{ij}^2 + {a'_{ij}}^2 + b_{ij}^2 - 1 + {b'_{ij}}^2 - 1)}\right\rangle\right)$$

Now it is clear that

$$\left(\left\langle \sqrt{min(\,1, \quad {a'_{ij}}^2 + a_{ij}^2 + {b'_{ij}}^2 - 1 + b_{ij}^2)}\right.\right.$$
$$\left.\left.\geq \sqrt{max(\,0, a_{ij}^2 + {a'_{ij}}^2 + b_{ij}^2 - 1 + {b'_{ij}}^2 - 1)}\right\rangle\right)$$

Hence from case (i) and (ii) $(\, A \wedge_L (A \rightarrow_L B)) \rightarrow_L B$ is an PFTM.

(ii)

$$A \rightarrow_L B$$
$$= \left(\left\langle \sqrt{min(\,1, \quad {a'_{ij}}^2 + b_{ij}^2)}, \sqrt{max(\,0, a_{ij}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right).$$
$$(A \rightarrow_L B) \wedge_L B^C$$
$$= \left(\left\langle \sqrt{min(\,1, \quad {a'_{ij}}^2 + b_{ij}^2)}, \sqrt{max(\,0, a_{ij}^2 + {b'_{ij}}^2 - 1)}\right\rangle\right)$$
$$\wedge_L \left(\langle b'_{ij}, b_{ij}\rangle\right)$$

Case (i) If $(A \rightarrow_L B) = (1,0)$ then

$$(A \rightarrow_L B) \wedge_L B^C = (1,0) \wedge_L \left(\langle b'_{ij}, b_{ij}\rangle\right) = \left(\langle b'_{ij}, b_{ij}\rangle\right)$$
$$= \left(\left\langle \sqrt{min(\,1, \quad b_{ij}^2 + {a'_{ij}}^2)}, \sqrt{max(\,0, {b'_{ij}}^2 + a_{ij}^2 - 1)}\right\rangle\right)$$
$$= (A \rightarrow_L B) = (1,0).$$

Hence $((A \rightarrow_L B) \wedge_L B^C) \rightarrow_L A^C$ is an PFTM.

IV. CONCULUSION

Defind Pythagorean fuzzy tautologial matrices and Pythagorean fuzzy cotautological matrices and some properties of Lukasiwicz implication operator over Pythagorean fuzzy tautologial matrices and Pythagorean fuzzy cotautological matrices are investigated. Also discussed the relation between implication with Lukasiewicz disjunction and conjunction operations of PFCMs and PFCTMs.

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