

## Modeling Heterogeneous Vehicle Routing Problem with Strict Time Schedule

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**Abstract**—Vehicle Routing Problem with time windows (VRPTW) is a well known combinatorial optimization problem normally to be used for obtaining the optimal set of routes used by a fleet of vehicles in logistic system. In VRPTW it is assumed that the fleet of vehicles are all homogeny. In this paper we consider a variant of the VRPTW in which the assumption of homogeny is dropped. Now the problem is called Heterogeneous VRP (HVRP). As the logistic company has so many customers, it puts a very strict restriction in time delivery for each vehicle used. Regarding to the structure of the problem we use integer programming approach to model the problem. A feasible neighbourhood method is developed to solve the model.

**Keywords**- Logistic system, Routing problem, Heterogeneous vehicle, Feasible neighbourhood search

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### I. INTRODUCTION

The vehicle routing problem (VRP) model has been applied to many real life problems such as transportation, and communications. Traditionally, the VRP is intended to deliver an amount of goods to the customers from a single distribution center along a set of routes. The objective of the problem is to find a set of minimum cost routes to fulfill customer demands. The cost of a route can be the total distance or time travelled by the vehicle associated with it. As the nature of the problem is combinatorial, it is not surprising that many researchers have been working in this area to obtain new approaches in selecting the best routes, particularly for large scale problems. Studies on this NP-hard problem have resulted in several exact and heuristic techniques of general applicability [1], [2], [3], [4]. Interesting review of VRP can be found in [5], or in a book by [6].

The VRP with time windows (VRPTW) is a well-studied version of VRP, in which the service time for customers is restricted with lower and upper bound. This version combines vehicle routing and scheduling problem which often arises in many real-world applications. The objective is to optimize the use of a number of vehicles that must make a number of stops to serve a set of customers, and to specify which customers should be served by each vehicle and in what order to minimize the cost, subject to vehicle capacity and service time restrictions [7]. The problem involves assignment of vehicles to trips such that the assignment cost and the corresponding routing cost are minimal.

Using graph, the VRPTW can be described as follows: Let  $G = (V, E)$  be a connected digraph, in which  $V$  is a set of  $n + 1$  nodes, the reachability of each node only within a specified time interval or time window, and  $E$  is a set of arcs with non-negative weights representing travel distances. One of the nodes of  $V$  is designated as the depot. Each node  $i$ , except the depot, requests a service of size  $q_i$ .

Due to the complexity nature of the problem the VRPTW has been the subject of intensive research, particularly in

developing new heuristic and exact optimisation approaches. Even though it is a well-known NP-hard problem, there are exact methods have been proposed. These exact methods are of type: branch and bound [8], branch and cut [9], column generation [10], [11], review of exact methods can be found in [12]. Because of the high complexity level of the VRPTW and its wide applicability to real-life situations, solution techniques capable of producing high quality solutions in limited time, i.e. heuristics, are the most popular. [13] propose a modified artificial bee colony algorithm to solve VRPTW. A hybrid metaheuristic based on Large Neighbourhood Search approach and Variable Neighbourhood Search technique was proposed by [14] to tackle the VRPTW.

The logistic company focused in this paper has various type of vehicle with different capacities in its operation to deliver demands for customers. The variant of VRP which considers mixed fleet of vehicles is called Heterogeneous VRP (HVRP), introduced firstly by [15]. This generalization is important in practical terms, for most of customers demand are served by several type of vehicles [16], [17]. The objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

From the literature, we would be able to observe that the most approach addressed for solving the HVRP is heuristics. [18] presented a deterministic annealing meta heuristic for the HVRPTW, and then [19] developed a linearly scalable hybrid threshold-accepting and guided local search meta heuristic for tackling large scale HVRPTW instances. [20] designed a trajectory search heuristic to solve a large-scale HVRPTW considering multi-trips. [21] presented an Adaptive Memory Programming solution approach for the HVRPTW that provides very good results in the majority of the benchmark instances examined. [22] proposed a hybrid algorithm for the problem which combine an Iterated Local Search (ILS) based heuristic and a Set Partitioning (SP) exact formulation. [23] present a modified column generation to solve HVRPTW. Recently, [24] addressed a constructive heuristic approach for solving time-dependent multi depot HVRPTW.

This paper concerns with HVRP with strict time schedule. The basic framework of the vehicle routing part can be viewed as a Vehicle Routing Problem with Time Windows (VRPTW) in which there are a limited number of vehicles, characterized by different capacities are available and the customers have a specified time windows for services. We address a large-scale mixed integer programming formulation to model the problem. A feasible neighbourhood heuristic search is presented to get the sub-optimal integer feasible solution

## II. PROBLEM STATEMENT

A large logistic company in Medan city, Indonesia, is facing a problem about how to deliver customers package efficiently and within a time frame schedule. The customer nodes are located within the whole are of Medan city. As the company has a lot of customer and order to be delivered it uses fleet of vehicles with different capacity and type. So this is a HVRPTW with strict time schedule, or it is called HVRPSST.

In terms of graph theory, HVRPSST can be described as follows. A complete directed graph  $G = (V, E)$  is defined,  $V = \{0, 1, \dots, n\}$  represents the customer nodes set and  $A = \{(i, j) : i, j \in V, i \neq j\}$  represents the set of vehicle route. For each route  $(i, j) \in A$  a distance (or travel cost)  $c_{ij}$  is defined.  $V = \{0\}$  is the depot node, center of service, where all vehicles are located. Define  $V_c$  is the set of customers' node. Each node  $i \in V_c$  has a known fixed demand  $w_i \geq 0$ , a service time  $s_i \geq 0$ , and a service time windows  $[a_i, b_i]$ . In particular, at depot the demand  $w = 0$  and service time  $t = 0$ .

As this is a heterogeneous problem, the fleet of  $K$  vehicles has  $m$  different type of vehicles, each with capacity  $Q_m$ . Let  $n_m$  be the number of vehicles available to be used, and let  $K_m$  be the set of vehicle type  $m$ . Each customer is served accordingly by exactly one vehicle. At the depot, we define a time window for vehicles to leave and to return to depot with  $[a_0, b_0]$ . Accordingly, the arrival time of a vehicle at customer  $i$  is  $a_i$  and its departure time is  $b_i$ . Each type of vehicle is imposed with a fixed cost,  $f_m$ . Further more, a fixed acquisition cost  $f_k$  is incurred for each of vehicle  $k$  in the routes. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer  $i$  before  $a_i$  and after  $b_i$  however, the vehicle can arrive before  $a_i$  and wait for service.

Each customer node  $i \in V_c$  has a known daily demand,  $w_i$ , service frequency,  $\sigma^t$  and a minimum service frequency,  $\sigma^t$ , measured in days,  $t$ , per period. The demand accumulated between visits,  $w_i$ , is a function of the daily demand of the node. The stopping cost at a node  $i$ ,  $\tau_i$ , is a function of the frequency of the schedule since more items accumulate with less frequent service and, therefore, require more time to load/unload. Associated with each arc  $(i, j) \in A$  is a known travel cost,  $c_{ij}$ . There is a set  $K$  of vehicles, each with

capacity  $C$ , and  $T$  is the set of workdays in the planning horizon,

The decision variables are defined as follows.

- $x_{ij}^k = \begin{cases} 1 & \text{if vehicle of type } k \in K \text{ is to deliver } i \in V_c \text{ to customer } j \in V_c; \\ 0 & \text{otherwise;} \end{cases}$
- $x_{ij}^m = \begin{cases} 1 & \text{if vehicle of type } m \in K_m \text{ is to deliver } i \in V_c, i \neq j; \\ 0 & \text{otherwise;} \end{cases}$
- $z_0^m = \begin{cases} 1 & \text{if vehicle type } m \in K \text{ is available;} \\ 0 & \text{active at depot;} \\ 0 & \text{otherwise;} \end{cases}$
- $l_i^m$  Arrival time for vehicle type  $m \in K_m$  at customer  $i \in V_c$  (non-negative continuous variable)
- $u_i^m$  Duration of service of vehicle type  $m \in K_m$  at customer  $i \in V_c$  (non-negative continuous variable)

## III. MATHEMATICAL MODELING

The objective is to minimize the total cost, expressed in Equation (1), which consists of the traveling cost of vehicle used, stopping cost and the cost for operating vehicles

$$\text{Minimize } \sum_{i \in V} \sum_{j \in V_c} c_{ij} \sum_{k \in K} x_{ij}^k + \sum_{(i,j) \in V_c} \sum_{m \in K_m} \tau_{ij} \sigma_{ij} x_{ij}^m + \sum_{m \in K_m} f_m z_0^m \quad (1)$$

Subject to several constraints

$$\sum_{k \in K} x_{0j}^k = 1, \quad \forall j \in V_c \quad (2)$$

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \quad \forall i \in V_c \quad (3)$$

$$\sum_{i \in V} x_{ij}^k - \sum_{i \in V} x_{ji}^k = 1; \quad \forall j \in V_c, \forall k \in K \quad (4)$$

$$x_{ij}^m \leq z_0^m, \quad (i, j) \in V_c, \forall m \in K_m \quad (5)$$

$$\sum_{j \in V_c} x_{1j}^k \leq 1; \quad \forall k \in K \quad (6)$$

$$\sum_{i \in V_c, i > 1} x_{i1}^k \leq 1; \quad \forall k \in K \quad (7)$$

$$\sum_{i \in V_c} d_i \sum_{j \in V_c} x_{ij}^m \leq Q_m; \quad \forall m \in K_m \quad (8)$$

$$x_{ij}^m (l_i^m + u_i^m + s_i + t_{ij} - l_j^m) = 0; \quad \forall m \in K_m, (i, j) \in A \quad (9)$$

$$l_i^m \leq a_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c \quad (10)$$

$$a_i \sum_{j \in V_c} x_{ij}^m \leq l_i^m + u_i^m \leq b_i \sum_{j \in V_c} x_{ij}^m; \quad \forall m \in K_m, i \in V_c \quad (11)$$

$$\sum_{j \in V_c} w_j x_{oj}^m \leq n_m; \quad \forall m \in K_m \quad (12)$$

$$\sum_{i \in V} \sum_{j \in V_c} \sum_{m \in K_m} x_{ij}^m \leq 1 \quad (13)$$

$$\sum_{i \in V_c} \sum_{j \in V} \sum_{m \in K_m} x_{ij}^m \leq 1 \quad (14)$$

$$x_{0j}^k, x_{ij}^m, z_0^m \in \{0,1\}; \quad \forall i \in V, \forall j \in V_c, \forall k \in K, \forall m \in K_m \quad (15)$$

$$l_i^m, u_i^m \geq 0; \quad \forall i \in V_c, \forall m \in K_m \quad (16)$$

Constraints (2) and (3) are to force that only one vehicle regardless their type are allowed to enter and depart once from every customer node and will be back to the depot. Constraint (4) describes a flow conservation in order to maintain the continuity of each vehicle route. Constraint (5) is to guarantee that each customer is served only by the available and active vehicle of the corresponding type. Constraints (6) and (7) will check the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the depot, not more than one, respectively. Constraints (8) are to force that each delivery does not exceed the capacity of each type of vehicle. Constraints (9) maintains the equilibrium among the arrival time, duration of service, service time and travel time at customers in the routes chosen. Constraints (10) and (11) present strict time window of service for each customer. Constraint (12) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot. Constraints (13) and (14) allow the unused vehicles to stay in the depot. Constraints (15) and (16) are to state the status of each variable.

#### IV. THE METHOD PROPOSED

Stage 1.

Step 1. Solve the relaxed problem. If the result is a feasible integer solution. Stop.

The original problem has been solved.

Otherwise go to step 2.

Step 2. Get row  $i^*$  such that  $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 3. Perform a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 4. Calculate the maximum movement of nonbasic  $j$ ,

$$\sigma_{ij} = v_{i^*}^T \alpha_j$$

With corresponds to

$$\min_j \left\{ \frac{d_j}{\alpha_{ij}} \right\}$$

Eventually the column  $j^*$  is to be increased form LB or decreased from UB. If none go to next  $i^*$ .

Step 5. Solve  $B\alpha_{j^*} = \alpha_{j^*}$  for  $\alpha_{j^*}$

Step 6. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic  $j^*$  from its bounds.

Step 7. Exchange basis

Step 8. If row  $i^* = \{\emptyset\}$  go to Stage 2, otherwise Repeat from step 2.

Stage 2.

Step 1 adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Step 2 adjust integer feasible superbasics. The objective of this phase is to conduct a neighbourhood search to verify local optimality.

interactive approach

#### V. CONCLUSION

This paper was intended to present a solution for one of the most important problems in Supply Chain Management and Distribution problems. The aim of this paper was to develop a model of heterogeneous vehicle routing problem (HVRP) with time windows considering service-choice. This problem has imposed a benefit that could be gained as incentive for offering serviced to customers. The proposed algorithm employs nearest neighbor heuristic algorithm for solving the model.

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