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Effect of Fractional Order of Repeated Roots in Pole Motion

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Abstract. There is relation between the impulse response characteristics and the location of poles of transfer function F(s). In this paper, we discuss impulse response for two roots, two times and three times repeated roots for different fractional values of q where $0.5 < q < 1$, $q = 1$, $1 < q < 1.5$ in pole motion. The different characters of the impulse response are shown in different numerical examples. The numbers of figures are presented to explain the concepts.

Keywords: *Transfer Function, Impulse response; Mat-lab.*

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I. INTRODUCTION

The transfer function G(s) is given by

$$G(s) = \frac{L_o}{L_i} \qquad (1)$$

where L denotes the Laplace transform. The frequency response function and the transfer function are interchangeable by the substitution $s = j\omega$ [1].

The frequency response function $G(j\omega)$ is

$$G(j\omega) = \frac{F_o}{F_i} \qquad (2)$$

where F denotes the Fourier transform.

TF has been used in many applications. One important application among them is monitoring the mechanical integrity of transformer windings (during testing and while in service).Mechanical deformation arises mainly due to short circuit forces, unskilled handling and rough transportation. Information related to winding deformation is embedded in the TF. Hence the first step should be the correct interpretation of TF [2]. TF can be used to describe a variety of filter or to express solution of linear differential equation accurately [3]. The TF of the system is analyzed and response curves are simulated [4].The location of poles and zeros gives the idea regarding response characteristics of a system.

II. SYSTEM STABILITY

If poles are in LHP, the system is stable; if poles are in RHP, the system is unstable and poles on imaginary axis then the system is marginally stable or limitedly stable [5, 6, 7].

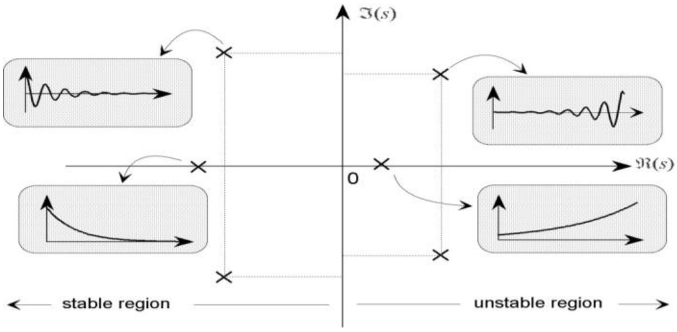

Fig.1. Stable and unstable region according to pole position

III. FRACTIONAL CALCULUS

Fractional calculus is a part of mathematics which deal with derivative of arbitrary order. The fractional order integral of an integral function f(t) with r €R.Thus the uniform formula of fractional order integral is defined as follows:

$$ {}_0D_t^{-q}x(t) = D^{-q}x(t) = \frac{1}{\Gamma(q)}\int_0^t (t-u)^{q-1}x(u)\,du \qquad (3)$$

where $q > 0$, f(t) is an arbitrary integrable function [8,9,10]

Laplace transforms for fractional – order integral operator is:[11]

$$L\{D^{-q}f(t)\} = s^{-q}F(s) \qquad (4)$$

IV. IMPULSE RESPONSE ANALYSIS

Impulse signals and their responses are commonly used in control systems analysis and design.

The inverse Laplace transform of

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480 – 483

$$F(s)=\frac{1}{\left(s^2+as+b\right)^q} \tag{5}$$

is derived by using the complex integral

$$f(t)=\Gamma(1-q)\, t^{q-1} e^{pt} \frac{\sin(q\pi)}{\Gamma(q)}=\frac{t^{q-1} e^{pt}}{\Gamma(q)} \tag{6}$$

Theorem 1: **For TF $F(s)=\frac{1}{(s^2+as+b)^q}$, When both of the poles lie in the open LHP, $\lim_{t\to\infty} f(t)=0$ with q > 0.[11]**

Consider transfer function with different fractional order then analyze impulse response of poles in LHP of functions $\{1/(s^2+as+b)^q\}$,$\{[1/(s^2+as+b)\ (s^2+as+b)]^q\}$, $\{[1/(s^2+as+b)\ (s^2+as+b)\ (s^2+as+b)]^q\}$ where a, b > 0 i.e. for two roots, two times repeated, three times repeated roots and MATLAB-based evaluation of impulse responses are given.

Numerical example 1: To observe the impulse response of transfer functions $\{1/(s^2+as+b)^q\}$,$\{[1/(s^2+as+b)\ (s^2+as+b)]^q\}$, $\{[1/(s^2+as+b)\ (s^2+as+b)\ (s^2+as+b)]^q\}$ where a, b > 0 i.e. for case i- two roots, Case ii- two times repeated roots, Case iii- three times repeated roots for the following positive values of a and b for q = 0.7, 1.0, 1.3 [9,10].

when a_1 = 1, b_1 = 1 then impulse response of two roots,[12] two and three times repeated roots $\lim_{t\to\infty} f(t)=0$ is demonstrated in figures 2,3,4 respectively.

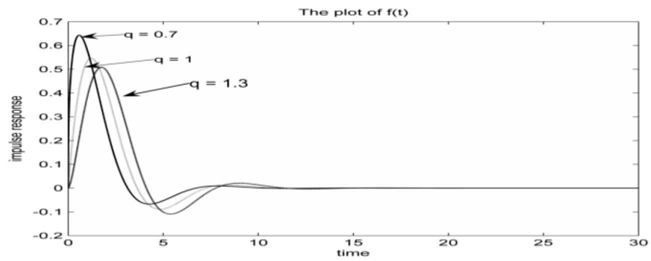

Fig 2 Impulse response of Case i

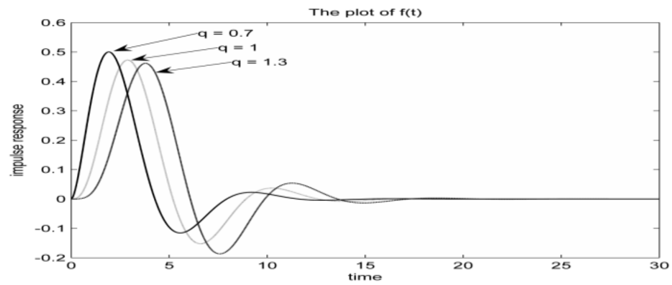

Fig. 3 Impulse response of case ii

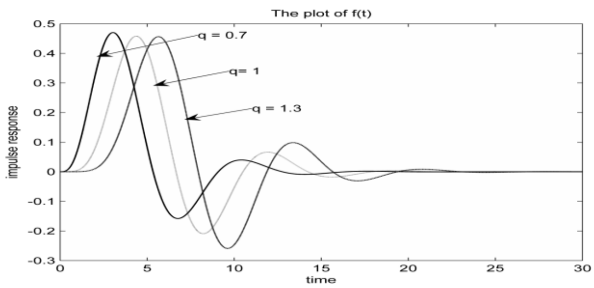

Fig 4 Impulse response of Case iii

When the number of roots increases for the same value of a,b then all three cases Impulse response tends to zero as t tends to infinity, rise time and negative impulse response go on increasing and peak time goes on decreasing. The peak time from a case I to case iii goes on decreasing. Rise time and settling time are greater for q = 1.3 as compared to q = 0.7 and q = 1, peak time is maximum at q=0.7 and peak time goes on decreasing from q = 0.7 to q = 1.3 for all three cases. But the difference between peak values of case i to case ii, case ii to case iii goes on decreasing for q = 0.7, q = 1, q = 1.3.

Numerical example 2: To observe the impulse response of transfer functions $\{1/(s^2+as+b)^q\}$,$\{[1/(s^2+as+b)\ (s^2+as+b)]^q\}$, $\{[1/(s^2+as+b)\ (s^2+as+b)\ (s^2+as+b)]^q\}$ where a, b > 0 i.e.for case iv- two roots, Case v- two times repeated roots, Case vi- three times repeated roots for the following values of a and b for q = 0.7, 1.0, 1.3 [9,10].

when a_2 = 2, b_2 = 2 then impulse response of two roots,[12] two and three times repeated roots $\lim_{t\to\infty} f(t)=0$ is demonstrated in figures 5,6,7 respectively.[13]

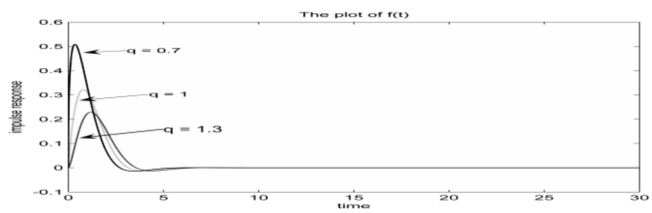

Fig. 5 Impulse response of case iv

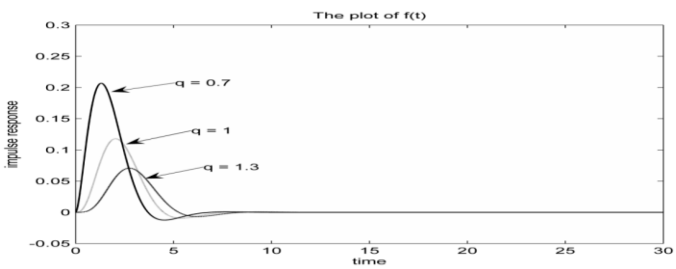

Fig.6 Impulse response of case v

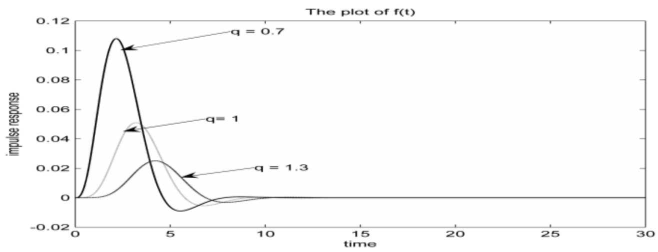

Fig 7 Impulse response of case vi

When poles move away from the origin in LHP then Impulse response tends to zero as t tends to infinity, rise time and negative impulse response go on increasing and peak time goes on decreasing from case iv to case vi. The peak time is minimum at q = 1.3 and maximum at q = 0.7 in all cases. Rise time and settling time are greater for q = 1.3 as compared to q = 0.7 and q = 1. Rise time goes on increasing for q = 0.7, q = 1, q = 1.3 from case iv to case vi.

Numerical example 3: To observe the impulse response of transfer functions $\{1/(s^2+as+b)^q\}$,$\{[1/(s^2+as+b)\ (s^2+as+b)]^q\}$, $\{[1/(s^2+as+b)\ (s^2+as+b)\ (s^2+as+b)]^q\}$ where a, b > 0 i.e. for case vii- two roots, Case viii- two times repeated roots, Case ix- three times repeated roots for the following values of a and b for q = 0.7, 1.0, 1.3 [9,10].

when a_2 = 3, b_2 = 3 then impulse response of two roots,[12] two and three times repeated roots $\lim_{t\to\infty} f(t)=0$ is demonstrated in figures 8,9,10 respectively.

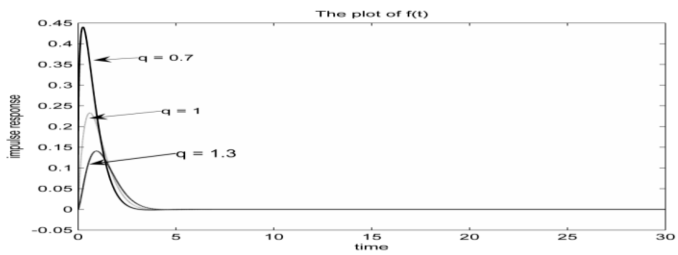

Fig. 8 Impulse response of case vii

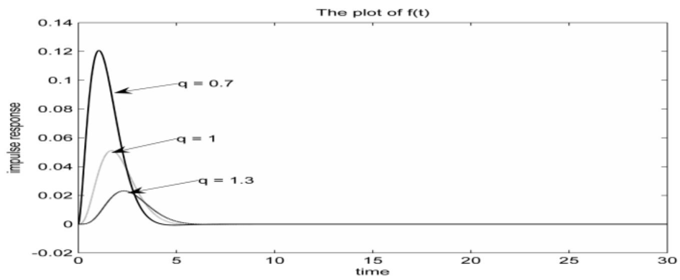

Fig. 9 Impulse response of case viii

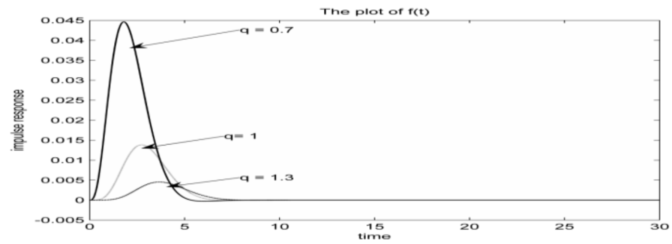

Fig 10 Impulse response of case ix

When poles move away from origin then there is no negative impulse response observed in any case. Impulse response tends to zero as t tends to infinity. Peak values go on decreasing, rise time and settling time goes on increasing when we increasing the repeated poles. Less impulse response observed at q = 1.3 as compared to other values of q.

CONCLUSION

When poles moves away from origin in LHP of functions $\{1/(s^2+as+b)^q\}$,$\{[1/(s^2+as+b)\ (s^2+as+b)]^q\}$, $\{[1/(s^2+as+b)\ (s^2+as+b)\ (s^2+as+b)]^q\}$ where a, b > 0 i.e. for two roots, repeated roots then impulse response tends to zero as t tends to infinity in pole motion in LHP. On increasing values of a and b, the peak value, rise time and settling time goes on decreasing from two roots to two times repeated roots and two times repeated roots to three times repeated roots. For positive values of a, b, peak time is greater for q = 0.7 and it is less for q = 1.3. For small values of a, b , the difference between peak values of q = 0.7 to 1, q = 1 to q = 1.3 is very less and as poles move away from origin in LHP; the differences goes on increasing. As on increasing values of a, b then peak time, rise time, settling time, negative impulse response goes on decreasing but for particular values of a, b peak time decreases and rise time, settling time increases.

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