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A Comparative Study of Sliding Mode Observers

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Abstract**—**In state space design, state feedback control is a powerful control technique. In state feedback control the Feedback of complete state vector gives the designer to have total control over the closed loop poles. Many times in state feedback control, all states are not available to feed back. To measure all unmeasurable states of the system the observer is needed. In real time system the system uncertainty will occur. So observer should estimate the state of the system in the presence of uncertainty. It is needed to control the system for which observer should be very robust to estimate the states of the system correctly. So here Utkin observer and Walcott zak observer`s are taken here. Their performances are compared to find the robustness against the system uncertainty. Utkin sliding mode observer has only the switching gain to minimize the observer error. But the Walcott-zak observer has static and nonlinear observer gain to minimize the observer error.

Index Terms**—***Utkin observer, state feedback control, Walcott zak observer.*

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I. INTRODUCTION

In state feedback control the Feedback of complete state vector gives the designer to have total control over the closed loop poles. Many times in state feedback control, all states are not available to feed back. To measure all unmeasurable states of the system the observer is needed. A key feature in the Utkin observer [5] is the introduction of a switching function in the observer to achieve a sliding mode and also stable error dynamics. This sliding mode characteristic which is a consequence of the switching function is claimed to result in system performance which includes insensitivity to parameter variations, and complete rejection of disturbances. Similar to the Utkin observer, the sliding mode functional observer also utilizes a switching function in its design and invariably will inherit the benefits of insensitivity to noise rejection as reported for the Utkin observer. The Utkin Sliding Mode Observer does not have a static observer gain in its structure and instead, the switching gain plays the role of stabilizing the error dynamics. However, the disadvantage of this sliding observer structure reveals it- self when there exist uncertainties and disturbances. In this case, the observer can only estimate the states with a bounded error and not asymptotically. The Walcott zak sliding mode Observer [4] has static and nonlinear observer gain it reduces error due to system uncertainty the Walcottzak observer algorithm explained in section III.

In section II discusses about Utkin observer. Section III discusses about walcottzak observer. In section IV the smart cantilever beam model is taken and the evaluation of the algorithms for smart cantilever beam is present.

II. UTKIN OBSERVER

Consider a continuous time linear system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

Where $A \in R^{n\times n}, B \in R^{n\times m}$ and $p \le m$ .Assume that the matrices B and C are of full rank and pair (A, C) is observable.

As the outputs are to be considered, it is logical to effect a change of coordinates so that the outputs appear as components of the states. One possibility is to consider the transformation $x \to T_c x$

Where, $$T_c = \begin{bmatrix} N_c \\ 0 \end{bmatrix} \tag{3}$$

Where the columns of $N_c \in R^{n\times(n-p)}$ span the null space of C. This transformation is nonsingular, and with respect to this new coordinate system, the new output distribution matrix is

$$CT_c^{-1} = \begin{bmatrix} 0 I_p \end{bmatrix} \tag{4}$$

Where

p is the number of output from the system.

n is the order of the system.

If the other system matrices are written as

$$T_C A T_C^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{5}$$

$$T_c B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad (6)$$

Then the nominal system can be written as

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1u(t) \quad (7)$$

$$y(t) = A_{21}x_1(t) + A_{22}y(t) + B_2u(t) \quad (8)$$

where

$$T_c x = \begin{bmatrix} x_1 \\ y \end{bmatrix} \quad (9)$$

The observer proposed by Utkin [1] has the form

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1u(t) + Lv \quad (10)$$

$$y(t) = A_{21}x_1(t) + A_{22}y(t) + B_2u(t) - v \quad (11)$$

Where (x_1, y_1) represent the state estimates for $(x_1, y_1), L \in R^{(n\times p)\times p}$ is a constant feedback gain matrix and the discontinuous vector v is defined component wise by

$$v_i = M \operatorname{sgn}(\hat{y}_i - y_i)$$

Where $M \in R_+$ If the errors between the estimates and the true states are written as a $e_1 = \hat{x}_1 - x_1$ and $e_y = \hat{y} - y$ then from equations (7)to (11) the following error system is obtained

$$\dot{e}_1(t) = A_{11}e_1(t) + A_{12}e_y(t) + Lv \quad (12)$$

$$\dot{e}_y(t) = A_{21}e_1(t) + A_{22}e_y(t) - v \quad (13)$$

Since the pair (A, C) is observable the pair (A_{11}, A_{21}) is also observable. As a consequence , L can be chosen to make the spectrum of $A_{11} + LA_{21}$ lie in C_-. Define a further change of coordinates

$$\tilde{T} = \begin{bmatrix} I_{n-p} & L \\ 0 & I_p \end{bmatrix} \quad (14)$$

and let $\tilde{e}_1 = e_1 + Le_y$. The error system with respect to the new coordinate can be written as

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) + \tilde{A}_{12}e_y(t) \quad (15)$$

$$\dot{e}_y(t) = A_{21}\tilde{e}_1(t) + \tilde{A}_{22}e_y(t) - v \quad (16)$$

Where

$\tilde{A}_{11} = A_{11} + LA_{21}, \tilde{A}_{12} = A_{12} + LA_{22} - \tilde{A}_{11}L$ and

$\tilde{A}_{12} = A_{22} - A_{21}L$.

It follows from (3.12) that in the domain

$$\Omega = \{(e_1, e_y) : \|A_{23}e_1\| + 0.5\lambda_{\max}(A_{22} + A_{22}^T)\|e_y\| < M - \eta$$

Where $\eta < M$ is some small positive scalar, the reachability condition

$$e_y^T \dot{e}_y < -\eta\|e_y\| \quad (17)$$

is satisfied. Consequently, an ideal sliding motion will take place on the surface

$$S_O = \{(e_1, e_y) : e_y = 0\} \quad (18)$$

It follows that after some finite time t_s, for all subsequent time, $e_y = 0$ and $\dot{e}_y = 0$

Equation (15) then reduces to

$$\dot{\tilde{e}}_1(t) = \tilde{A}_{11}\tilde{e}_1(t) \quad (19)$$

Which, by choice of L, represents a stable system and $\tilde{e}_1 \to 0$ consequently, $\hat{x}_1 \to x_1$ as $t \to \infty$.

III. WALCOTT ZAK OBSERVER

SYNTHESIS OF A DISCONTINUOUS OBSERVER

Consider the dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t) + B\xi(t, x, u) \quad (20)$$

Where $A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{p\times n}$ *and* $P \geq m$ in addition the matrixes B and Care assumed to be of full rank. The function $f : R_+ \times R^n \times R^m \to R^n$ is unknown and represents the system uncertainty. A natural problem to consider initially is the special case when the uncertainty is matched: suppose

$$f(t, x, u) = B\xi(t, x, u) \quad (21)$$

Where the function $\xi : R_+ \times R^n \times R^m \to R^m$ is unknown, but bounded, so that

$$\|\xi(t, x, u)\| \leq r_1\|u\| + \alpha(t, y) \quad (22)$$

Where r_1 is a known scalar and $\alpha : R_+ \times R^p \to R_+$ is a known function.

Suppose that there exists a linear change of coordinates T_o so that the system can be written as

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1u(t)$$

$$\dot{x}_1(t) = A_{21}x_1(t) + A_{22}y(t) + B_2u(t) + D_2\xi \quad (23)$$

Where $x_1 \in R^{n-p}, y \in R^p$ and the matrix A_{11} has stable eigenvalues. Consider an observer of the form

$$\dot{\hat{x}}_1(t) = A_{11}\hat{x}_1(t) + A_{12}\hat{y}(t) + B_1u(t) - A_{12}e_y(t) \quad (24)$$

$$\dot{\hat{y}}_1(t) = A_{21}\hat{x}_1(t) + A_{22}\hat{y}(t) + B_2u(t) - (A_{12} - A_{22}^s)e_y(t) + v \quad (25)$$

Where A_{22}^s is a stable design matrix and $e_y = \hat{y} - y$ (26)

$$v = \begin{cases} -\rho(t, y, u)\|D_2\| \dfrac{P_2 e_y}{\|P_2 e_y\|} & if \quad e_y \neq 0 \\ 0 & else \end{cases} \quad (27)$$

$$\rho(t, y, u) \geq r_1\|u\| + \alpha(t, y) + \gamma_o$$

$$e_1(t) = \hat{x}_1(t) - x_1(t)$$

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$$\dot{e}_1(t) = A_{11}e_1(t) \quad (28)$$

$$\dot{e}_y(t) = A_{21}e_1(t) + A_{22}^s e_y(t) + v - D_2\xi \quad (29)$$

There exists a family of symmetric positive definite matrices P_2 such that the uncertain dynamical error system above is quadratically stable. x

Let $Q_1 \in R^{(n-p)\times(n-p)}$ and $Q_2 \in R^{p\times p}$ be symmetric positive definite design matrices and define $P_2 \in R^{p\times p}$ to be the unique symmetric positive definite solution to the Lyapnouv equation.

$$P_2 A_{22}^s + A_{22}^{s\,T} P_2 = -Q_2 \quad (30)$$

Using the computed value of P_2 define

$$\hat{Q} = A_{22}^T P_2 Q_2^{-1} P_2^T A_{22} + Q_1 \quad (31)$$

and notice that $\hat{Q} \in \hat{Q}^T > 0$.

Let $P_1 \in R^{(n-p)\times(n-p)}$ be unique symmetric positive definite solution to the Lyapunov equation

$$A_{11}^T P_1 + P_1^T A_{11} = -\hat{Q} \quad (32)$$

Consider the quadratic form given by

$$V(e_1, e_y) = e_1^T P_1 e_1 + e_y^T P_2 e_y \quad (33)$$

As a candidate Lyapunov function. The derivative along the system trajectory

$$\begin{aligned}\dot{V} &= -e_1^T \hat{Q}_1 e_1 + e_1^T A_{21}^T P_2 e_y + e_y^T A_{21}^T P_2 e_1 \\ &\quad - e_y^T Q_2 e_y - 2e_y^T P_2 v - 2e_y^T P_2 D_2 \xi\end{aligned} \quad (34)$$

It is easy to verify that

$$\begin{aligned}&(e_y - Q_2^{-1} P_2 A_{21} e_1)^T Q_2 (e_y - Q_2^{-1} P_2 A_{21} e_1)^T \equiv \\ &e_y^T Q_2 e_y - e_1^T A_{21}^T P_2 e_y - e_y^T A_{21}^T P_2 e_1 \\ &+ e_1^T A_{21}^T P_2 Q_2^{-1} P_2 A_{21} e_1\end{aligned} \quad (35)$$

Substituting the identity (35) into equation (34) and writing for notational convenience $(e_y - Q_2^{-1} P_2 A_{21} e_1)$ as $\tilde{e}_y$ then

$$\dot{V} = -e_1^T \hat{Q}^T e_1 + e_1^T A_{21}^T P_2 Q_2^{-1} P_2 A_{21} e_1 - \tilde{e}_y^T Q_2 \tilde{e}_y + 2e_y^T P_2 v - 2e_y^T P_2 D_2 \xi$$

$$= -e_1^T Q_1 e_1 - \tilde{e}_y^T Q_2 \tilde{e}_y + 2e_y^T P_2 v - 2e_y^T P_2 D_2 \xi$$

$$= -e_1^T Q_1 e_1 - \tilde{e}_y^T Q_2 \tilde{e}_y - 2e_y^T P_2 D_2 \xi - 2\rho(t,y,u)\|D_2\|\|P_2 e_y\|$$

Using uncertainty bound and the bound for $\rho(\bullet)$ from equation (25) in the inequality above

$$\dot{V} \le -e_1^T Q_1 e_1 - \tilde{e}_y^T Q_2 \tilde{e}_y - 2\rho(t,y,u)\|D_2\|\|P_2 e_y\|$$

$$+ 2(r_1\|u\| + \alpha(t,y))\|D_2\|\|P_2 e_y\|$$

$$\le -e_1^T Q_1 e_1 - \tilde{e}_y^T Q_2 \tilde{e}_y - 2\gamma_o \|D_2\|\|P_2 e_y\|$$

$$< 0 \;\; for\; (e_1, e_y) \ne 0$$

And hence the error system is quadratically stable.

Consider the hyper plane in the error space given by

$$S_o = \{e \in R^n : Ce = 0\} \quad (36)$$

If $\hat{x}$ represents the state estimate for and $e = \hat{x} - x$ then the robust observer can conventiently be written as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) - G_1 Ce(t) + G_n v \quad (37)$$

Where the linear gain

$$G_l = T_o^{-1}\begin{bmatrix} A_{12} \\ A_{22} - A_{22}^s \end{bmatrix} \quad (38)$$

and the nonlinear gain

$$G_n = \|D_2\| T_o^{-1}\begin{bmatrix} 0 \\ I_p \end{bmatrix} \quad (39)$$

and

$$v = \begin{cases} -\rho(t,y,u)\|D_2\| \dfrac{P_2 Ce}{\|P_2 Ce\|} & if \quad FCe \ne 0 \\ 0 & else \end{cases} \quad (40)$$

Even in the special case when $D = B$ the observer formulation (37) to (40) is different from that of walcott & żak (3) for the case when $p > m$ since their results guarantee sliding will take place on the surface in the error space given by $\{e \in R^n : FCe = 0\}$. In the above formulation this is guaranteed.

Let (A, D, C) represent the linear part of the uncertain system in (20) which represents the propagation of the uncertainty through to the output. Consider the problem of constructing an observer for the uncertain system of the form

$$\dot{z}(t) = Az(t) + Bu(t) - G_1 Ce(t) + G_n v \quad (41)$$

where $e = z - x$, v is discontinuous about the hyperplane $S_o = \{e \in R^n : Ce = 0\}$ and $G_l, G_n \in R^{(n\times p)}$ are appropriate gain matrices. The purpose of this section is determining the class of systems for which the observer (41) provides quadratic stability of the error system despite the presence of bounded matched uncertainty. The canonical form from the section will provide an intermediate step for establishing the form in section from which the observer was designed.

Let G_l and G_n be appropriate gain matrices so that $A_o = A - G_l C$ is stable, and assume an ideal sliding mode insensitive to uncertainty exists on the hyperplane in the error space given by S_o. The error system satisfies

$$\dot{e}(t) A_o e(t) - D\xi(t,x,u) - G_n v \quad (42)$$

For a unique equivalent control to exist, $\det(CG_n) \ne 0$.

$$\begin{aligned}\dot{e}(t) &= (I - G_n(CG_n)^{-1}C)e(t) \\ &+ (I - G_n(CG_n)^{-1}C)D\xi(t,x,u)\end{aligned} \quad (43)$$

To be insensitive to the uncertainty it follows that

$$(I - G_n(CG_n)^{-1}C)D = 0$$

or equivalently

$$D = G_n (CG_n)^{-1} C \quad (44)$$

Since by assumption $rank(D) = q$. Therefore it c ab be assumed without loss of generality that the system (A, D, C) is in the canonical form. If the nonlinear gain matrix is partitioned so that

$$G_n = \begin{bmatrix} G_n \\ G_2 \end{bmatrix} \quad (45)$$

Then $CG_n = TG_2$ and so det $\det(G_2) \neq 0$. From equation (45). It follows that the poles of the (linear) reduced order motion are given by

$$\lambda\left((A_o)_{11} - G_1 G_2^{-1} (A_o)_{21}\right) \quad (46)$$

where $(A_o)_{11}$ and $(A_o)_{21}$ represent the top left and bottom left sub-blocks of the closed-loop matrix A_o partitioned in a compatible way to the canonical form. By definition the matrix $A_o = A - G_l C$, so

$$(A_o)_{11} = A_{11} - (G_l C)_{11}$$

Where $(G_l C)_{11}$ represents the top left sub-block of the square matrix $G_l C$. However, it is easy to check that $(G_l C)_{11} = 0$ for all $G_l \in R^{r \times r}$ and so $(A_o)_{11} = A_{11}$. Similarly it can be shown that $(A_o)_{21} = A_{21}$ and consequently.

$$\lambda\left((A_o)_{11} - G_1 G_2^{-1} (A_o)_{21}\right) = \lambda\left(A_{11} - G_1 G_2^{-1} A_{21}\right) \quad (47)$$

From equation (44) it follows that

$$G_1 G_2^{-1} \overline{D}_{21} = 0$$

Which after considering the structure of $\overline{D}_2$ implies

$$G_1 G_2^{-1} = \begin{bmatrix} \overline{G} & 0 \end{bmatrix}$$

Where $\overline{G}_1 \in R^{(n-p)\times(p-q)}$ and therefore from the definition of A_{11} it follows that

$$A_{11} - G_1 G_2^{-1} A_{21} = A_{11} - \overline{G} A_{211}$$

By construction the pair (A_{11}, A_{211}) is such that

$$\{zeros\,of\ (A, D, C)\} = \lambda A_{11}^o \subset \lambda(A_{11} - \overline{G} A_{211}) \text{ for all } \overline{G} \in R^{(n-p)\times(p-q)}$$

Let (A, D, C) represent the system and suppose $rank(CD) = q$ and any invariant zeros lie in C_-. Without loss of generality it can be assumed that the system is already in the canonical where the matrix A_{11}^o is stable. As a consequence there exists a matrix $L \in R^{(n-p)\times(p-q)}$ such that $(A_{11} - LA_{211})$ is stable. Define nonsingular transformation as

$$T_L = \begin{bmatrix} I_{n-p} & \overline{L} \\ 0 & T \end{bmatrix} \quad (48)$$

Where $\overline{L} \in \begin{bmatrix} I & 0_{(n-p)\times q} \end{bmatrix}$

After changing coordinates with respect to T_L, the new output distribution matrix becomes

$$c = C\overline{T}_L^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix}$$

From the definition of L and $\overline{\mathrm{D}}_2$

$$\mathrm{L}\overline{\mathrm{D}}_2 = [\mathrm{L} \quad 0] \begin{bmatrix} 0 \\ \mathrm{D}_2 \end{bmatrix} = 0$$

and so the uncertainty distribution matrix is given by

$$\mathcal{D} = \mathrm{T_L D} = \begin{bmatrix} \mathrm{L}\overline{\mathrm{D}}_2 \\ \mathrm{T}\overline{\mathrm{D}}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \mathrm{T}\overline{\mathrm{D}}_2 \end{bmatrix} = 0$$

Finally, if $\mathrm{A} = \mathrm{T_L A T_L^{-1}}$, it can be shown by direct evaluation that

$$\mathcal{A}_{11} = \mathrm{A}_{11} + \mathrm{L A}_{211}$$

This is stable by choice of L. The system triple (A, D, C) is now in the canonical form (24) a robust observer exists.

In the special case where $\mathrm{p} = \mathrm{q}$ an observer of the form (45) which is insensitive to the uncertainty in (20) exists if one only if

$$\det(\mathrm{CD}) \neq 0. \quad (49)$$

The invariant zeros of (A, D, C) lie in $\mathbb{C}_-$.(49).That is, the triple (A, D, C) is minimum phase and relative degree 1. In this case the restriction that $\det(\mathrm{CD}) \neq 0$ guarantees the existence of exactly $\mathrm{n} - \mathrm{p}$ invariant zeros and therefore the reduced order sliding motion is totally determined by these zeros.

IV. SIMULATION RESULTS

State space model of Smart cantilever beam with uncertainty is considered in this work

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 92.1084 & 64.5070 & -39.8911 & 65.1749 \\ -159.5286 & 14.3813 & 112.5734 & -118.4229 \\ 116.4182 & -111.6173 & -15.247 & 160.9807 \\ -63.1027 & 39.0227 & -63.7560 & -93.4438 \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} -0.5220 \\ 0.2457 \\ -0.3766 \\ 0.7240 \end{bmatrix} u(t) + \begin{bmatrix} -0.0141 \\ -0.0387 \\ 0.0421 \\ 0.0058 \end{bmatrix} (-10 * \sin(190t))$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_4(t) \end{bmatrix}^T$$

Assume initial conditions of the actual system is

$$x(0) = \begin{bmatrix} 1.5 & 1.5 & 0 & 0 \end{bmatrix}^T$$

Assume initial conditions of the estimator system is

$$\hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

System uncertainty is $\xi(x, t, y) = -\sin(190t)$;

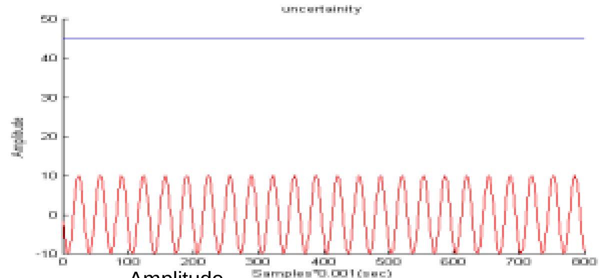
Assume that input to the actual system and observer system as same. That is u=0 or known.

Observe the system states using Utkin observer

The Utkin observer is explained in II.

Applying the Utkin observer for the above system. The results obtained as

Assume M=1 obtains L as $\begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}$



Amplitude

Figure.1 System uncertainty

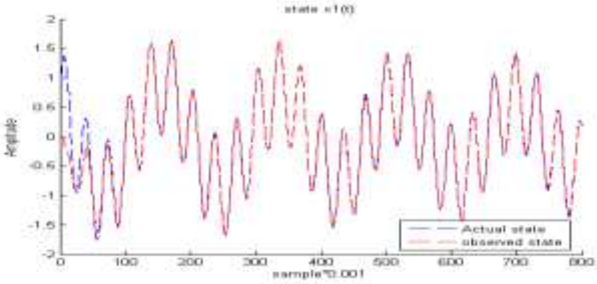


Figure.2 Actual state $x_1(t)$ vs estimated state $\hat{x}_1(t)$

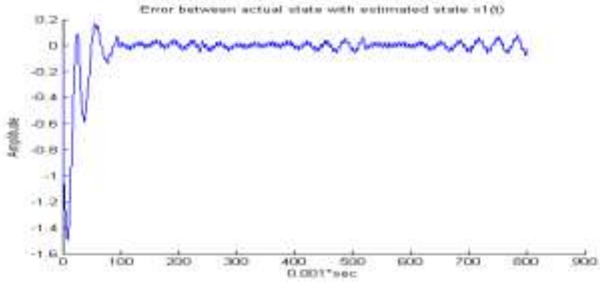
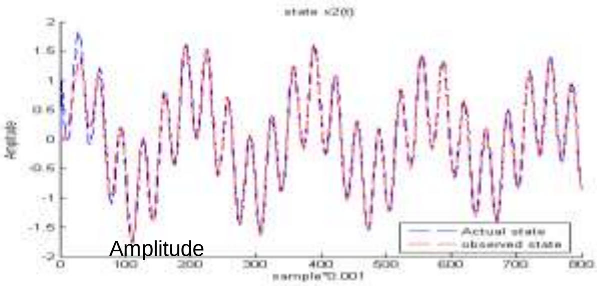
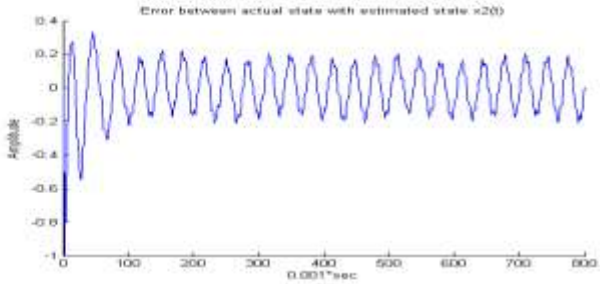


Figure.3 Estimation error between actual state $x_1(t)$ vs estimated state $\hat{x}_1(t)$



Amplitude

Figure.4 Actual state $x_2(t)$ vs estimated state $\hat{x}_2(t)$



FiFigure.5 Estimation error between actual state $x_2(t)$ vs estimated state $\hat{x}_2(t)$

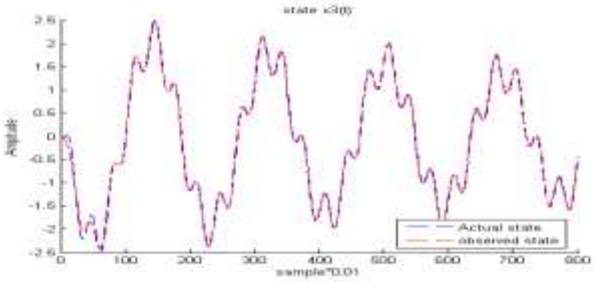


Figure.6 Actual state $x_3(t)$ vs estimated state $\hat{x}_3(t)$

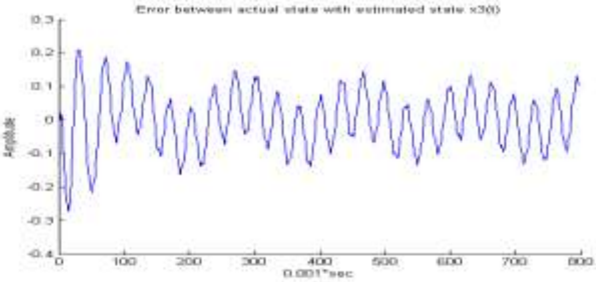


Figure.7 Estimation error between actual state $x_3(t)$ vs estimated state $\hat{x}_3(t)$

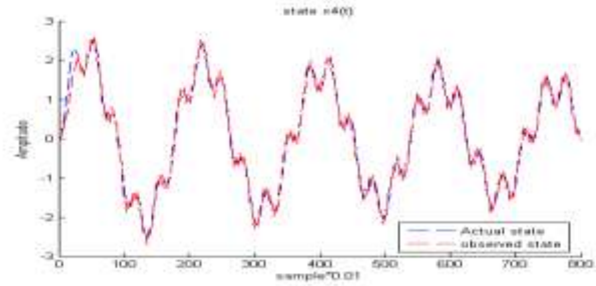


Figure.8 Actual state $x_4(t)$ vs estimated state $\hat{x}_4(t)$

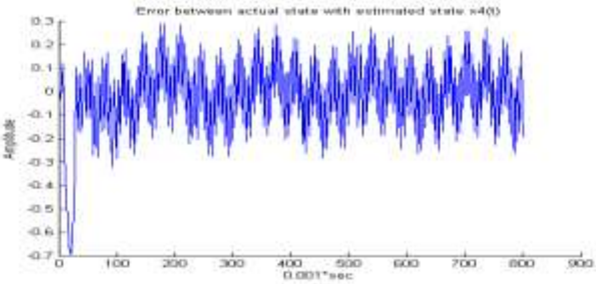


Figure.9 Estimation error between actual state $x_4(t)$ vs estimated state $\hat{x}_4(t)$

The observer starts at 0 sec but it response to track the actual state after 2 sec only. It tries to approach the actual state and it continuously minimizes the error but due to system uncertainty present in actual system it cannot able to track.

Observe The System States Using Walcott Zak Observer

The Walcott Zak observer is expressed as.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) - G_1Ce(t) + G_n v$$

Assumptions

Stable design matrix $A_{22}^s = -10$

$$\rho(t, y, u) = 15$$

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From the algorithm Linear gain

$$G_l = [376 \quad -366 \quad 255 \quad 266]^T$$

Nonlinear gain $G_n = [0.0141 \quad 0.0387 \quad -0.0421 \quad -0.0058]^T$

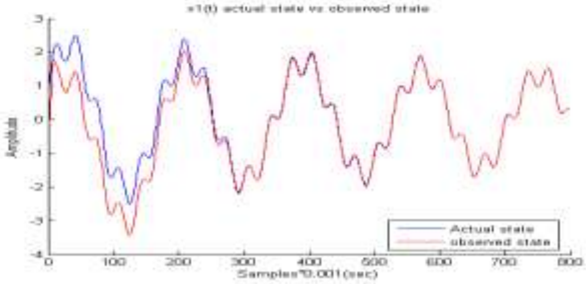

Figure.10 Actual state $x_1(t)$ vs estimated state $\hat{x}_1(t)$

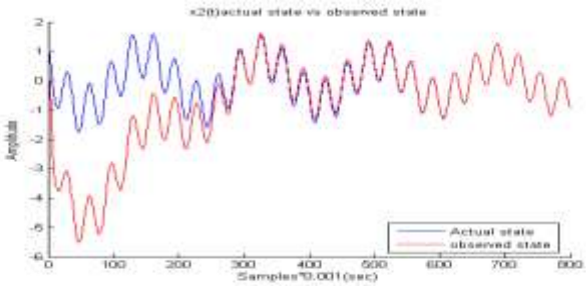

Figure.11 Actual state $x_2(t)$ vs estimated state $\hat{x}_2(t)$

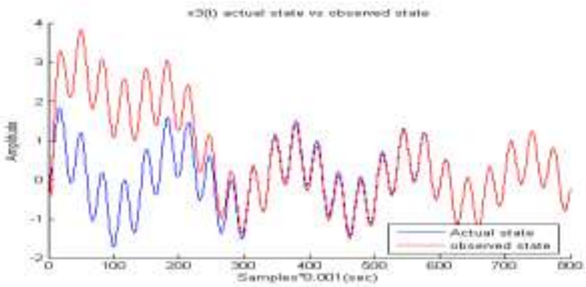

Figure.12 Actual state $x_3(t)$ vs estimated state $\hat{x}_3(t)$

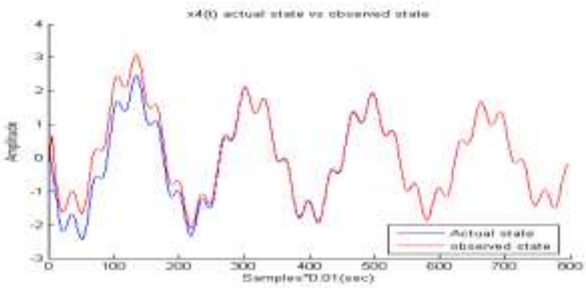

Figure.13 Actual state $x_4(t)$ vs estimated state $\hat{x}_4(t)$

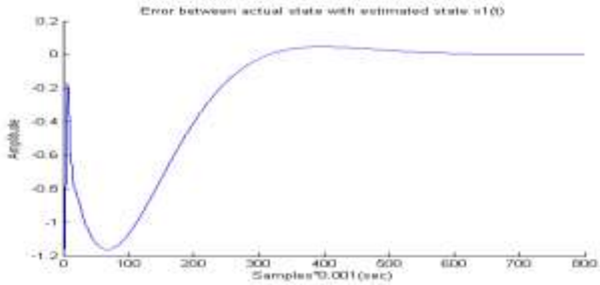

Figure.14 Estimation error between actual state $x_1(t)$ vs estimated state $\hat{x}_1(t)$

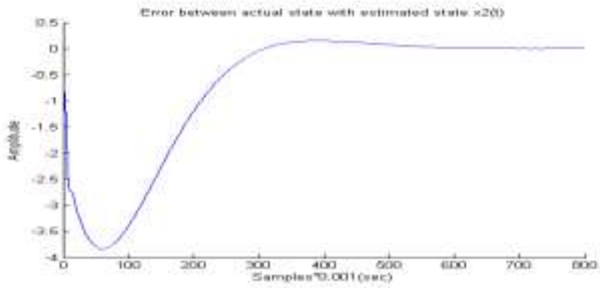

Figure.15 Estimation error between actual state $x_3(t)$ vs estimated state $\hat{x}_3(t)$

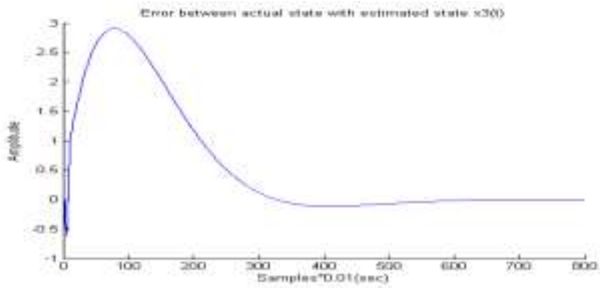

Figure.16 Estimation error between actual state $x_4(t)$ vs estimated state $\hat{x}_4(t)$

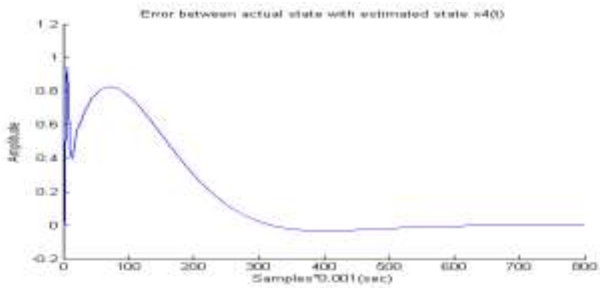

Fig.17 Estimation error between actual state $x_2(t)$ vs estimated state.

The observer starts at 0 sec but it response to track the actual state after 6 sec. It tries to approach the actual state and it continuously minimizes the error and it eliminates the error due to system uncertainty.

V. CONCLUSION

From the results obtained, the Utkin observer starts to track the actual system states after 1 sec and it is unable to approach the actual system states in presence of uncertainty. This can be seen from the simulation results. Because the Utkin Sliding mode observer does not have a static observer gain in its structure and instead, the switching gain Lv plays the role of stabilizing the error dynamics. So it is unable to minimize the error due to system uncertainty.

From the results obtained, the Walcott Zak observer starts to track the actual system states after 1 sec and it is able to approach the actual system states after 6 sec. Because the Walcott Zak observer has two error stabilizing components one is feedback the estimation error and another one is the switching function. The switching function v has the range of upper bound value of the system uncertainty. The estimation error is feedback to the observer with linear gain G_l and the switching function is feedback with the non linear gain G_n these two feedback error gains play an important role to minimize the error due to system uncertainty very effectively and observe the system, for this the system uncertainty is assumed to be an unknown function but with bounded range. So the Walcott Zak observer is robust against system uncertainty when compared to the Utkin observer.

Both Utkin observer and Walcott Zak observer are having one disadvantage. The problem of observing the states of a system, some of whose inputs are not available for measurement. Utkin observer and Walcott Zak observer cannot observe the system states, under such conditions. Both are suitable for only the input to the system is available and zero. In future the unknown input observer can be designed with controller.

VI. REFERENCES

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