Laplace Transform & It's Applications to Solve Ordinary Differential Equation (ODE) By Using MATLAB

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Abstract— Laplace transform is an integral transform method which is widely used by scientists & Engineers with the increasing complexity Of Engineering problems .Laplace transform help in solving complex problems with a very simple approach just like the applications of Transfer functions to solve ordinary differential equations.
This paper introduce new technological approach for solving ODE by using MATLAB.

Keywords- *Laplace Transform , ODE, MATLAB.*

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I. INTRODUCTION

1) Laplace transform is effectively used in solving linear ordinary & Partial differential equations . Laplace transform reduces an ordinary differential equation into algebraic equations. It gives the direct

Solution of differential equation with given initial conditions. To compute quickly the solution of differential equation , we use Mathematical software.
One of such powerful software package is MATLAB that contains many easy to use tools & built in functions to solve or simulate Differential equation by Laplace Transform.

Definition- Laplace Transform-

Let $f(t)$ be a function of t defined for all $t > 0$. Then the Laplace Transform of $f(t)$ denoted by $L(f(t)$isdefined by,

$$L\big(f(t)\big) = \int_0^\infty e^{-st} f(t)dt$$

Where is a parameter which is either real or a complex quantity.

Theorem - Sufficient condition for existence of Laplace Transform-
If $f(t)$ is piecewise continuous in every finite interval in t Range $t \geq 0$ & is of exponential order, then its Laplace transform.

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2) Method to solve linear ordinary differential (ODE) by using Laplace transform –

The advantage of using the Laplace transform is that it yields the Particular solutions directly without the necessity of first finding .The general solution & then evaluating the arbitrary constants.

Step -1 Start with differential equation with initial condition.

Step-2 Take Laplace Transform on both sides of equation obtain subsidiary equation.

Step -3 Take inverse transform & simplify.

Example- i) solve the equation

$$\boldsymbol{y'' - 3y' + 2y = 12e^{-2t}}$$

$$\boldsymbol{given\ that\ y(0) = 2, y'(0) = 6.}$$

Solution-

Consider $y'' - 3y' + 2y = 12e^{-2t}$

Taking Laplace transform on both sides & Let $L\big(y(t)\big) = Y(s)$.

$$L(y'') - 3L(y') + 2L(y) = 12L(e^{-2t})$$

Using formulae

$$\{s^2Y(0) - sy(0) - y'(0)\} - 3\{sY(s) - y(0)\} + 2Y(s) = \frac{12}{s+2}$$

Substituting given conditions, $y(0) = 2, y'(0) = 6.$

We have

$$(s^2 - 3s + 2)Y(s) - 2s = \frac{12}{s+2}$$

$$Y(s) = \frac{1}{(s^2 - 3s + 2)}(2s + \frac{12}{s+2})$$

$$Y(s) = \frac{2s^2+4s+12}{(s^2-3s+2)(s+2)}$$

Using Partial Fractions, We get

$$Y(s) = -\frac{6}{s-1} + \frac{7}{s-2} + \frac{1}{s+2}$$

Taking inverse Laplace transforms,

$$L^{-1}(Y(s)) = -6L^{-1}\left(\frac{6}{s-1}\right) + L^{-1}(\frac{7}{s-2}) + L^{-1}(\frac{1}{s+2})$$
$$y(t) = -6e^{t} + 7e^{2t} + e^{-2t}.$$

3) To solve ordinary differential equations by Laplace transform by using MATLAB –

MATLAB has a powerful features for solving differential equations of all types . We will explore some of these features using Laplace transform. We use symbolic Math Toolbox. The result will be the form of the function & it may be readily Plotted with MATLAB.

MATLAB program me to solve above example (i) above -

Example- solve the equation

$y'' - 3y' + 2y = 12e^{-2t}$ $given\ that\ y(0) = 2, y'(0) = 6$

Program me –

Syms s t y

$$f = 12 * \exp(-2 * t)$$

$$F = Laplace(f, t, s)$$
$$y_1 = s * y - 2$$
$$y_2 = s * y_1 - 6$$

$$\therefore\ y_2 = s * (s * y - 2) - 6$$

$$sol = solve(y_2 - 3 * y_1 + 2 * y - F, y)$$
$$sol = ilaplace(sol, [0,15])$$

Grid on,('GRAPHIC DISPLAY OF SOLUTION OF ODE BY LAPLACE TRANSFORM')

X label ('time'), y label (f(t))

Legend ('Laplace transform')

Output:$f = 12 * exp(-2 * t)$

$$sol = 7 * exp\ (2 * t) + exp(-2 * t) - 6 * exp(t)$$

Also we get graphic display of solution of ODE by Laplace Transform.

Here we consider the two approaches to solve an initial value Problem with the general method & technological method using MATLAB & it is observed that both gives same solution.

4) CONCLUSION

MATLAB which is mathematical software saves a lot of time in routine calculation for engineers & scientists.

This paper represents an example of MATLAB applications in Mathematical computation including generating the graph of respective functions using Laplace Transform.

Authors Contribution-

The author personally run the program me & checked output . Also read & approved the final manuscript.

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