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An insight into the Role of Phase Factors in Gauge Theories

Dr Dev Raj Mishra
Department of Physics,
R.H.Government Post Graduate College,
Kashipur, U.S.Nagar, Uttarakhand -244713 INDIA
e-mail: dr_devraj_mishra@yahoo.co.in

Abstract:The pedagogical introduction to abelian and non-abelian phase factors, gauge invariance, Aharonov–Bohm effectis presented. Parallel transport is studied as a function of the Phase factors and in Yang–Mills theory, itdetermines the field components. These components are the proper curvature in an internal color space and associated four potential is the connection.

Keywords:*Gauge Theory, Non-Abelian Symmetry, Parallel Transport, .*

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I. INTRODUCTION

The principle of local gauge invariance and ananalog of the phase factor was first introduced by H Weyl in 1919 [1] for describing the electromagnetic interaction in analogy with general covariance in Einstein's theory of gravitation. An extension to non-Abelian gauge groups was given by Yang and Mills in 1954 [2]. A crucial role in gauge theory is played by the phase factor which is associated with parallel transport in an external gauge field[3,4]. In general parallel transport is a vector around a closed loop, used to compare the phases of a wave function at two different points. Parallel transport is supposed to be curved space generalization of the concept of keeping the vector constant as we move along the path.The phase factors are observable in quantum theory, in contrast to classical theory. This is analogous to the Aharonov–Bohm effect for the electromagnetic field [5].

In gauge theory, the transformation between the possible gauges, form a symmetry group G. The elements of the subset S of the group G are called group generators of the field represented byT_α . These generators are commutative, that is $[T_a T_b] = if_{abc} T_c = 0$,where the structure constants f_{abc} are zero. Then the gauge bosons are not self-interacting.Thefield is called abelian gauge field. This is the case of the quantum electrodynamics (QED) theory in which photons play the role of mediators and are not self-interacting.The group generators in QED commute with each other and therefore QED is an abelian gauge theory with symmetry group U(1). However,the mediators in quantum chromodynamics (QCD) theory are quarks which are of self-interacting nature.The group generators in QCD anti-commute with each other i.e. $[T_a T_b] = if_{abc} T_c \neq 0$. That means the structure constants f_{abc} are non-zero. Therefore QCD is a non abelian gauge theory and is governed by a symmetric group SU(3).

II. INVARIANCE UNDER GAUGE TRANSFORMATION

A local symmetry is a symmetry that depends on spacetime, x. A gauge symmetry is a local symmetry where the symmetry group is continuous, e.g. U(N), SU(N).Let us say we have an N component column vector of fermion fields with identical masses

$$\chi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \vdots \\ \Psi_N(x) \end{pmatrix} \qquad (1)$$

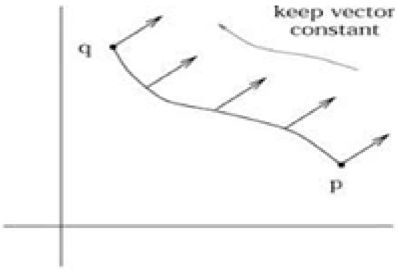

Figure1.Parallel transport between two points

The Lagrangian for the fermion fields can be written as $\mathcal{L} = \bar{\chi}(x)\left[\left(i\gamma^\mu \partial_\mu - m\right)I_{N\times N}\right]\chi(x)$ (2)
where γ^μ is an 4×4 matrices and a scalar with respect to the spinor doublet χ (x).

χ (x) is a N component spinor-column vector, where each component is a 4-component column vector Ψ. Therefore χ (x) is an4N component column vector. Further $\left(i\gamma^\mu \partial_\mu - m\right)$is a scalar and $\left(i\gamma^\mu \partial_\mu - m\right)I_{N\times N}$ is a scalar multiplied by the identity matrix. The I_{NxN} can be left out but we shall leave it in, to be explicit.

Let the Lagrangian is invariant under a global U(N) transformation given by

$$\chi(x) \rightarrow \chi'(x) = \Omega(x)\chi(x)$$
$$\text{where} \quad \Omega(x) \in SU(N)$$
$$\chi^\dagger(x) \rightarrow \chi'^\dagger(x) = \chi^\dagger(x)\Omega^\dagger(x) \qquad (3)$$

Here $\Omega(x) \in G$ with G being a semisimple Lie group which is called the gauge group (G=SU(3) for QCD).In above equation $\chi(x)$ belongs to the fundamental representation of G. This is an unitary gauge group when

$$\Omega^{-1}(x) = \Omega^\dagger(x) \qquad (4)$$

In analogy with QCD, the gauge group G =SU(N) is usually associated with color. The proper index of $\chi(x)$ is called the color index.

The gauge transformation of the matter field $\chi(x)$in Eq.(3) is compensated by a transformation of the non-Abelian gauge field $A_\mu(x)$ which belongs to theadjoint representation of G.$A_\mu(x)$ is an element of the Lie algebra of SU(N) and so can be expanded in basis of it.
Therefore

$$A_\mu(x) = A_\mu^a(x)T^a \qquad (5)$$

where T^a are a basis (of matrices) of the Lie algebra for SU(N) and $A_\mu^a(x)$ are the gauge fields co-efficients to the matrix generators T^a. The number of gauge fields is equal to the dimension of the SU(N) group i.e. dim (SU(N)). This can further be generalised that the number of gauge fields is equal to the dimension of Group G i.e. dim (G) if the gauge group is some compact group G. The equation (5) can be further written in terms of the ij components of the corresponding matrices, as

$$[A_\mu(x)]^{ij} = g\sum_a A_\mu^a(x)[t^a]^{ij} \qquad (6)$$

The matrices $[t^a]^{ij}$ are thecomponents of the generators $T^a$of $G(a = 1, \ldots\ldots, N^2 - 1)$ for SU(N).The generators of the representation of SU(N) are normalised such that

$$Tr_R(T^aT^b) = T(R)\delta^{ab} \qquad (7)$$

where T(R) is the Dynkin index of the representation. Notation in the literature for the trace varies. The term "Tr" is commonly used for the trace performed in the adjoint representation and the term "tr" is used for the trace on the fundamental representation. For a trace on a general R-dimensional representation, Tr_R is commonly used. The value of Dynkin index T(R) is 1/2 for the fundamental representation and N for the adjoint representation of SU(N). The matter fields are normally in the fundamental representation of SU(N) in the standard model with an exception of only the right handed fields in the electroweak theory. Therefore quite often $tr(T^aT^b) = \frac{\delta^{ab}}{2}$is used in literature. The factor ½ is used for historical reasons, in particular $t^a = \frac{\sigma^a}{2}$ for SU(2) group, where σ^a are the Pauli matrices. This results in the redefinition of the coupling constant, $\bar{g}^2 = 2g^2$.

Multiplying Eq.(6) by $[t^b]^{ji}$ and summing over ij, we get

$$\sum_{i,j}[A_\mu(x)]^{ij}[t^b]^{ji} = g\sum_{i,j}\sum_a A_\mu^a(x)[t^a]^{ij}[t^b]^{ji}$$

$$\text{Or} \; trA_\mu(x)t^b = g\sum_a tr(T^aT^b)A_\mu^a(x)$$

$$\sum_a \delta^{ab}A_\mu^a(x) = gA_\mu^b(x)(8)$$

Therefore we get

$$trA_\mu(x)t^b = gA_\mu^b(x) \qquad (9)$$

Dividing by g and changing b to a, we have

$$A_\mu^a(x) = \frac{1}{g}trA_\mu(x)t^a(10)$$

Let us consider the ordinary derivative in some direction n^μ:

$$\partial_\mu\chi(x) = \lim_{\epsilon\to 0}\frac{1}{\epsilon}(\chi(x+\epsilon) - \chi(x)) \qquad (11)$$

whereϵ is a small displacement in n^μ direction. The problem is that derivative ∂_μdoes not transform covariantly.

$$\partial_\mu(\chi(x)) \nrightarrow \Omega(x)\,\partial_\mu\chi(x)$$

under the transformation $\chi(x) \to \Omega(x)\chi(x)$.

However, we have to construct a covariant derivative, say D_μ which will transform covariantly in order to make our Lagrangian invariant under the transformations. Therefore

$$D_\mu(\chi(x)) \to \Omega(x)D_\mu\chi(x) \qquad (12)$$

The covariant derivative is defined as

$$D_\mu\chi(x) = \lim_{\epsilon\to 0}\frac{1}{\epsilon}[\chi(x+\epsilon) - U(x+\epsilon,x)\chi(x)](13)$$

By construction as we have defined each term to transform the same under the gauge transformation, therefore now it can be shown that $D_\mu(\chi(x)) \to \Omega(x)D_\mu\chi(x)$.

The parallel transport $U(y,x)$used in Eq.(13) with y=x+εis given by

$$U(y,x) = \exp\left(ig\int_\Gamma A_\mu(x)dx^\mu\right)$$

whereΓ is a path joining y to x. (14)

It transforms as

$$U(y,x) \to \Omega(y)U(y,x)\Omega^\dagger(x)(15)$$

Since $U(y,x) \in SU(N), A_\mu(x)$ is an element of the Lie algebra of SU(N) with coupling g. Choosing y= x + ε, we get for infinitesimal path Γ from x to y = x + ε

$$U(x+\varepsilon,x) \approx \exp(ig\epsilon A_\mu(x))$$

$$\approx 1 + ig\epsilon A_\mu(x) + O(\epsilon^2) \qquad (16)$$

and also from Eq.(15)

$$U(x+\varepsilon,x) \to \Omega(x+\varepsilon)U(x+\varepsilon,x)\Omega^\dagger(x)(17)$$

Using Eqs.(16) and (17) and using expansion $\Omega(x+\varepsilon) = \Omega(x) + \varepsilon\,\partial_\mu\Omega(x) + O(\epsilon^2)$, It can be shown that the transformation equation for the gauge field $A_\mu(x)$is

$$A_\mu(x) \to A_\mu'(x) = \Omega(x)A_\mu(x)\Omega^\dagger(x) - \frac{i}{g}\partial_\mu\Omega(x)\Omega^\dagger(x)(18)$$

where g is the coupling constant.

Let the transformation is expressible as an exponential function of the form

$$\Omega(x) = e^{i\alpha(x)}(19)$$

where$\alpha(x)$ is a function of spacetime.We have for small $\alpha(x)$

$$\Omega(x) = e^{i\alpha(x)} = 1 + i\alpha(x)$$

and $\Omega^\dagger(x) = e^{-i\alpha(x)} = 1 - i\alpha(x)(20)$

Substituting in Eq. (18), we have

$$= (1+i\alpha(x))A_\mu(x)(1-i\alpha(x)) - \frac{i}{g}(\partial_\mu(1+i\alpha(x))(1-i\alpha(x))$$

$$= A_\mu(x) + i\alpha(x)A_\mu(x) + A_\mu(x)(-i\alpha(x)) - \frac{i}{g}i\,\partial_\mu\alpha(x)(1-i\alpha(x))$$

$$= A_\mu(x) + i\alpha(x)A_\mu(x) - iA_\mu(x)\alpha(x) + \partial_\mu\alpha(x)$$

(retaining only the first order terms in $\alpha(x)$) (9)

That is$A_\mu(x) \to A_\mu'(x) = A_\mu(x) + \partial_\mu\alpha(x) - i[A_\mu(x),\alpha(x)]$

Or we can write $\delta A_\mu(x) = D_\mu^{adj}\alpha$

Where $D_\mu^{adj}\alpha = \partial_\mu\alpha - i[A_\mu,\alpha]$ (21) (10)

is the covariant derivative in the adjoint representation of G, while$D_\mu^{fun}\psi = \partial_\mu\psi - i[A_\mu,\psi](22)$

is that in the fundamental representation.

It can be shown that$D_\mu^{adj}B(x) = [D_\mu^{fun},B(x)]$where B(x) is a matrix − valued function of x.(23)

The QCD action is given in matrix notation as$S[A,\psi,\overline{\psi}]$is given by $\int D^4x\left[\overline{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi + m\overline{\psi}\psi + \frac{1}{4g^2}tr\mathcal{F}_{\mu\nu}^2\right]$ (24)

where$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$(25)

$\mathcal{F}_{\mu\nu}$ isthe matrix of the non-abelian field strength.

This action is invariant under the local gauge transformation as

$\mathcal{F}_{\mu\nu}(x) = \Omega(x)\mathcal{F}_{\mu\nu}(x)\Omega^\dagger(x)$(26)

For an infinitesimal gauge transformation

$$\delta\mathcal{F}_{\mu\nu}(x) \to -i[\mathcal{F}_{\mu\nu}(x), \alpha(x)] \quad (27)$$

III. PARALLEL TRANSPORT

The abelian phase factor is defined by the formula

$$U(y,x) = e^{ig\int_\Gamma A_\mu(x)dx^\mu} \quad (28)$$

whereΓ is a path joining y to x.

It transforms as

$$U(y,x) \to \Omega(y)U(y,x)\Omega^\dagger(x) \quad (29)$$

Substituting$\Omega(x) = e^{i\alpha(x)}$,

$$U(y,x) \to e^{i\alpha(y)}U(y,x)e^{-i\alpha(x)} \quad (30)$$

which is the transformation law for the phase factor $U(y,x)$ under the transformation$\Omega(x)$.

The wave function $\chi(x)$transforms as in Eq.(3)

$$\chi(x) \to \Omega(x)\chi(x) \quad \text{where} \quad \Omega \in SU(N)$$

$$\chi^\dagger(x) \to \chi^\dagger(x)\Omega^\dagger(x)$$

Substituting $\Omega(x) = e^{i\alpha(x)}$, we have

$$\chi(x) \to e^{i\alpha(x)}\chi(x)$$

$$\chi^\dagger(x) \to \chi^\dagger(x)e^{-i\alpha(x)} \quad (31)$$

Let us see the transformation for the product$\chi(y)\chi(x)$, which is as under

$$\chi(y)\chi(x) \to e^{i\alpha(y)}\chi(y)\chi^\dagger(x)e^{-i\alpha(x)} \quad (32)$$

A comparison of Eqs. (30) and (32) shows that $U(y,x)$ transforms like the product $\chi(y)\chi^\dagger(x)$. It can be mathematically written as

$$U(y,x) \sim "\chi(y)\chi^\dagger(x)" \quad (33)$$

Also it can be seen that the wave function at point x transforms like the wave function at point y after multiplication by the phase factor

$$U(y,x)\chi(x) \sim "\chi(y)\chi^\dagger(x)"\chi(x) \sim \chi(y) \quad (34)$$

And analogously $\chi^\dagger(y)U(y,x) \sim "\chi^\dagger(x)"$(35)

The phase factor plays the role of a parallel transporter in an electromagnetic field, and to compare phases of a wave function at points x and y, we should first make a parallel transport along some contourΓ_{yx}. The result is Γ-dependent except when $A_\mu(x)$ is a pure gauge (vanishing field strength$F_{\mu\nu}(x)$). Certain subtleties occur for not simply connected spaces (the Aharonov–Bohm effect).

IV. OBSERVING PHASE DIFFERENCE (AHARONOV–BOHM EFFECT)

It is well known that the transverse components of the electromagnetic field describe photons and the longitudinal components relates to the gauging of the phase of the wave function. Therefore longitudinal components permit one to compare its values at different space-time points when an electron is placed in an external electromagnetic field. The phase of the wave-function itself is unobservable in quantum mechanics. However the phase differences are observable e.g. via interference phenomena. The phase difference depends on the value of the phase factor for a given pathΓ_{yx} along which the parallel transport is performed. The phase factors $U(y,x)$ as their differences are,therefore indirectly observable in the quantum theory. This is in contrast to the classical theory. The phase difference is visible in the Aharonov–Bohm effect [5].

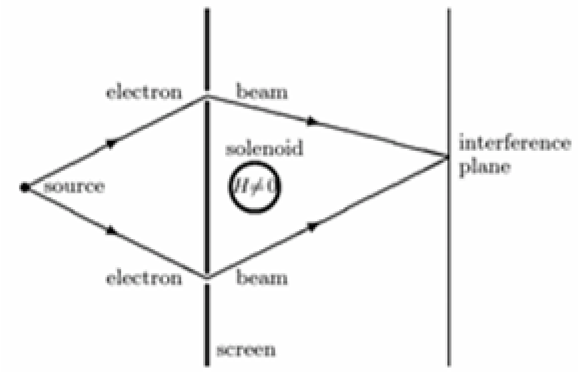

Figure2. Observing interference effects of the Aharonov–Bohm experiment.

The experimental arrangement consists of passing a coherent beam through a solenoid as shown in Figure.2. As the coherent beam of electrons passes through a solenoid; it splits into two parts. It is observable that the electrons do not pass inside the solenoid where the magnetic field is concentrated. The phase difference arises between the electron beam passing through the slits. The interference picture also changes with the value of the electric current. The phase difference depends on (the real part of) the phase of the wave function along the path because the dependent vector potential is non-zero. i.e.$\vec{A} \neq 0$.The wave function can be written as

$$\psi = \psi_0 \exp\left[\frac{ie}{\hbar c}\int A_\mu(z)dz^\mu\right] \quad (36)$$

Where ψ_0 is the free case wave function, and as we know $A_\mu(z)$is the covariant potentialand z = (x, y, z, t) and $dz^\mu = (cdt, d\vec{z})$.

To compute interference pattern consider the wave functions along the path $y \to x$ and path $x \to y$

$$\psi_{\Gamma_{yx}^+} = \psi_{\Gamma_{yx}^+,0}\exp\left[\frac{ie}{\hbar c}\int_{\Gamma_{yx}^+} A_\mu(z)dz^\mu\right] \quad (37)$$

$$\psi_{\Gamma_{xy}^-} = \psi_{\Gamma_{xy}^-,0}\exp\left[\frac{ie}{\hbar c}\int_{\Gamma_{xy}^-} A_\mu(z)dz^\mu\right] \quad (38)$$

Therefore, path difference is given by

$$= e^{ig\int_{\Gamma_{yx}^+} A_\mu(x)dx^\mu} - e^{ig\int_{\Gamma_{xy}^-} A_\mu(x)dx^\mu}$$

$$= e^{ig\oint_\Gamma A_\mu(x)dx^\mu}$$

$$= e^{ig\oint_\Gamma F_{\mu\nu}d\sigma^{\mu\nu}} \quad \text{using strokes theorem}$$

where the closed contour Γ is composed of paths $\Gamma_{yx}^+$and $\Gamma_{xy}^-$and $\sigma^{\mu\nu}$is the area enclosed by the closed contour. Therefore the phase difference is equal to

$$= e^{igHS} \quad \text{as} \quad \oint_\Gamma F_{\mu\nu}d\sigma^{\mu\nu} = HS \quad (39)$$

It shows the phase difference does not depend on the shape of the contour Γ or of the paths $\Gamma_{yx}^+$and Γ_{xy}^- but depends only on HS, the magnetic flux through the solenoid.

V. NON ABELIAN QCD ACTION

An extension to non abelian gauge group was given by Yang and Mills in 1954[2]. Yang Mills theory explains the term 'gauge invariance'.The term "gauging"literally means fixing a scale. This theory describes the behaviour of elementary particles using the non abelian lie groups. It explains the unification of weak and electromagnetic force i.e. [U(1)×SU(2)] and thereforethe standard model.

In the case of non-abelian group SU(N) local gauge transformation given as

$$\Psi(x) \overset{g.t.}{\rightarrow} \Psi'(x) = \Omega(x)\Psi(x) \quad (40)$$

Here $\Omega(x) \in G$ with G being a semisimple lie group which is called the gauge group.

The gauge transformation of field Ψ gives the transformation of non abelian gauge field

$$A_\mu(x) \overset{g.t.}{\rightarrow} A'_\mu(x) \quad (41)$$

$$= \Omega(x)A_\mu(x)\Omega^\dagger(x) - \frac{i}{g}\Omega(x)\,\partial_\mu\Omega^\dagger(x) \quad (42)$$

where g is coupling constant.$A_\mu$is defined as

$$A_\mu = A_\mu^a T_a = \sum_{a=1}^{N^2-1} A_\mu^a T_a \quad (43)$$

$T_a$introducethe generators of the group G(a=1 ...$N^2 - 1$) for SU(N),can be normalised such that- $\mathrm{Tr}\, T_a T_b = \delta ab$where Tr is the trace over the generators. (44)

The covariant derivative is given as-

$$D_\mu = \partial_\mu - igA_\mu(x) \quad (45)$$

The QCD action in the matrix form may be given as-

$$S = \int d^4x \left[\frac{1}{2g^2}\left(F_{\mu\upsilon}F_{\mu\upsilon}\right) + \overline{\Psi}(\gamma_\mu D_\mu + m)\Psi\right] (46)$$

where

$$F_{\mu\upsilon} = D_\mu A_\upsilon - D_\upsilon A_\mu$$

$$= \partial_\mu A_\upsilon - \partial_\upsilon A_\mu - i\,[A_\mu, A_\upsilon] \quad (47)$$

is the Hermitian matrix of the non-abelian field strength.

VI. NON ABELIAN PARALLEL TRANSPORT

The comparison of phases of wave functions at distinct points need the non-abelian extension of the parallel transporter.The proper extension of the Abelian formula given in equation (28) is -

$$U[\Gamma_{yx}] = Pe^{ig\int_{\Gamma_{yx}} dz^\mu A_\mu(z)} \quad (48)$$

The symbol **P**refers for path-ordering.Substituting

$$dz^\mu = dt\dot{z}^\mu, \quad (49)$$

theintegral,withtime t as parameter is obtained

$$U[\Gamma_{yx}] = Pe^{ig\int_0^\tau dt\dot{z}^\mu(t)A_\mu(z(t))} \quad (50)$$

Here t varies from 0 to τ as path Γ_{yx} is traced.

Using the concept of integral as limit to the sum, we can write above equation as an exponential function of the summation

$$U[\Gamma_{yx}] = e^{ig\sum_{t=0}^{\tau} dt\dot{z}^\mu(t)A_\mu(z(t))}$$

$$= \prod_{t=0}^{\tau}\left(1 + igdt\dot{z}^\mu(t)A_\mu(z(t))\right) \quad (51)$$

Using Eq.(49), we can rewrite it as

$$= \prod_{z\epsilon\Gamma_{yx}}\left(1 + i\,g\,dz^\mu A_\mu(z(t))\right) \quad (52)$$

The discretisation into M points of the contourΓ_{yx},converts Eq.(52) to the following approximationvalue for the non-abelian phase factor

$$U[\Gamma_{yx}] = \lim_{M\to\infty}\prod_{i=1}^{M}\left[1 + ig(z_i - z_{i-1})^\mu A_\mu\left(\frac{z_i + z_{i-1}}{2}\right)\right] (53)$$

Eq.(52) is obviously reproduced with the limit$z_{i-1} \rightarrow z_i$.

The non-abelian phase factor in Eq.(48) is an element of the gauge group G itself while $A_\mu$belongs to the Lie algebra of G.Matrices are rearranged in inverse order under Hermitian conjugation

$$U^\dagger[\Gamma_{yx}] = U[\Gamma_{xy}] \quad (54)$$

It is evident that the notation Γ_{yx} means the orientation of the contour from xto y, while Γ_{xy}denotes the opposite orientation from y to x. Therefore these two result in opposite orders of multiplication for the matrices in the path-ordered product.The phase factors obey the backtracking condition

$$U[\Gamma_{yx}]U[\Gamma_{xy}] = 1 \quad (55)$$

The gauge field A_μ in the finitely discretized phase factor (53) is chosen at the center of the ith interval in order to satisfy Eq. (55).

Since factors in Eq.(52) transform under the Gauge transformation in Eq.(42) as

$$[1 + igdz^\mu A_\mu(z)] \rightarrow [1 + igdz^\mu A'_\mu(z)]$$

$$= \Omega(z + dz)[1 + ig\,dz^\mu A_\mu(z)]\Omega^\dagger(z) \quad (56)$$

Substituting for$A_\mu(z)$ from Eq.(42) in Eq.(52) and cancelling $\Omega^\dagger(z)$ and$\Omega(z)$ at the intermediate points z, it can be shown that the non-abelian phase factor $U[\Gamma_{yx}]$transforms as

$$U[\Gamma_{yx}] \rightarrow \Omega(y)U[\Gamma_{yx}]\Omega^\dagger(x) \quad (57)$$

Similar to abelian phase factor as discussed above in Eq.(34),$\psi(x)$ is transported by the non-abelian phase factor matrix $U[\Gamma_{yx}]$ to the point yand therefore, $U[\Gamma_{yx}]$ is, indeed, a parallel transporter?

$$U[\Gamma_{yx}]\psi(x) \sim \text{"}\psi(y)\text{"} \quad (58)$$

Further it can be proven that $\overline{\psi}(y)U[\Gamma_{yx}]\psi(x)$is gauge invariant

$$\overline{\psi}(y)U[\Gamma_{yx}]\psi(x) \rightarrow \overline{\psi}(y)U[\Gamma_{yx}]\psi(x) \quad (59)$$

And also under transformation Eq.(57), the trace of the phase factor for a closed contour Γcan be shown to be gauge invariant:

$$\mathrm{trP}e^{ig\oint_\Gamma dz^\mu A_\mu(z)} \rightarrow \mathrm{trP}e^{ig\oint_\Gamma dz^\mu A_\mu(z)} \quad (60)$$

This result is also quite similar to the Abelian phase factor. The vanishing of $\mathcal{F}_{\mu\nu}$like the curvature tensor in general theory of relativity is the sufficient and necessary condition for the phase factor to be independent on a local variation of the path. Therefore$\mathcal{F}_{\mu\nu}$ in Yang–Mills theory is the proper curvature in an internal color space while A_μ is the connection.

CONCLUSION

The phase factor plays the role of a parallel transport in an electromagnetic field.The Wilson loop which is the trace of the non-abelian path ordered parallel transport is gauge invariant.The gauge invariance of Wilson loopsplays an important role in QCD studies especially in the phenomenon of quark confinement.

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Dr Dev Raj Mishra Gold Medallist at Graduation,did M.Sc. from M.D. University, Rohtak (Haryana).Did his Ph.D. in High Temperature Superconductivity from Cryogenics Group in National Physical Laboratory and University of Delhi, Delhi.Joined as Lecturer in the Department of Higher Education, Government of Uttarakhand in Jan. 2000.Presently posted at Department of Physics, R.H. P.G. College, Kashipur, U.S.Nagar, India.