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Study of Different algorithm in Euler Graph

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Abstract: In this paper we discuss about the algorithm to constructing an Euler Path in Euler Graph .There are many algorithm available to constructing an Path in Euler Graph, but in this paper we study mainly two algorithm i.e. Fleury's & Hierholzer's. Both are very important algorithm for determination of Euler path in Euler graph. Also both algorithms are different and more effective than simple algorithm.

Key Word- Vertices, *Edges, Graph, Trail, Walk, Paths, Circuit.*

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I. Introductions

Graph theory was born in 1736 with Euler's famous paper in which he solved the Konigsberg problem. Euler posed a more general problem that problem in which type of graph G is possible to find a closed walk running through every edge of G exactly once? A closed trail containing all points and lines is called an Euler trail. A graph having an Euler trail is called an Euler graph.

Obviously in Euler graph, for every pair of points u and v there exist at least two edge disjoint u-v trails and consequently there are at least two edge –disjoint a u-v paths available to constructing an Euler Path like simple algorithm, Fleury's algorithm & Hierholzer's algorithm.

Simple algorithm contain four simple steps but in Fleury's algorithm to construct a trail that grows to the desired Euler path. In Hierholzer's algorithm to start with closed trail in G attach to it systematically, detour trail unit all edges of graph are used up.

II. 1.2Basic concept of Euler Graph

First we prove a simple properties that is needed to study of Euler graph

Properties1: If G is a graph in which the degree of every vertex is at least two then G contains a cycle.

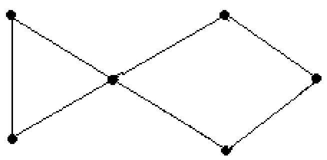

Fig.1

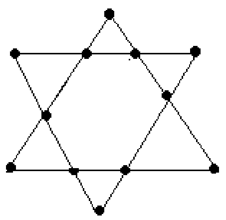

Fig.2

Proof. Construct a sequence v, v_1, v_2 ....of vertices as follows. Choose any vertex v. Let v_1 be any vertex adjacent to vertex v. Let v_2 be any vertex adjacent to v_1 other than v . At any stage, if vertex v_i, $i \geq 2$ is already chosen, then choose v_{i+1} to be any vertex adjacent to v_i other than v_{i-1} Since degree of each vertex is at least 2, the existence of v_{i+1} is always guaranteed.

Since G has only a finite number of vertices, at some stage we have to choose a vertex which has been chosen before. Let v_k be the first such vertex and let $v_k = v_i$ where $i < k$. The $v_i v_{i+1} \ldots\ldots v_k$ is a cycle.

The following properties answer the problem. In what type of graph G is it possible to find a closed trail running through every edge of G?

Properties 2

The following statements are equivalent for a connected graph G.

(a) If G is Euler graph
(b)Every point of G has even degree.
(c)The set of edges of G can be portioned into cycles.

Proof. (a) ⇨ (b): Let T be an Euler trail in G, with origin u. Every time a vertex v occurs in T in a place other than the origin and end, two of the edges incident with v are accounted for.

Since an Euler trail contains every edges of G, d (v) is even for every $v \neq u$. For u, one of the edges incident with u is accounted for by the origin of T, another by terminus of T and others are accounted for in pairs

Hence d (u) is also even.

(b) ⇨ (c) : Since G is connected and nontrivial every vertex of G has degree at least 2. Hence G contains a cycle Z. The removal of the lines of Z results in a spanning sub graph G_1 in which again every vertex has even degree. If G_1 has no edges, then all the lines of G form one cycle and hence (c) holds.

Otherwise, G_1 has a cycle Z_1 , Removal of the lines of Z_1 from G_1 results in spanning sub graph G_2 in which every vertex has even degree. Containing the above process, when a graph G_n with no edge is obtained, we obtain a partition of the edges of G into n cycles.

(c) ⇨ (a):If the partition has only one cycle, then G is obviously Euler, since it is connected. Otherwise let Z_1 ,$Z_{2,}$… Z_n be the cycles forming a partition of the lines of G. Since G is connected there exists a cycle $Z_i \neq Z_1$ having a common point v_1 with Z_1. Without loss of generality let it be Z_2. The walk beginning at v_1 and consisting of the cycle Z_1 and Z_2 in succession is closed trail containing the edges of these two cycles. Continuing these process, we can construct the edges of these two cycles. Continuing this process, we can construct a closed trail containing all the edges of G. Hence G is Euleiran.

Note. The above properties and its proof hold for pseudo graphs also. Even otherwise, a pseudo graph G^* becomes a graph G when we introduce two points of degree 2 on each loop and a point of degree 2 on every other edge. Every vertex of G is of even degree iff every vertex of G^* is of degree.

The proof of the above properties gives a method for finding a Eulerian trail when such a trail exists.

III. Simple Algorithm (When every vertices of graph have even degree)

Choose any vertex randomly from the Graph G.

Step 1:- construct a circuit C in graph G with cover every edge exact one. If all the edge of G used in C then we are done otherwise.

Step 2:- Since C does not contain every edge in G, we have not found an euler tour. Delete all the edge in C form G, find a sub graph G of G. Since C is Euler. We know that the components of G are as well. It is possible, depending on the choice of C_1, that G_1 could be disconnected, meaning that it has at least two components. We also know that because G is connected, there must exist at least one vertex that is common to both C&G_1.

Step 3:- Choose any such vertex. Let us start at the beginning of G and traverse from start to end, let's choose the first vertex in C which has unused edges G, let's G be the cycle in G.

Step 4:- Delete all edges in C1 from G_1 & splice them into C. if there are no unused edges, we are done.
It not, we are left with a sub graph of G we will call G_2 that is smaller than G_1, return to step 2 until all edges are used.

IV. Simple Algorithm (when two vertices of odd degree)

With above properties it is clear that Euler path exist in any graph iff there are two odd degree vertices. Here we discuss the algorithm of constructing Euler path. Let any graph G be given with two odd degree vertices x& y in G. According to simple algorithm Euler Path may be starting with x and ending with y, covering all edges of G, or starting with y and ending with x, covering all edges of G

We have following steps

Step 1:- Identify two vertices of odd degree in given graph

Step 2:- Construct one edge we will call e incident to x & y. Now the degree of both vertices in
even.

Step3:- Run Euler circuit algorithm starting at x.

Step 4:- Remove edge e. We have just constructed an Euler Path from x to y

V. Fleury's Algorithm

There is a good algorithm, due to Fleury, to construct an Eulerain trail in an Eulerian graph. Algorithm approach is to construct a trail that grows to desired Euler line, the Algorithm says.

Step 1. Choose an arbitrary vertex V_0 and set $W_0 = v_o$

Step 2. Suppose that trail W has been chosen. Then choose an edge from in such a way that

(i) e_{1+1} is incident with v_i

(ii) Unless there is no alternative, e_{i+1} is not a bridge of G – (e_1, $e_{2,....}$ e_i)

Step 3. Stop when step 2 can no longer be implemented.

Obliviously, Fluryr's algorithm constructs a trail in G. It can be proved that if G is Euler, then any trail in G constructed by Fleruy's algorithm is an Euler trial in G.

VI. Hierholzer's algorithm

There is another algorithm developed by . It's method to start with any closed trail in G and "attach" to trail until all edges of G used up.

Step 1. Choose any vertex v in graph G and choose any closed trail W_0 in G starting and ending at v .Set i = 0

Step 2. If E (W_1) = E (G) step. Since then W_i is an Euler tour of G. Here, E(W_i)denotes the set of edges of a sub graph W_i .Otherwise choose a vertex v_i on W_i which is incident with an edge in G which is not in W_i . now choose a close trail W*.

Step 3. Let W_{i+1} be the closed trail consisting of the edges of both W_i and W_i*.Obtained by starting at vertex v_i traversing the trail W_i until v_i is reached, then traversing the closed trail W_i* and, on returning to v_i completing the rest of the trail W_i .
Set i = i+1 and return to step 2.

VII. Conclusion-

Many physical problems can be represent by graph and solved by observing the relevant properties of the corresponding graph. If any system like as Euler graph then using of above properties we can find the optimize many thing. Chinese Postman problem is an example. There are many algorithms available to constructing an Euler path in Euler graph, one of them is Fluryr's algorithm. This type of approach is to construct a trail that grows to the desired Euler path.

In this paper we discuss about the algorithm to constructing an Euler Path in Euler Graph.Here we study mainly two algorithm i.e. Fleury's & Hierholzer's. Both are very important algorithm for determination of Euler path in Euler graph. Also both algorithms are different and more effective than simple algorithm. During Comparative Study of Fleury's & Hierholzer's algorithm in Euler Graph we find that Fleury's algorithm in more complex than Hierholzer's algorithm. So we can say Hierholzer's algorithm is easiest algorithm for determination of Euler path in Euler graph**.**

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