

An Optimization Model for Integrated Capacity Management and Bed Allocation Planning of Hospitals

Suryati Sitepu
University HKBP Nomensen
Medan, Indonesia

Pasukat Sembiring
Department of Mathematics
University of Sumatera Utara
Medan, Indonesia

Herman Mawengkang
Department of Mathematics
University of Sumatera Utara
Medan, Indonesia
hmawengkang@yahoo.com

Abstract— Hospitals are facing with increasing demands of unlimited needs of people health. On the other hand, due to the rising cost of healthcare services, hospitals need to put more effort in order to overcome these two problems. This paper deals with proposing an integrated strategy for solving these problems. We address an integer optimization model which integrate capacity staff management problem and bed allocation planning problem. We solve the model using a direct approach, based on the notion of superbasic variables.

Keywords- Capacity management, bed allocation planning, modeling, superbasic variables.

I. INTRODUCTION

Hospitals are a very important institution to provide health care for people. It is not surprising that nowadays the people's demand for hospitals is increasing. This condition occur particularly for the people who live in big city, such as, Medan city, the capital of North Sumatera province, Indonesia. Obviously, one of the the main cause of this rise lies in the ageing populations who are putting heavy demands for health care. The direct impact of this booming is the number of hospitals are also increasing. Unfortunately, there are more and more people from Medan who seek health care from neighbour countries, such as, Malaysia and Singapore. Undoubtedly, the urgent need to tackle this situation is to improve health service performance of hospitals.

All operations related to the health service performance in hospitals are limited in terms of capacity. Therefore, in order to fulfill the patients' demand for health care, the hospitals management should have a planning and controlling the capacity of the operations ([3], [4], [5]).

Capacity planning decisions are important to any industry, especially to health care industry because not only it relates to the management of highly specialized and costly resources (i.e., nurses, doctors, and advanced medical equipment), but also it makes a difference between life and death in critical conditions [16]. Public hospitals have, in general, more demand for health services than available capacity. Therefore it is important to forecast and manage demand with good precision, in order to adjust capacity or take alternative courses of action for example transfer demand to other facilities. Demand forecasting and management is part of a larger design that intents to provide a systemic solution to global hospital management. Such solution is commonly based on the design of a general process structure for hospitals and

which defines the management processes that are needed to optimize the use of resources in doing so and to ensure a predefined service level for patients. The general process structure allowed us to determine the key processes where implementation of new practices would generate most value ([18], [17]).

For inpatient care facilities in a hospital, this requires information on bed capacity and nursing staff capacity, on a daily as well as annual basis. Quantitative models can be used to calculate capacity needs for different planning purposes and for short, medium and long term planning issues. Although several useful models are described in the international literature ([6], [7], [8]) many of them are difficult to apply in practice because they require a great deal of data and clerical work [6].

Healthcare executives and managers are always searching for better ways to improve production capacity for medical treatment and thus, improving operational efficiency. Obviously, many times, capacity in a health care organization is a vague and hard-to-measure concept which varies with local economic conditions and over time [25]. In any hospital, resources are scarce or limited and they are mostly dissimilar in nature. Such dissimilarity nature of capacity for different forms of resources makes the comparison of capacities very important to determine the exact capacity of the system taken as a whole. An inappropriate capacity comparison would lead to inaccurate system capacity and so resulting in inefficiencies in the system – observed in excessive waiting, poor capacity utilization across different resources and poor bottleneck management [28] Therefore, when capacity management is done properly, it can lead to lean service models in healthcare by minimizing almost all the wastage and inefficiencies.

Practically, in order to be able to apply capacity management, the models must consider different

functions, such as hospital staff planning, roster scheduling support, and daily assignment of nurses to wards [7-8]. In addition, strategic decisions are sometimes mentioned as a separate planning level [6, 9, 10]. Models based on mathematical optimization techniques from operations research are generally focused on short-term scheduling [11, 12]. Models that do integrate different planning horizons (daily, periodical (1–2 months), and annual) are for example described by [12] and [15]. These models contain connected models for periodical staff planning and daily scheduling. However, models incorporating operational planning issues with tactical and strategic decisions or operational scheduling support with annual staff planning were not found in the literature. [26] use stochastic optimization model to solve hospital nursing – staff management problem.

In general, capacity models are aimed at calculating the number of nurses needed, whereas capacity management models should ideally also give insight into opportunities for improving capacity usage.

Regarding to bed resources capacity planning most approach reported in the literature fall into the use of mathematical programming and simulation.

[27] present a mathematical programming model approach. They propose a multi-objective decision aiding model for solving allocation of beds problem in a hospital. Their model is based on queuing theory and goal programming (GP). In order to obtain some essential characteristics of access to various departments within a hospital, a waiting line model is explored. The results are used to construct a goal programming frame work, taking account of targets and objectives related to customer service and profits from the hospital manager and all department heads. [23] create a mathematical model based on a dynamic dispatching approach for bed resources allocation considering hospitalization demands, bed capacity and income. The objectives of their model are to maximize income and to minimize the costs to use supplementary beds. Another goal programming model approach for solving re-allocation bed in a hospitals was proposed by [2]. The constraints considered in their model are total number of beds, nursing work hours, waiting time, the definite bed allocation to patient, and the definite bed allocation to ward. An integer linear programming model approach is featured by [24] to solve a hospital bed management problem constrained with budgetary cuts. They implemented their result in French hospitals.

In terms of simulation technique for solving bed allocation problem, the works in the literatures belong to the use of simulation only and a combined simulation-optimization. Due to the inherent uncertainties in the problem, it is reasonable to use simulation technique only. In particular, the use of simulation we should be able to get useful insights on bed allocation. These results can be found in Harper and Shahani (2002), Akkerman and Knip (2004), and Cochran and Bharti (2006). Laker et al., (2014) use discrete event simulation for solving bed allocation problem in the emergency department of a hospital. An interesting review of using simulation in healthcare is addressed in [1]. However the use of only simulation may not reach the best solution, particularly

when the problems involve combinatorial nature. It is not surprising that some literatures propose a combine strategy simulation and optimization. Keshtkar et al., (2015) propose a simulation optimization approach for solving resource allocation in an emergency department. The use of multi-objective optimization combined with simulation to tackle the bed allocation problem can be found in [21], and [27]

II. FRAMEORK OF THE OPTIMIZATION MODEL

An optimisation model referred to as a mathematical programming model represents a real-world problem in which problem choices are represented by decision variables and the problem is solved by finding values of these decision variables that maximises or minimizes a function called an objective function. In real world situation, the decision variables are restricted with some conditions which are termed as constraints of the model. Principally in optimization model we need to have decision variables, parameters, objective functions and constraints.

In the following, the framework of optimization model is briefly described. Mathematically the model is expressed as follows:

$$\begin{aligned} & \text{minimize} && f(x) \\ & \text{subject to} && x \in X \end{aligned} \tag{P}$$

f is called the objective function and X is a region which contains all the possible solution of (P), called feasible region. It is assumed that the region is a subset of R^n . Therefore f is a real-valued function on X .

If f is a linear function and the feasible region is generated by a set of linear equations then problem (P) with nonnegative restriction is given to the decision variables, then the model is called a linear programming model. Frequently in real world application, some of the decision variables restricted to take integer value, then the model can be called as integer linear programming problem.

III. DEFINITION OF THE PROBLEM

Increasing demand for healthcare through hospitals created heavy challenges to their managers and decision makers. The challenges involve high costs, limited budget, and limited resources. Most of hospitals in Medan are encountered with some pressures, such as, shortages of qualified healthcare professionals, limited hospital equipments and facilities, increasing operational costs.

Capacity planning, for hospitals in particular, is concerned with making sure of balancing the quality of health care delivered with the cost of providing that care. Such planning involves predicting the quantity and particular attributes of resources required to deliver health care service at specified levels of cost and quality. The most fundamental measure of hospital capacity

planning is the number of inpatient beds accordingly the number of doctors and the number of nurses. Hospital bed capacity decisions have traditionally been made based on target occupancy levels. Certain units in the hospital, such as, intensive care units (ICUs) are often run at much higher utilization levels because of their high costs.

The other important thing that should be considered in order to enhance the service performance of hospitals is waiting time, due to the bed capacity allocation system ([2], [24]).

Alternative strategy to overcome this situation is to have a well coordinated hospital capacity management along with bed allocation system.

IV. MATHEMATICAL MODEL FORMULATION

The decision problem, for our problem, is to maximally coordinate the utilization of multifold resources within the hospital. In our case the multifold resources are doctors, nurses, beds and rooms.. In this case the most appropriate model to be created is a linear integer programming problem.

Firstly, we define the notations to be used in the model

Sets

- I Set of departments
- J Set of doctors type
- K Set of nurses type
- L Set of available beds
- R Set of rooms

Decision variables

- DA_{ij} : Initial number of type doctors in department
- SA_{ij} : Initial number of type nurses in department
- SBA_{ij} : Initial number of type nurse-aids in department
- D_{ij} : Type doctors added in department
- S_{ij} : Number of type nurses added in department
- SB_{ij} : Number of type nurs-aids added in department
- TPA_{il} : Initial number of beds in department
- TP_i : Number of beds added in department

Binary variables

- Equals to 1, if bed are allocated for room in department ,
- equals to 0 otherwise.

- y_r^i Equals to 1 if room $r \in R$ is used in department $i \in I$
- Equals to 0 otherwise

Parameters

- bd_{ij} : Cost of $j \in J$ type doctors in department $i \in I$
- bs_{ik} : Cost of type $k \in K$ nurses in department $i \in I$
- bsa_{ik} : Cost of type $k \in K$ nurse-aids in department $i \in I$
- bt_{il} : Cost of operating beds $l \in L$ in department $i \in I$
- bw_{il} : Waiting cost for beds $l \in L$ in department $i \in I$
- ba_{lr}^i : Cost for allocating bed $l \in L$ for room $r \in R$ in department $i \in I$
- cr_i^r : Cost to operate room $r \in R$ in department $i \in I$
- md_{ij} : Maximum number of doctor for each type can be allocated to each department
- mn_{ik} : Maximum number of nurse each type can be allocated to each department
- mna_{ik} : Maximum number of nurse-aids each type can be allocated to each department
- mb_{il} : Maximum number of beds $l \in L$ can be allocated to each department
- ρ_i : Maximum fund available for department $i \in I$

A. Objective Function

The objective of this problem is to minimize total operating cost, which consists of

i) costs related to doctors

$$CD = \sum_{i \in I} \sum_{j \in J} bd_{ij} (DA_{ij} + D_{ij}) \quad (1)$$

ii) costs related to nurses

$$CN = \sum_{i \in I} \sum_{k \in K} bs_{ik} (SA_{ik} + S_{ik}) \quad (2)$$

(2)

iii) costs related to nurse – aids

$$CNA = \sum_{i \in I} \sum_{k \in K} bsa_{ik} (SBA_{ik} + SB_{ik}) \quad (3)$$

(3)

iv) costs related to bed

$$CB = \sum_{i \in I} \sum_{l \in L} bt_{il} (TPA_{il} + TP_{il}) \quad (4)$$

(4)

v) costs related to waiting for beds

$$CWB = \sum_{i \in I} \sum_{l \in L} bw_{il} (TPA_{il} + TP_{il})$$

(5)

vi) costs related to allocating bed if bed $l \in L$ is allocated to room $r \in R$

$$CAB = \sum_{l \in L} \sum_{r \in R} \sum_{i \in I} ba_{lr}^i x_{lr}^i \quad (6)$$

vii) costs related to operating a room if it is chosen

$$CR = \sum_{i \in I} \sum_{r \in R} cr_r^i y_r^i, \quad \forall i \in I, \forall r \in R$$

(7)

The objective function can be expressed as

$$\text{Minimize } CD + CN + CNA + CB + CWB + CAB + CR \quad (8)$$

Constraints:

$$DA_{ij} + D_{ij} \leq md_{ij}, \quad \forall i \in I, \forall j \in J$$

(9)

Constraints (9) state that the number of doctor should not be greater the maximum number of doctors can be allocated to each department.

$$SA_{ik} + S_{ik} \leq mn_{ik}, \quad \forall i \in I, \forall k \in K$$

(10)

$$SBA_{ik} + SB_{ik} \leq mna_{ik}, \quad \forall i \in I, \forall k \in K$$

(11)

$$TPA_{il} + TP_{il} \leq mb_{il}, \quad \forall i \in I, \forall l \in L \quad (12)$$

Constraints (10) – (12) are just an analogy to Constraints (9), respectively, for the number of nurse, the number of nurse-aids and the number of beds.

$$\sum_{l \in L} x_{lr}^i \leq y_r^i, \quad \forall i \in I, \forall r \in R \quad (13)$$

Constraints (13) are to make sure that beds are allocated to the available room.

$$\sum_{i \in I} \sum_{j \in J} bd_{ij} D_{ij} + \sum_{i \in I} \sum_{k \in K} bs_{ik} S_{ik} + \sum_{i \in I} \sum_{k \in K} bsa_{ik} + \sum_{i \in I} \sum_{l \in L} bt_{il} TP_{il} +$$

$$\sum_{i \in I} \sum_{l \in L} bw_{il} TPA_{il} + \sum_{i \in I} \sum_{r \in R} \sum_{l \in L} ba_{lr}^i x_{lr}^i + \sum_{i \in I} \sum_{r \in R} cr_r^i y_r^i \leq \sum_{i \in I} \rho_i$$

Constraints (14) state that the budget spent should not be greater than the fund provided.

$$DA_{ij}, D_{ij}, S_{ik}, SA_{ik}, SB_{ik}, SBA_{ik}, TPA_{il}, TP_{il} \geq 0 \text{ and integer} \quad (15)$$

$$\forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L$$

$$x_{lr}^i, y_r^i \in \{0,1\} \quad \forall i \in I, \forall l \in L, \forall r \in R \quad (16)$$

Expressions (15) and (16) are integrity constraints.

The model is in the form of a large scale integer programming problem.

V. FEASIBLE NEIGHBORHOOD HEURISTIC SEARCH THE ALGORITHM

Branch-and-bound approach is a general method for solving linear integer programming problem. However, for

large-scale problems such a procedure would be prohibitively expensive in terms of total computing time, and frequently the algorithm terminates without solving the problem. We have adopted the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

This may be implemented within the structure of a program by marking all integer variables at their bounds at the continuous solution as nonbasic and solving a reduced problem with these maintained as superbasic..

The procedure may be summarized as follows:

Step 1 : Solve the problem ignoring integrality requirements.

Step 2 : Obtain a (sub-optimal) integer feasible solution, using heuristic rounding of the continuous solution.

Step 3 : Divide the set I of integer variables into the set I_1 at their bounds that were nonbasic at the continuous solution and the set $I_2, I = I_1 + I_2$.

Step 4 : Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2 .

Step 5 : At the solution obtained in step 4, examine the reduced costs of the variables in I_1 . If any should be released from their bounds, add them to set I_2 and repeat from step 4, otherwise terminate.

The above summary provides a framework for the development of specific strategies for particular classes of problems. For example, the heuristic rounding in step 2 can be adapted to suit the nature of the constraints, and step 5 may involve adding just one variable at a time to the set I_2 .

At a practical level, implementation of the procedure requires the choice of some level of tolerance on the bounds on the variables and also their integer infeasibility. The search in step 4 is affected by such considerations, as a discrete step in a super basic integer variable may only occur if all of the basic integers remain within the specified tolerance of integer feasibility.

In general, unless the structure of the constraints maintains integer feasibility in the integer basic variables for discrete changes in the superbasic, the integers in the set I_2 must be made superbasic. This can always be achieved since it is assumed that a full set of slack variables is included in the problem.

VI. CONCLUSIONS

This paper presents a capacity nursing-staff management model which integrate bed allocation planning.. The result model would be a large scale integer problem. We propose a feasible neighbourhood integer search for solving the integer model

REFERENCES

- [1] S. Almagoshi, "Simulation modeling in healthcare: Challenges and trends," *Procedia Manufacturing*, Vol. 3, pp. 301-307, 2015.
- [2] F. Ataollahi, M. A. Bahrami, M. Abesi, and F. Mobasheri, "A goal programming model for reallocation of hospitals inpatient beds," *Middle east Journal of Scientific Research*, Vol. 18(11), pp. 1537-1543, 2013.

- [3] S. G. Elkhuisen, G. Bor, M. Smeenk, N. S. Klazinga, and P. J. M. Bakker, "Capacity management of nursing staff as a vehicle for organizational improvement," *BMC Health Services Research*, 7:196, 2007.
- [4] M. Aarabi, and S. Hasanian, "Capacity planning and control: a review," *Int. Journal of Scientific & Engineering Research*, Vol. 5, Issue 8: 975-984, 2014.
- [5] W.P. Pierskalla, and D. Brailer, "Application of operations research in health care delivery. In: Pollock S, Barnett A, Rothkopf M (eds) Beyond the profit motive: public sector applications and methodology," *Handbooks in OR&MS*, vol 6. North-Holland, New York, 1994.
- [6] V. L. Smith-Daniels, S. B. Schweikhart SB, and D. E. Smith-Danielsn, "Capacity management in health care services", review and future research directions *Decision Sci*, Vol. 19, pp. 899-918, 1988.
- [7] K. Hurst, "Selecting and Applying Methods for Estimating the Size and Mix of Nursing Teams", Leeds, Nuffield Institute for Health, pp. 1-19, 2003.
- [8] E. J. Halloran, and P. E. H. Vermeersch, "Variability in Nurse Staffing Research data Collection and the Method of Reporting", *Journal of Nursing Administration*, pp. 17:26-34, 1987.
- [9] S. P. Siferd SP, and W.C Benton, "Workforce Staffing and Scheduling- Hospital Nursing Specific Models", *Eur J Oper Res*, Vol. 60, pp. 233-246, 1992.
- [10] M. J. Brusco, J. Futch, and M. J. Showalter, "Nurse Staff Planning Under Conditions of a Nursing Shortage", *J Nurs Adm*, Vol. 23, pp. 58-64, 1993.
- [11] E. P. C. Kao, G. G. Tung, "Aggregate Nursing Requirement Planning in a Public Health Care Delivery System", *Socioecon Plann Sci*, Vol. 15, pp. 119-127, 1981.
- [12] W. J. Abernathy, N. Baloff, J. C. Hershey, and S. Wandel, "A Three-Stage Manpower Planning and Scheduling Model-A service sector example", *Oper Res*, Vol. 21, pp. 693-711, 1973.
- [13] L. E. Brown and B. A. Lewin, "Supplemental Staffing Agencies: Friend or Foe?", *Nurs Manage*, Vol. 13, pp. 37-47, 1982.
- [14] S. K. Bordoloi, and E. J. Weatherby, "Managerial Implications of Calculating Optimum Nurse Staffing in Medical Units. Health Care", *Manage Rev*, Vol. 24, pp. 35-44, 1999.
- [15] P. D. Wright PD, K. M. Bretthauer, and M.J. Cote, "Reexamining the Nurse Scheduling Problem: Staffing Ratios and Nursing Shortages", *Decision Sciences*, Vol. 37, pp. 39-70, 2006.
- [16] E. W. Hans, M. Van Houdenhoven and P. J. Hulshof, "A framework for healthcare planning and control," *Handbook of Healthcare System Scheduling* (Springer), 2012.
- [17] G. Ma and E. Demeulemeester, "A multi level integrative approach to hospital case mix and capacity planning," *Computers and Operations Research*, Vol. 40, pp. 2198-2207, 2013.
- [18] L. E. Swayne, W. J. Duncan and P. M. Ginter, "Strategic Management of Health Care Organization," John Wiley and Sons, 2012.
- [19] L. F. Laker, C. M. Froehle, C. J. Lindsell, and M. J. Ward, "The flex track: Flexible partitioning between low and high acuity areas of an emergency department," *Annals of Emergency Medicine*, Vol. 64(6), pp. 591-603, 2014.
- [20] L. Keshtkar, K. Salimifard, and N. Faghih, "A simulation optimization approach for resource allocation in an emergency department," *QScience Connect*, Vol. 1, 2015.
- [21] Y. Wang, L. H. Lee, E. P. Chew, S. S. W. Lam, S. K. Low, M.E.H. Ong, and H. Li, "Multi-objective optimization for a hospital inpatient flow process via discrete event simulation," In *Proceedings of the 2015 Winter Simulation Conference*, edited by L. Yilmaz, W. K. V. Chan, I. Moon, T. M. K. Roeder, C. Macal, and M. D. Rosseti, 3622-3631. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc, 2015.
- [22] N. A. Pujowidianto, L. H. Lee, C. H. Chen, and H. Li. "Constrained optimization for hospital bed allocation via discrete event simulation with nested partitions," *Proceedings of the 2016 Winter Simulation Conference*, edited by T. M. K. Roeder, P. I. Frazier, F. Zhou, T. Huschka, and S. F. Chick, 2016.
- [23] T. Wang, A. Guinet, and B. Besombes, "A sizing tool for allocation planning of hospitals bed resources," S. McClean et al. (Eds.): *Intelligent Patient Management*, SCI 189, pp. 113-125, 2009.
- [24] R. B. Bachouch, A. Guinet, S. Hajri-Gabouj, "An integer linear model for hospital bed planning," *Int. J. Production Economics* 140, pp. 833-843, 2012.
- [25] B. Cardoen, E. Demeulemeester, J. Bellen. Optimizing a multiple-objectivesurgical case sequencing problem. *International journal of Production Economics* 119, 354-366, 2009.
- [26] S. Sitepu, H. Mawengkang. A two-stage stochastic optimization model of hospital nursing staff management prblm. *International Journal of Advanced Research in Computer Engineering & Technology*, Vol. 4(1), 44-47, 2015.
- [27] X. Li, P. Beullens, D. Jones, M. Tamiz. An integrated queueing and multi-objective bed allocation model with application to a hospital in China. *Journal of the Operational Research Society*. Vol. 60, 330-338, 2009.
- [28] B. Rechel, S. Wright, J. Barlow, M. Mckee. Hospital capacity planning from measuring stocks to modelling flow. *Bulletin of the World Health Organization*, Vol. 88, 632-636, 2010.