Studies on the Dynamics of two Mutually Coupled Colpitts Oscillators

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Abstract**—**In this paper, we have thoroughly examined the dynamics of a system of two mutually coupled Colpitts oscillators. Two bias-tuned Colpitts oscillators with identical structure are bilaterally coupled through direct coupling scheme.Numerically solving the system equations, it is observed that both the Colpitts oscillators oscillating in periodic mode become perfectly synchronized for some values of the coupling factors. For higher values of coupling factors, the system become chaotic.We have also studied the dynamics of two mutually coupled non-oscillatory Colpitts oscillators. With the increase of the coupling factors between two oscillators, birth of periodic oscillation observed. For increased value of coupling factors, system dynamics become chaotic.We develop conditions for periodic bifurcation in parameter space analytically and verified it through numerical solution of system equations. We also perform a simulation experiment using PSPICE circuit simulator in the radio frequency range with prototype Colpitts oscillator circuits and the experimental observations are in agreement with the numerical simulation results.

Keywords-*Colpitts oscillators; Bilateral direct coupling; birth of oscillation; synchronized oscillations; chaotic oscillations.*

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I. INTRODUCTION

During last few decades a huge numbers of research works have been done on the nonlinear dynamics of a system involving two or more mutually coupled oscillators [1-5]. Several interesting dynamical phenomena have been detected in coupled system of two or more than two oscillators due to inherent nonlinearity of the oscillating device. Occurrence of nonlinear phenomena like quasi periodicity, chaos, intermittency etc. are well documented in the literature [6]. An interesting phenomenon, namely amplitude death, occurs when two coupled oscillators drive each other to fixed points and stop oscillating [7]. Recent experiments have led to increase interest in the origin and dynamics of Chimera state in two or more mutually coupled oscillators system [8-10]. Coupled electronic oscillators have important applications in various electronics and communication systems, e.g. spectrally pure signal generators, coherent modulators and detectors, power mixers, lock in amplifiers and filters, frequency synthesizers etc. [11, 12]. But to the knowledge of the authors, the phenomenon of manifestation of oscillations in two mutually coupled oscillators, initially both in non-oscillatory states, is not thoroughly examined. Here our intension is to observe such phenomenon, may be called as birth of oscillation, in a system of two mutually coupled bias-tuned Colpitts oscillators. This paper includes a comprehensive study of the dynamics of two mutually coupled Colpitts oscillators. Here, we have first observed the dynamics of two periodic COs coupled in a bilateral way (mutually coupled PCO-PCO system) and thoroughly studied the effect of variation of coupling coefficients using direct coupling scheme. In the second part of our study, we kept both the COs in non-oscillatory mode by properly adjusting their bias current. Then they are coupled bidirectionally with the same coupling scheme. Initiating with non-oscillatory COs in steady equilibrium state, we get limit cycle oscillations for some critical coupling parameters. The phenomenon has similarity with rhythmogenesis referred in nonlinear dynamics literature [13, 14]. Obviously, dynamics of such a system has practical importance and it needs serious study. A very few work in this direction could be found in the literature [15]. We have also observed generation of chaotic oscillation in such a system. An experimental verification of numerically obtained results has been done using a circuit simulation experiment. Our main intention is to explore the possibility of inducing periodic oscillation in two coupled COs kept at a non-oscillatory states.

The paper has been organized in the following way. Description of the bilaterally coupled Colpitts oscillator (BCCO) system has been discussed in section II. The process of the setup of BCCO system and its circuit theoretic model have been discussed here. We formulate the equations describing dynamics of a system of bidirectionally coupled COs using the equations of bias current controlled CO derived in our earlier works [16,17]. The stability of the system dynamics has been studied in section III. The conditions for birth of periodic oscillations from a stable non-oscillatory state has been formulated in section III-A. In section IV, we have shown that numerical simulations of the system dynamics agree with our analytical predictions. It also indicates that the system can exhibit nonlinear chaotic oscillations for larger value of coupling factors. The observations are also verified with simulation experiment using PSPICE circuit simulator. The results of this experiment have been described in section V. Finally some concluding remarks are included in section VI.

II. DESCRIPTION OF THE BCCO SYSTEM AND FORMULATION OF THE SYSTEM EQUATIONS

The BCCO system under study is implemented by connecting two current controlled BJT based COs as described in our earlier works [16,17] through two coupling networks in bidirectional way. We inject a fraction of output signal of first oscillator into the input of the second one through a coupling network and a fraction of the output of the second CO into the input of the first one through another similar coupling network. A simplified block diagram of BCCO system is shown in Fig.1. The details of the practical coupling arrangement are discussed in Section 5. The effect of this coupling is examined by varying the coupling factors.

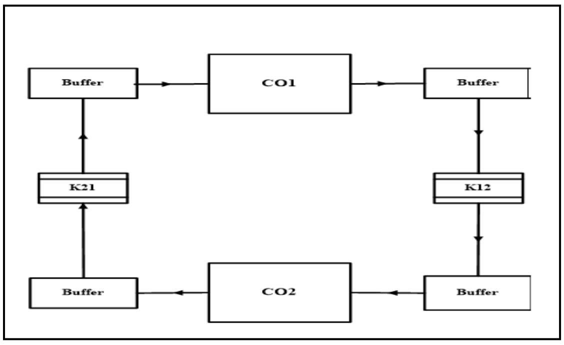


Figure 1. A simplified block diagram of the bilaterally coupled CO system.

As shown in Fig.2, to form the bidirectional coupling, collector of the each CO is coupled with the emitter of the other CO by the resistors R_{s1} and R_{s2}. By changing the values of the resistances R_{s1} and R_{s2} the coupling strength can be varied.

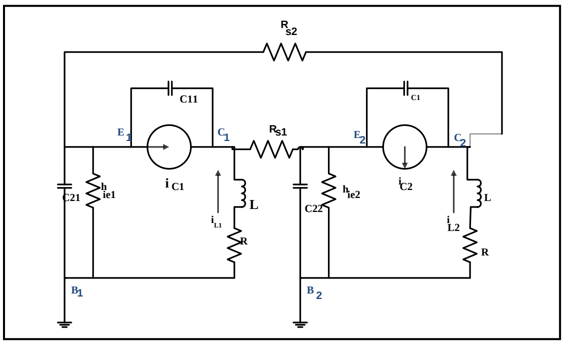


Figure 2. ac equivalent circuit of the BCCO system.

The system equations of the bidirectionally coupled oscillators system are formulated by considering the ac equivalent circuit as shown in Fig.2. The state variables of the first CO and the second CO are taken as (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively. Applying Kirchoff's law to both the loop in the Fig.2 a set of first order nonlinear autonomous differential equations formed which describe the dynamics of the BCCO system. The dynamics is described by the following set of differential equations. Here the first three are for the first CO and the rest three are for the second CO. The equations are:

$$\frac{dx_1}{d\tau} = -\frac{g_1(z_1 - a_1 y_1 + b_1 y_1^3)}{Q_1(1-k_1)} \quad (1a)$$

$$\frac{dy_1}{d\tau} = \frac{g_1(k_{F21}(x_2+y_2) - z_1) - h_{r1} y_1}{Q_1 k_1} \quad (1b)$$

$$\frac{dz_1}{d\tau} = \frac{Q_1 k_1 (1-k_1)(x_1+y_1)}{g_1} - \frac{z_1}{Q_1} \quad (1c)$$

$$\frac{dx_2}{d\tau} = -\frac{g_2(z_2 - a_2 y_2 + b_2 y_2^3)}{Q_2(1-k_2)} \quad (2a)$$

$$\frac{dy_2}{d\tau} = \frac{g_2(k_{F12}(x_1+y_1) - z_2) - h_{r2} y_2}{Q_2 k_2} \quad (2b)$$

$$\frac{dz_2}{d\tau} = \frac{Q_2 k_2 (1-k_2)(x_2+y_2)}{g_2} - \frac{z_2}{Q_2} \quad (2c)$$

The system equation (1b) of the first CO and (2b) of the both COs are modified by the inclusion of the forcing signal term $k_{F21}(x_2 + y_2)$ and $k_{F12}(x_1 + y_1)$ respectively obtained from the other CO. Here k_{F12} and k_{F21} are the coupling factors mentioned before. Since two oscillators are considered nearly identical in structure, in the simulation study we take the system parameters $g_1 = g_2 = g$, $Q_1 = Q_2 = Q$, $h_{r1} = h_{r2} = h_r$, $k_1 = k_2 = k$. Suitably adjusting a and b, both the COs are made to operate in the oscillatory or non-oscillatory stable mode.

III. STABILITY ANALYSIS OF SYSTEM EQUATIONS

In the analytical study of the BCCO dynamics, first we evaluate the equilibrium points $(x_1^*, y_1^*, z_1^*, x_2^*, y_2^*, z_2^*)$ in the six dimensional state space by equating the time derivatives of the state variables $x_1, y_1, z_1, x_2, y_2, z_2$ to zero in equations (1) and (2). This gives three possible equilibrium points P1, P2 and P3 respectively, where P1 is (0, 0, 0, 0, 0, 0). While P2 and P3 are given by expressions written as

$$\left(\mp\left(\frac{n_1}{Q_1 k_1(1-k_1)} + 1\right)\sqrt{\frac{a_1 + \frac{n_1}{m_1}}{b_1}}, \pm\sqrt{\frac{a_1 + \frac{n_1}{m_1}}{b_1}}, \mp\frac{n_1}{m_1}\sqrt{\frac{a_1 + \frac{n_1}{m_1}}{b_1}}, \mp\sqrt{\frac{a_2}{b_2}}, \pm\sqrt{\frac{a_2}{b_2}}, 0\right).$$

Where $n_1 = \frac{h_{r1}}{Q_1}$, $m_1 = \frac{g_1}{Q_1}$, $n_2 = \frac{h_{r2}}{Q_2}$, $m_2 = \frac{g_2}{Q_2}$,

The stability of steady state points is examined with the help of (6×6) transformation Jacobian matrix J(X) of the system evaluated at a particular non-trivial steady state point as represented in equation (3).

J=

$$\begin{pmatrix} 0 & (a_1 - 3b_1 y_1^{*2}) & -1 & 0 & 0 & 0 \\ 0 & -\frac{n_1}{k_1} & -\frac{m_1}{k_1} & k_{F21}\frac{m_1}{k_1} & k_{F21}\frac{m_1}{k_1} & 0 \\ \frac{k_1(1-k_1)}{m_1} & \frac{k_1(1-k_1)}{m_1} & -\frac{1}{Q_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (a_2 - 3b_2 y_2^{*2}) & -1 \\ k_{F12}\frac{m_2}{k_2} & k_{F12}\frac{m_2}{k_2} & 0 & 0 & -\frac{n_2}{k_2} & -\frac{m_2}{k_2} \\ 0 & 0 & 0 & \frac{k_2(1-k_2)}{m_2} & \frac{k_2(1-k_2)}{m_2} & -\frac{1}{Q_2} \end{pmatrix} \quad (3)$$

Here, y_1^* *and* y_2^* indicate the value of y_1 *and* y_2 respectively at a particular equilibrium point. Obeying standard notations of nonlinear dynamics and representing X as the state vector having six components ($x_1, y_1, z_1, x_2, y_2, z_2$), the characteristics equation is written as det(J(X_s) $-\boldsymbol{\lambda}$I) = 0, where, X_s is the state vector at an equilibrium point, I is the identity matrix and $\boldsymbol{\lambda}$ is Eigen value of Jacobian matrix. So for the system under study we get characteristics equation in the form as given in equation (4),

$$P_6\lambda^6 + P_5\lambda^5 + P_4\lambda^4 + P_3\lambda^3 + P_2\lambda^2 + P_1\lambda + P_0 = 0 \quad (4)$$

Here, the coefficients P_0, P_1, P_2, P_3, P_4, P_5 and P_6 will depend on the nature of coupling, values of system parameters and the location of fixed point of the concerned system.

The dynamics of BCCO system can be achieved by inspecting coefficients of characteristic equation (4) without explicitly finding roots [18]. The nature of the roots of the characteristic equation (4) is obtained by applying the Routh-Hurwitz's criteria [19], and hence the stability of a particular equilibrium point can be predicted. The equilibrium point would be stable if the roots of (4) have negative real parts.

A. Condition for birth of oscillation

We can achieve the condition of bifurcation to stablenon-oscillatory state in terms of P_i's by applying the method defined in [18]. The conditions of periodic oscillatory bifurcation are as follows.

$$P_0 > 0 \qquad (5a)$$
$$P_1 > 0 \qquad (5b)$$
$$(P_1P_2 - P_0P_3) > 0 \qquad (5c)$$
$$P_1(P_2P_3 - P_1P_4) + P_0(P_1P_5 - P_3^2) > 0 \qquad (5d)$$

The coefficients in equation (4) can be evaluated by solving the above equations (given in the appendix). Where we have assumed that two COs are identical which makes all the parameters in both the oscillators equal and the coupling coefficients are also equal, i. e.$k_{F12} = k_{F21} = c$.

For the equilibrium point (0,0,0,0,0,0), from the condition in (5a) we can get

$$c < \frac{(1-k)\left(a+\frac{n}{m}\right)kQ}{\sqrt{am}} \qquad (6)$$

The relation stated in (6) is also valid in the condition in (5b).

By substituting the values of the other parameters (as stated in [16],[17]) except the coefficient of coupling, c in the condition (5c) and neglecting the higher order terms involving c, it has been found that c should be greater than 0.056 to get into periodic oscillatory bifurcation.

So we get a range of coupling factors for which periodic Hopf bifurcation is possible.

These results are consistent with intuitive predictions that a third or higher order system may become oscillatory if damping term is reduced to zero. Here because of the process of coupling, we get situations when damping term of a system is effectively minimized due to forcing term from other oscillator. This makes the coupled system oscillatory.

IV. EVALUATION OF SYSTEM DYNAMICS THROUGH NUMERICAL SIMULATIONS

The dynamics of the BCCO system is studied by numerical integration of (1) and (2) for a carefully chosen set of system parameters. We adopt 4th order Runge–Kutta integration algorithm in the normalized time domain with a step size h = 0.01. Initial transients in the numerical solution are excluded by discarding more than 50% of the data points close to initial time and thus steady state values of the state variables are obtained. The values of the parameters k, g, Q and h_r of both the Colpitts oscillators have been taken in accordance with the designed experimental circuit (described in Section 5). They are taken as $g = 1.32, Q = 4.0,\ k = 0.5, h_r = 0.04, b = 0.2$ for both the oscillators.

A. Dynamics of mutually coupled PCO-PCO system

At first, we would consider the effect of direct mutual coupling in a system of two periodic Colpitts oscillators. A simultaneous solution of (1) and (2) with properly chosen parameters gives the dynamics of mutually coupled PCO-PCO system. The parameter a_1 of the first CO is taken as 2.20, which gives its periodic condition of oscillation. The parameter a_2 of the second CO is taken as 2.22. This ensures that its oscillation in periodic mode [16, 17] and the case of mutually coupled PCO-PCO system is obtained. Then, varying the value of the coupling parameters k_{F1} and k_{F2} we examine the effect of periodic perturbation on both the oscillators' dynamics. The dynamics of the system can attain different states depending upon the values of coupling factors and bias current parameter, a of two individual oscillators. In case of very small magnitude of c (we have chosen$k_{F12} = k_{F21} = c$), the system dynamics attains a nearly uncorrelated state. The BCCO system achieves a synchronized state of two oscillators for a range of values of c. Both the oscillators have a common frequency of oscillation with a relative phase shift. The system becomes completely synchronized for a particular range of values of c. Fig. 3 shows the state- space trajectories in y - z plane of two oscillators when coupling is very weak. Fig. 3 (c) pointed out that the oscillations of two COs are uncorrelated.

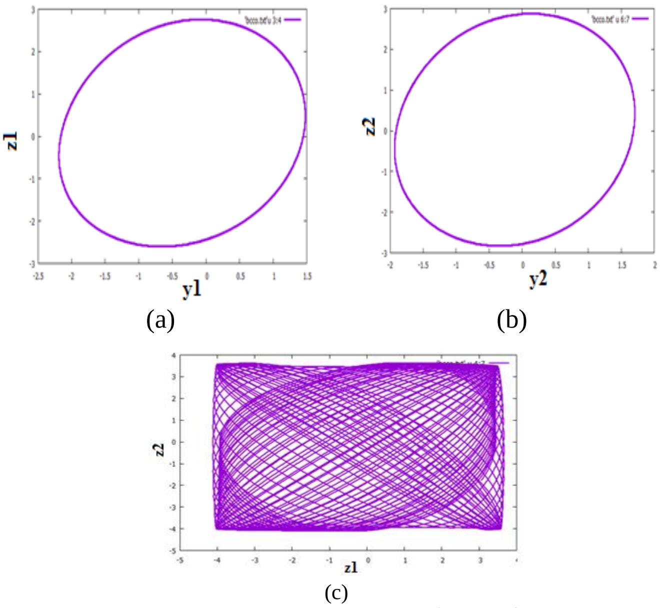


Figure 3. Numerically obtained phase plane plots of $y - z$ of (a) CO1, (b) CO2 and (C) mutual phase plane plot of state variables z1 –z2 of BCCO system for very weak coupling, $k_{F12} = k_{F21}$=c = 0.0006 indicating uncorrelated states. (The system parameters area_1 = 2.20, a_2 = 2.22, g = 1.32, Q = 4.0, k = 0.5, h_r = 0.04, b = 0.2.).

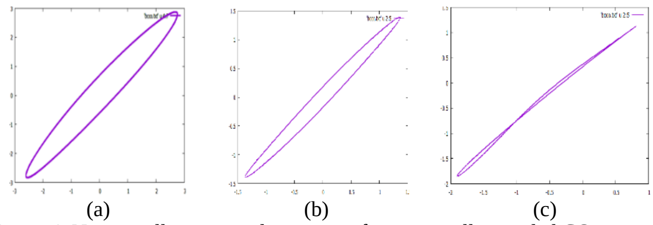
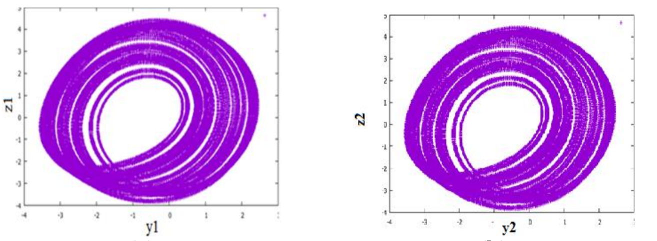


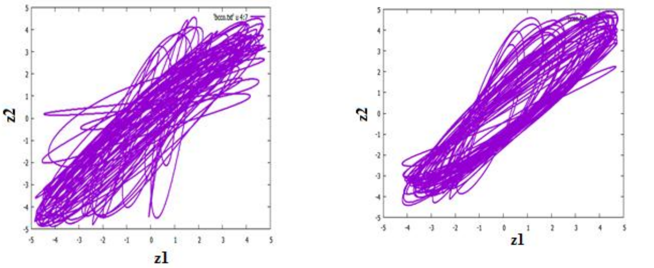
Figure 4. Numerically computed response of two mutually coupled COs system showing the relation between z_1 and z_2 for different values of the coupling factors. The system parameters area_1 = 2.20, a_2 = 2.22, g = 1.32, Q = 4.0, k = 0.5, h_r = 0.04, b = 0.2. (a) c= 0.04, (b) c= 0.012, (c) c= 0.15.

Fig. 4 states that as coupling strength increases a state of synchronized oscillation approaches and at $k_{F12} = k_{F21} = c =$ 0.15, a state of perfect synchronization happens. For further increase in the coupling factors the dynamics of mutually coupled system become apparently random. The nature of the phase plane trajectories between $y - z$ of both the oscillators indicate chaos and the elliptical pattern of mutual phase space plot of CO1 and CO2 indicates the tendency of phase synchronization between two chaotic oscillations generated in two COs. Fig. 5 and Fig. 6 show the numerically obtained results.



(a) (b)

Figure 5. Numerically calculated phase plane plot of $y - z$ of BCCO system in direct coupling scheme for (a)CO1 and (b) CO2 respectively for coupling strength , $k_{F12} = k_{F21} = c =$ 0.3 (The values of other system parameters are g = 1.32, Q = 4.0, k = 0.5, h_r = 0.04, a=0.6, b = 0.2).



(a) (b)

Figure 6. Mutual phase plane plot of state variable z1 –z2 of BCCO system in direct coupling scheme for different coupling factors, (a) $k_{F12} = k_{F21} = c =$ 0.25 and (b) $k_{F12} = k_{F21} = c =$ 0.3 respectively. (The values of other system parameters are g = 1.32, Q = 4.0, k = 0.5, h_r = 0.04, a=0.6, b = 0.2).

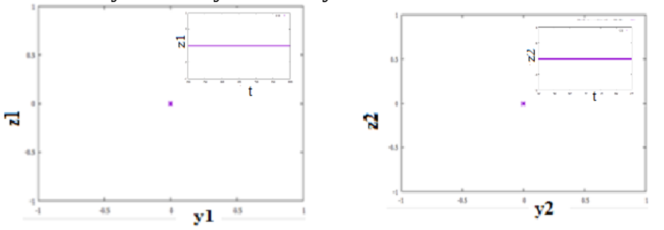
B. *System of two mutually coupled non-oscillatory CO circuits*

In the next part of our numerical study, we repeat the simulation study witha_1 = 0.65 and a_2=0.65. Here, we would consider the effect of coupling factors on the dynamics of two under-biased non-oscillatory Colpitts oscillators. A simultaneous solution of (1) and (2) with properly chosen parameters gives the dynamics of the BCCO system. The parameters a_1 and a_2 of both COs are taken also as 0.65. This ensures from [16] and [17] that both the oscillators are in stable non-oscillatory states. Then, varying the value of the coupling parameters k_{F12} *and* k_{F21} we examine the effect of direct coupling on BCCO system dynamics. The values of other CO parameters are taken as similar as in the previous case.

C. *Birth of oscillation*

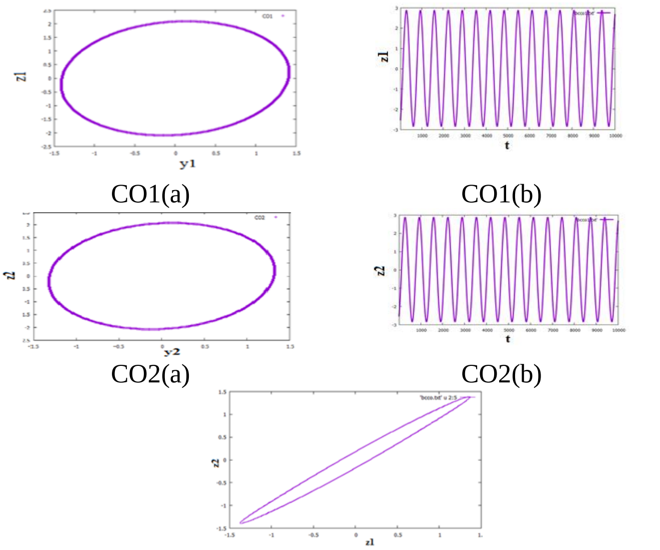
In this case appearance of periodic oscillating states take place for c =0.05 which is in agreement with the predicted conditions of periodic bifurcation by analytical method. It is observed from the results depicted in Fig. 7 that the BCCO system remains in non-oscillatory state until the coupling strength reaches the threshold value.

Taking coupling factor c (same for two COs) as control variables we get different dynamical states of BCCO system. For each value of parameter c, we obtain time series data set and phase plane trajectories of state variables. Fig 8 confirms that the BCCO system undergo periodic oscillatory state after a critical value of the coupling factor as stated in the analytical study. The critical value of c = 0.05 which is almost identical as predicted by the analytical study.



(a) (b)

Figure 7. Numerically calculated phase plane plots of state variables y-z and Time series (inset) of BCCO system in direct coupling scheme for very small coupling factor, c = 0.005. The values of other system parameters are g = 1.32, Q = 4.0, k = 0.5, h_r = 0.04, a=0.6, b = 0.2. (a) CO1 and (b) CO2.



CO1(a) CO1(b)

CO2(a) CO2(b)

(c)

Figure 8. Numerically calculated (a) phase plane plots of state variables of y-z, (b) time series plots and (c) mutual phase plane plot of state variables z1 –z2 of BCCO system in direct coupling scheme for small coupling factor, c = 0.08. The values of other system parameters are g = 1.32, Q = 4.0, k = 0.5, h_r = 0.04, a=0.6, b = 0.2. Upper plots for CO1 and lower plots for CO2.

The phase relation between the periodic oscillations in two COs is examined by noting the relative time domain evolution of the state variables z_1(t) and z_2 (t). For this purpose we plot them along x and y axes respectively of x–y plane as shown in Fig. 8 (c). It has been detected from the Fig. 8 (c) that the phase plane plot of generated periodic outputs from two COs of BCCO is an elliptic pattern. This elliptical mutual phase plane indicates that there exists a constant phase relation between two outputs from two COs. For a small range of c values, a regular pattern of the curve is observed. This phenomenon is similar as predicted in [15].

D. *Generation of chaotic oscillation*

Numerical simulations also indicate that BCCO system shows different chaotic dynamics for further increase of coupling factors. Fig. 9 and Fig. 10 show the nature of the system dynamics in time domain and phase plane plots for different values of coupling factors. Both the nature of phase plane and time domain plots approve that the dynamics of the BCCO system is aperiodic for large coupling.

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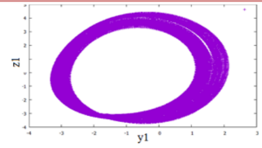
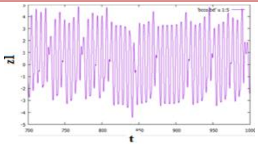
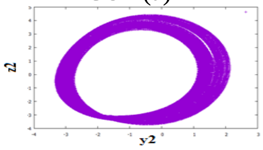


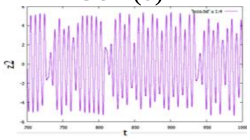
CO1 (a)



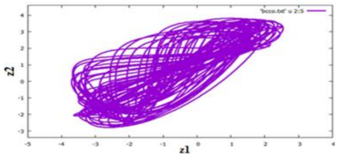
CO1 (b)



CO2 (a)

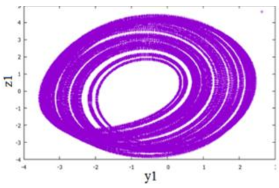


CO2 (b)

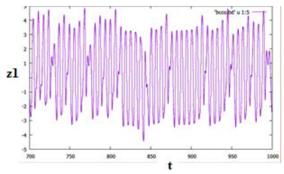


(c)

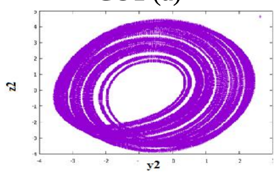
Figure 9. Numerically calculated (a) phase plane plot $(y - z)$, (b) time series plots of state variable z and (c) mutual phase plane plot of state variable z1 –z2 of BCCO system in direct coupling scheme for moderate coupling factor, c = 0.15. The values of other system parameters are g = 1.32, Q = 4.0, k = 0.5, h_r= 0.04, a=0.6, b = 0.2. Upper plots for CO1 and lower plots for CO2.



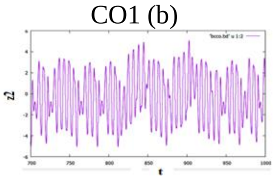
CO1 (a)



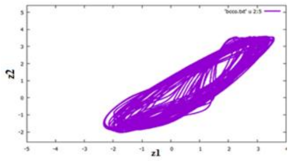
CO1 (b)



CO2 (a)



CO2 (b)



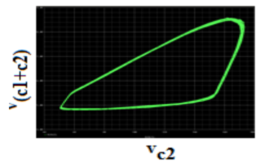
(c)

Figure 10. Numerically calculated (a) phase plane plot, (b) time series plots and (c) mutual phase plane plot of z1 –z2 of BCCO system in direct coupling scheme for high coupling factor, c = 0.35. The values of other system parameters are g = 1.32, Q = 4.0, k = 0.5, h_r= 0.04, a=0.6, b = 0.2. Upper plots for CO1 and lower plots for CO2.

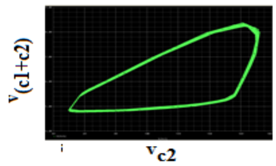
V. Experimental Studies

Dynamics of two mutually coupled COs is studied experimentally in radio frequency regions by using PSPICE circuit simulation software. The design procedure of both the COs are similar as reported in [16] and [17]. Two such COs has been coupled together properly to form a BCCO system. The coupling network has also been designed using op-amps. The couplings of two COs are done in such a way that the dc operating conditions of the oscillators are not disturbed. We have applied direct coupling scheme by feeding a fraction of output signal of one CO directly into the other CO through a buffer amplifier and vice versa. The coupling factor is controlled by varying the value of the resistor used for controlling the gain of the buffer amplifier. Moreover, circuit parameters of both the oscillators are taken nearly equal, but the dc bias currents of CO1 and CO2 are chosen differently in order to drive two oscillators oscillating in different modes of oscillations. Two variable resistors are used to control the coefficient of coupling between the COs (k_{12} and k_{21})independently. The observations are summarized as follows:

(i) Keeping both CO1 and CO2 in periodic mode but a little different in bias current ($I_1 = 5.5mA$ and $I_2 = 5.4mA$), we choose non zero value of k_{12} andk_{21} to obtained the bilaterally coupled system. The frequency of oscillation of CO1 and CO2 are obtained as 105.0 and 102.0 kHz, respectively. In BCCO, system behaviors of individual oscillators are modified from their free running mode due to coupling with other oscillator. . The voltages at the collector terminal and the emitter terminal are chosen as the state variables to be applied at the x and y input of the oscilloscope to plot the experimental phase-plane diagram. Changing the value of the coupling resistors R_{S1} and $R_{S2,}$ the strength of the injected current to one CO taken from another CO is varied. This ensures the variation of k_{F12} and k_{F12}. We perform simulation experiment on this circuit using the circuit simulator PSPICE. A few representative results are depicted in Fig. 11 to Fig. 13. The dynamics of the system can attain different states depending upon the strength of coupling and operating bias current of two individual oscillators. In case of very small magnitude of c the system dynamics reaches a nearly uncorrelated state. The BCCO system achieves a synchronized state of two oscillators for a range of values of c. Both the oscillators have a common frequency of oscillation with a relative phase shift. The system becomes completely synchronized for a particular range of values ofk_{12} andk_{21}.



(a)



(b)

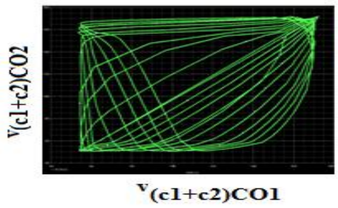
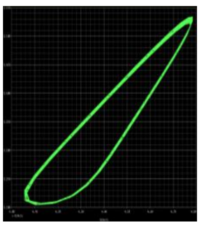
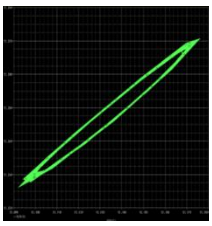


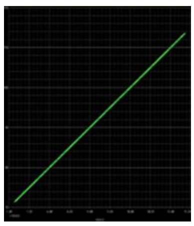
Figure 11. Experimentally (PSPICE) obtained phase plane plots of $v_{(c1+c2)}$ - $v_{c2}$for (a) CO1, (b) CO2 and (c) mutual phase plane for output taken from CO1 and CO2 of BCCO system for coupling strength $k_{F12} = k_{F21}$=.0006, indicating uncorrelated state.



(a)



(b)



(c)

Figure 12. Experimentally (PSPICE) obtained mutual phase plane for output taken from CO1 and CO2 of BCCO system for different coupling strength (a)$k_{F12} = k_{F21}$=0.042, (b)$k_{F12} = k_{F21}$=0.09, (c) $k_{F12} = k_{F21}$=0.16.

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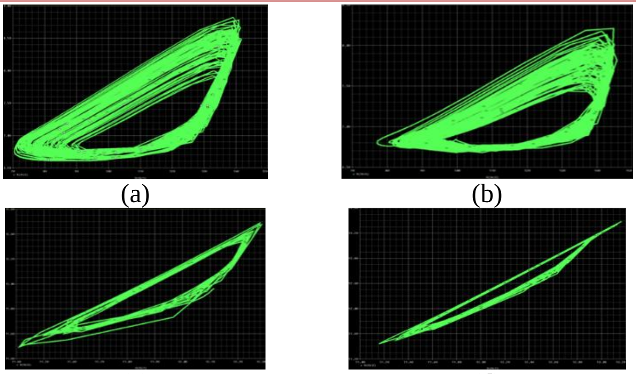


(a) (b) (c) (d)

Figure 13. Experimentally (PSPICE) obtained phase plane plot of $v_{(c1+c2)} - v_{c2}$ of BCCO system in direct coupling scheme for (a) CO1 and (b) CO2 respectively for coupling strength $c =$ 0.3 and (c), (d) are mutual phase plane for output taken from CO1 and CO2 of BCCO system for different coupling strength (c) $k_{F12} = k_{F21}$=0.26, (d) $k_{F12} = k_{F21}$=0.32.

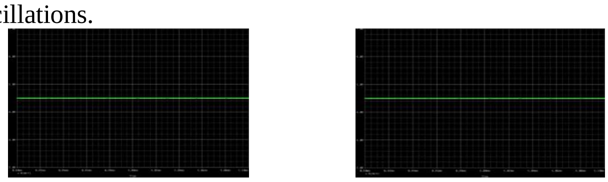
(ii) For some critical value of coupling factors the dynamics become apparently random. The nature of the phase plane trajectory specifies chaos and the elliptical pattern of mutual phase space of CO1 and CO2 indicates phase synchronization between two chaotic oscillations generated in two COs.

(iii) In the next phase, we have chosen that bias current, I = 1.2mA for both the oscillators which confirms non-oscillatory state of COs from the experimental works as described in [16] and [17]. The coupling factor k_{F12} and k_{F12} are taken equal and they are slowly increased by equal amounts (i.e. always taking $k_{F12} = k_{F21}$ = c (say)). We observe the variation of out puts of two COs in phase plane plot, time and frequency domain for different coupling factors. Experimentally obtained results are depicted in Fig. 14 to Fig. 17.

Setting both CO1 and CO2 in non-oscillatory mode (but both are just on the verge of oscillation), we choose non zero value of k_{12} and k_{21}. For a particular rang of values of coupling factors both the oscillators can be brought into periodic oscillation state. Observations of simulation experiment indicate that sustained oscillation is possible in non-oscillatory COs when two similar oscillators are coupled together. Farther increase in ks, the system exhibits nonlinear chaotic behavior.

We observe that a stable periodic oscillation begins when c= 0.05. The observation is consistent with predicted analytical condition, which states the condition of birth of oscillation in this case is c =0.056. The condition has been verified for other values of I also and results obtained are satisfactory. We get chaotic oscillations when c > 0.15. This indicates that experimental results are in good agreement with analytical predictions as well as numerical simulation observations.

We also investigate experimentally the existence of any types of synchronization between the stable oscillations in two COs. We have done this by plotting their outputs along X and Y direction of the same Cartesian plane as shown in Fig. 15(c), Fig. 16(c), and Fig. 17(c). This gives a closed elliptic curve which confirms constant phase difference between two oscillations.



(a) (b)

Figure 14. Experimentally obtained (PSPICE) time domain plots for (a) CO1 ($V_{(c1+c2)}$)and (b) CO2 ($V_{(c1+c2)}$) when coupling is very week coupling, $k_{F12} = k_{F21} = c = 0.005$.

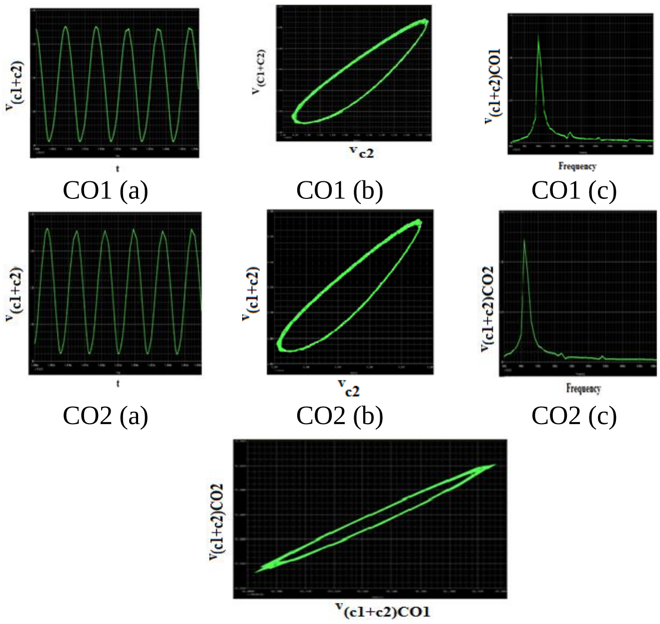


Figure 15. Experimentally (PSPICE) obtained (a) time domain, (b) phase plane ,(c) frequency domain plots and (d) mutual phase plan ($V_{C1} - V_{C3}$) plots of BCCO system for small coupling, $k_{F12} = k_{F21} = c$ = 0.05. Upper plots for CO1 and lower plots for CO2

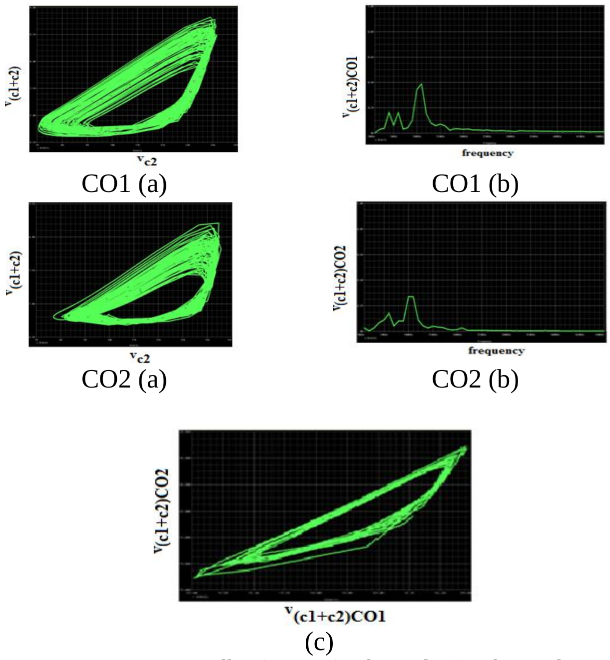
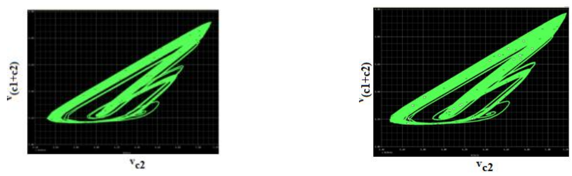
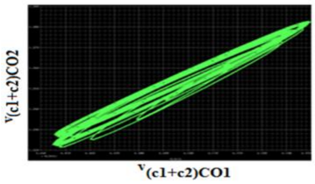


Figure 16. Experimentally (PSPICE) obtained (a) phase plane, (b) frequency domain plots and (c) mutual phase plan ($V_{C1} - V_{C3}$) plots of BCCO system for moderate coupling, $k_{F12} = k_{F21}$ c = 0.152. Upper plots for CO1 and lower plots for CO2.



(a) (b)



(c)

Figure 17. Experimentally (PSPICE) obtained (a) phase plane, (b) frequency domain plots and (c) mutual phase plan ($V_{C1} - V_{C3}$) plots of BCCO system for moderate coupling, $k_{F12} = k_{F21} = c = 0.35$.

VI. CONCLUSIONS

In this paper, we have done a detailed investigation on the dynamics of the BCCO system through numerical simulation as well as PSPICE simulation experimental studies. It is observed that two periodically oscillating COs when bilaterally coupled can enter into different states depending on the values of the coupling strengths. When both the coupling factor are small, i.e., when two COs interact very weakly with each other, their dynamics although affected but remains uncorrelated. However, the interaction of periodic COs for higher values of coupling factors (one or both) leads to qualitative synchronization of the periodic oscillations of the COs. It is observed that the dynamics of two mutually coupled COs become chaotic for some critical values of coupling factors. Numerical simulation of the mathematical model of the BCCO system reveals the influence of the coupling factors on the dynamics of the system. A phenomenon may be called as birth of oscillation in a bidirectional coupled non-oscillatory CO system also has been studied in this chapter. Using standard technique of stability analysis of stable equilibrium points of the coupled system, the required amount of coupling factor for periodic bifurcation has been predicted. We have considered direct coupling schemes in this study. The analytically predicted values are in close agreement with numerical simulation and circuit simulation experimental results. Beyond this onset value of coupling factor, we observe chaotic oscillations in this system through numerical simulation for higher values of coupling factors. Time series plots and phase plane trajectories of the system support the existence of chaos. Finally the analytical and numerical predictions are verified with simulation experiment using PSPICE simulation software. Presence of chaos like aperiodic state has been obtained for different values of coupling factors numerically as well as experimentally. For small amplitudes of coupling leads to periodic oscillations; but increased amount of coupling results into aperiodic chaotic oscillations. This present work is an evidence that the generation of oscillation in coupled systems is possible where all the nodes are in under threshold conditions. Qualitatively, we may understand that coupling between two oscillators in non-oscillatory state with proper phase and amplitude may reduce the internal damping of isolated oscillators and at some critical values of coupling factors internal damping becomes zero. In these conditions, oscillations are generated. Details studies with other coupling schemes (e. g. diffusive) along with a large number of coupled oscillators where some of the oscillators are in non-oscillatory state have enormous practical importance in variety of relevant fields.

APPENDIX

The coefficients in equation (4)can beevaluated by solving the characteristics equation stated in section III. They are:-

$$P_6 = 1 \tag{A.a}$$

$$P_5 = 2\left(\frac{n}{k} + \frac{1}{Q}\right) \tag{A.b}$$

$$P_4 = \frac{4n}{kQ} + \frac{n^2}{k^2} + \frac{1}{Q^2} + 2\frac{k(1-k)}{m} + 2(1-k) - \frac{m^2C^2}{k^2Q} \tag{A.c}$$

$$P_3 = \frac{2n^2}{k^2Q} + \frac{2n}{kQ^2} + \frac{2n(1-k)}{k} + \frac{3n(1-k)}{m} + \frac{2(1-k)}{Q} + 2n(1-k) + \frac{k(1-k)}{mQ} + (a-3by_1{}^{*2})(1-k) - \frac{mC}{k^2Q} - \frac{(a-3by_1{}^{*2})mC^2}{k^2} - \frac{m^2C^2}{k^2} - (a-3by_2{}^{*2})(1-k) \tag{A.d}$$

$$P_2 = \frac{n^2}{k^2Q^2} + \frac{n(1-k)}{kQ} + \frac{4n(1-k)}{mQ} + \frac{n^2(1-k)}{mk} + \frac{k(1-k)}{mQ} + \frac{(1-k)}{kQ} + \frac{(a-3by_1{}^{*2})(1-k)n}{k} + \frac{(a-3by_1{}^{*2})(1-k)}{Q} + \frac{n^2(1-k)}{mk} + \frac{k^2(1-k)^2}{n^2} + (1-k)^2 - \frac{n(1-k)(a-3by_2{}^{*2})}{k} - \frac{(a-3by_2{}^{*2})(1-k)}{Q} - \frac{m^2C^2}{k^2} - \frac{m(1-k)C^2}{k} - \frac{(a-3by_1{}^{*2})(a-3by_2{}^{*2})m^2C^2}{k^2} - \frac{2(a-3by_2{}^{*2})mC^2}{k^2Q} \tag{A.e}$$

$$P_1 = \frac{2n^2(1-k)}{mkQ} + \frac{n(1-k)\left(a-3by_1{}^{*2}\right)}{kQ} + \frac{\left(a-3by_1{}^{*2}\right)k(1-k)^2}{m} + \frac{n(1-k)}{m} + \frac{k(1-k)^2}{m} + \frac{2kn(1-k)^2}{m^2} + \left(a-3by_1{}^{*2}\right)(1-k)^2 - \frac{mn(1-k)\left(a-3by_2{}^{*2}\right)}{kQ} - m(1-k)^2\left(a-3by_2{}^{*2}\right) - \frac{n(1-k)^2}{m} - \frac{mQ(1-k)C^2}{k} - \frac{2(a-3by_1{}^{*2})(a-3by_2{}^{*2})m^2C^2}{k^2Q} - \frac{(a-3by_1{}^{*2})m^2C^2}{k^2Q^2} \tag{A.f}$$

$$P_0 = \frac{\left(a-3by_1{}^{*2}\right)(1-k)^2n}{m} + \frac{\left(a-3by_2{}^{*2}\right)(1-k)^2n}{m} + \frac{n^2(1-k)^2}{m^2} + \left(a-3by_1{}^{*2}\right)\left(a-3by_2{}^{*2}\right)(1-k)^2 - \frac{\left(a-3by_1{}^{*2}\right)mC^2}{k^2Q^2} \tag{A.g)}$$

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