A Multiple-Buyer Single-Vendor in a Continuous Review Inventory Model with Ordering Cost Reduction Dependent on Lead Time

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Abstract:- In this competitive environment, integration between two or more business entities is an important way to gain competitive advantages as it lowers supply chain cost. This paper presents a multiple-buyer single-vendor integrated inventory system with ordering cost reduction on lead time. The options of ordering cost reduction included lead time of every buyer can be reduced at an added crash cost. Lead time plays a vital role in supply chain management and inventory management system. A lead time means that time gap between the placing of an order and its actual arrival in the inventory. In this paper, we study a continuous review model. The model is formulated to integrated total cost of the vendor-buyers system to determine the optimal solutions of order quantity, ordering cost, lead time and the number of deliveries between the vendor and buyers in a production cycle. Finally, a numerical example and effects of key parameter are included to illustrate the results of the proposed model.

Keywords: *Integrated inventory model; Multiple-buyer; Lead time crashing cost; Ordering cost reduction.*

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1. Introduction

Inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role in choosing the best inventory policy. To meet the needs of customers, business must maintain higher inventory levels to avoid shortages. However, Continuous review inventory system is record of the inventory level to checked continuously until a specified point is reached where a new order is placed. This system is also called fixed order quantity system.

Integrated inventory management system is common practice in the global markets and provides economic advantages for both the vendor and buyer. In recent years, most integrated inventory system form a strategic alliance to minimize their own cost or maximize their own profit, then trading parties can collaborate and share information to achieve improve benefits. In the situation of multiple buyer is common in real life, a vendor may supply a product to different buyers to fulfil their requirements. For example, In health care industries, the vendor suplies instruments to different hospitals according to fulfill their needs.

In this literature of continuous review inventory model, a single vendor multiple buyer supply chain system to placing an order whenever inventory level falls on the re-order point. The proposed model describes ordering cost reduction on lead time under linear function case and the lead time components of each buyer can be reduced to an added crash cost. The buyer level demand is assumed to be normally distributed. Our objective is to determine the integrated total cost of vendor-buyers system.

In this paper extends the work of Vijayashree and Uthayakumar (2017) have presented a ordering cost reduction on lead time with single-vendor single-buyer integrated inventory system. In this study, by assuming the multiple-buyer single-vendor integrated model based on equal shipment cycle time, ordering cost reduction and lead time reduction. The solution procedure is to determine the optimal solutions of order quantity, ordering cost, lead time and the number of shipments under integrated model. For this reason, inventory models are developed and numerical example are presented to illustrate the models.

The rest of the paper proceeds as follows, section 2 presents a review of related literature, section 3 defines notations and assumptions, section 4 formulate the mathematical model, section 5 presents numerical example and section 6 conclude the paper.

2. Literature review

The co-ordination between vendor and buyer for improving the performance of inventory control has received a great deal of attention. Goyal (1976) is among the first who analyzed an integrated inventory model for a single buyer system. Banerjee (1986) modified Goyal's (1976) presented a JELS where a vendor produces for a buyer to order on a lot for lot basis. Pan and Yang (2002) generalized Goyal's (1988) model by considering lead time as a decision variable and obtained a lower joint total expected cost and shorter lead time. In practices, the lead time and ordering cost reduction may be closely related (see Silver and

Peterson (1985), Ouyang et al. (2005), Chen et al. (2001)). Yang and Pan (2004) considered variable lead time and quantity improvement investment with normal distributional demand in the model proposed in Pan and Yang (2002). Ouyang et al. (2004) extend Pan and Yang (2002) developed integrated inventory model under lead time is stochastic.

Liao and Shyu (1991) presented a probabilistic model in which the order quantity was pre-determined and lead time was a unique decision variable. Li et al. (2012) investigated on a supply chain consisting of a vendor and a buyer controllable lead time (see also Chang et al. (2006)). Jha and Shanker (2013) presented an integrated production inventory model where a vendor produces an item and supplies it to set of buyers. The buyer level demand is normally distributed and lead time of each buyer can be added at crash cost. Yi and Shanker (2013) also used controllable lead time in a buyer –vendor system. Hoque (2008) describes synchronization in the single-manufacturer multi buyer integrated inventory supply chain. Vijayashree and Uthayakumar (2014) have presented a two stage supply chain model with selling price dependent demand and investment for quality improvement. Vijayashree and Uthayakumar (2013) have discussed vendor–buyer integrated inventory model with quality improvement and negative exponential lead time crashing cost. Vijayashree and Uthayakumar (2015)have developed two echelon supply chain with controllable lead time. Vijayashree and Uthayakumar (2014) developed an integrated inventory model with controllable lead time and setup cost reduction for defective and non-defective items. Vijayashree and Uthayakumar (2016) have considered an optimizing integrated inventory model with investment for quality improvement and Setup cost reduction. Hemapriya and Uthayakumar (2016) have developed ordering cost dependent lead time in integrated inventory model. Vijayashree and Uthayakumar (2017) have developed a single-vendor single-buyer integrated inventory model with ordering cost reduction dependent on lead time.
The above mentioned paper concerned on developing integrated inventory model of vendor and buyers system.

3. Notations and Assumptions

To develop the proposed model, the notations and assumptions are similar to Pan and Yang (2002). This paper considers a situation where single-vendor multiple-buyer under ordering cost reduction on lead time based on the work of Vijayashree and Uthayakumar (2017) under the following notations and assumptions.

3.1 Notations

The notations are divided into two subsection variables and parameters are used to develop the model.

Variables

m Number of lots in which the product is delivered from the vendor to the buyer in one production cycle, a positive integer.

For the ith buyer ($i = 1,2,3 \dots \dots . N$)

Q Shipment lot size in each delivery to meet the demand of all the buyers, $Q = \sum_{i=1}^{N} Q_i$

L_i Length of lead time for the buyer i

A_i Ordering cost per order for buyer i, $0 \leq A \leq A_0$

Parameters

D_i Average demand per unit time

N Number of buyers

P Production rate, $P > D (D = \sum_{i=1}^{N} D_i)$

S Vendor's setup cost per setup

c_v Unit production cost paid by the vendor($c_v < c_{b_i}, \forall i$)

c_{b_i} Unit purchase cost paid by the buyer i

r Annual inventory holding cost per dollar invested in stocks

R_i Reorder point of the buyer i

k_i Safety factor for buyer i

A_0 Original ordering cost (before any investment is made)

ITC Integrated total cost for the single vendor and the multiple buyer

3.2 Assumptions

To develop the model, we adopt the following assumptions

1. The system consist of multiple buyers who are supplied with a single-item by a single vendor.
2. The buyer i orders a lot of size Q_i and the vendor manufactures mQ with a finite production rate $P(P > D)$ in one setup but ships in quantity Q over m times to meet the demands of all the buyers such that $Q_i = D_i Q/D$
3. The vendor incurs a setup cost S for each production run and the buyer incurs an ordering cost A_i for each order of quantity Q_i.
4. The demand Xduring lead time L_i follows a normal distribution with mean $\mu_i L_i$ and standard deviation $\sigma_i\sqrt{L_i}$.
5. The inventory is continuously reviewed. The buyer places the order when the one hand inventory reaches the reorder point R_i
6. The lead time L_i of buyer i has n_i mutually independent components. The lth component of lead time of buyer i has a minimum duration $a_{i,l}$, a normal duration $b_{i,l}$ and a crash cost per unit time $c_{i,l}$. For convenience, we arrange $c_{i,l}$ such that $c_{i,1} \le c_{i,2} \le \ldots\ldots \le c_{i,n}, \quad \forall\, i$
7. The lead time components of each buyer are crashed one at a time starting with the least crash cost $c_{i,l}\ \forall\, i$ component and so on.
8. Let $L_{i,0} = \sum_{j=1}^{n_i} b_{i,j}$, $\forall\, i$ denote the maximum duration of lead time for buyer i and $L_{i,l}$ be the length of lead time for buyer i with components $i = 1,2 \ldots l$, crashed to their minimum duration, then $L_{i,l} = \sum_{j=l+1}^{n_i} b_{i,j} + \sum_{j=1}^{l} a_{i,j}$, $l = 1,2 \ldots\ldots n_i, \forall\, i$. The lead time crashing cost $R_i(L_i)$ per cycle of the ith buyer for a given $L_i \epsilon [L_{i,l}, L_{i,l-1}]$, is given by $R_i(L_i) = c_{i,l}(L_{i,l-1} - L_i) + \sum_{j=1}^{l-1} c_{i,j}(b_{i,j} - a_{i,j}),\ \forall i.$
   In addition, the length of lead time is equal for all shipping cycles and the lead time crashing cost occur in each shipping cycle. Liao and Shyu (1991), Li et al. (2012), Vijayashree and Uthayakumar (2014,2016).
9. The reduction of lead time L_i accompanies a reduce of ordering cost A_i and A_i is firmly concave function of L_i, i.e) $A'(L_i) > 0$ and $A''(L_i) < 0$. (Ouyang et al. 2005; Chen et al. 2001).
10. If extra cost incurred by the vendor will be fully transferred to the buyer if shortened lead time is required (Pan and Yang 2002)

4.Mathematical formulation

The total cost per unit time for the ith buyer as

$TC\,(Q_i, L_i)$ = ordering cost + holding cost +lead time crashing cost

$$TC\,(Q_i, L_i) = \frac{A_i D_i}{Q_i} + rc_{b_i}\left(\frac{Q_i}{2}\right) + rc_{b_i} k_i \sigma_i \sqrt{L_i} + \frac{D_i}{Q_i} R_i(L_i) \qquad \ldots\ldots(1)$$

Order lot size Q_i of buyer i should be in proportion of their demand of shipment lot size Q, That is $Q_i = \frac{D_i Q}{D}$

Therefore substitution of Q_i in (1) gives,

$$TC\,(Q, L_i) = \frac{A_i D}{Q} + rc_{b_i}\left(\frac{Q}{2D} D_i + k_i \sigma_i \sqrt{L_i}\right) + \frac{D}{Q} R_i(L_i) \qquad \ldots\ldots(2)$$

The total cost per unit time for the vendor is,

$TC(Q, m)$ = Setup cost + Holding cost

$$TC(Q, m) = \frac{S}{m}\frac{D}{Q} + \frac{Q}{2} r \left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right) c_v \qquad \ldots\ldots(3)$$

Integrated total cost per unit for the vendor – buyers integrated system is the sum of the total cost of the buyer and the total cost of the vendor, which is given by

$$ITC(Q,L_i,m) = \frac{D}{Q}\left(\frac{s}{m} + \sum_{i=1}^{N}\left(A_i + R_i(L_i)\right)\right) + \frac{Q}{2}r\left(\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + \sum_{i=1}^{N}\left(\frac{c_{b_i}D_i}{D}\right)\right) + \sum_{i=1}^{N} rc_{b_i}k_i\sigma_i\sqrt{L_i} \quad \ldots\ldots(4)$$

Linear function case:

We assume that lead time and ordering cost dependently with following relation, (Chen et al. 2001; Chiu 1998; Ouyang et al. 2004)

$\frac{L_0 - L}{L_0} = \omega\left(\frac{A_0 - A}{A_0}\right)$ where $\omega > 0$ is a scaling parameter

From this, the ordering cost A can be written as a linear function of L

$i.e)\ A(L) = x + yL$

where $x = \left(1 - \frac{1}{\omega}\right)A_0$ and $y = \frac{A_0}{\omega L_0}$

For ith buyer, $A_i(L_i) = x + yL_i$

Substitute above case in (4), gives

$$ITC(Q,L_i,m) = \frac{D}{Q}\left(\frac{s}{m} + \sum_{i=1}^{N}\left((x + yL_i) + R_i(L_i)\right)\right) + \frac{Qr}{2}\left(\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)c_v + \sum_{i=1}^{N}\left(\frac{c_{b_i}D_i}{D}\right)\right) + \sum_{i=1}^{N} rc_{b_i}k_i\,\sigma_i\sqrt{L_i},$$

for $L_i \epsilon [L_{i,l}\ , L_{i,l-1}]\ \ \forall\, i$

First order partial derivatives of $ITC(Q,L_i,m)$ with respect to Q, L_i, m

$$\frac{\partial ITC(Q,L_i,m)}{\partial Q} = 0$$

$$Q = \sqrt{\frac{2D\left(\frac{s}{m} + \sum_{i=1}^{N}\left((x+yL_i) + R_i(L_i)\right)\right)}{r\left(\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)C_v + \sum_{i=1}^{N}\frac{c_{b_i}D_i}{D}\right)}} \qquad L_i \epsilon [L_{i,l}\ , L_{i,l-1}]\ \ \forall\, i$$

Similarly

$$\frac{\partial^2 ITC(Q,L_i,m)}{\partial Q^2} = \frac{2D}{Q^3}\left(\frac{S}{m} + \sum_{i=1}^{N}\left((x + yL_i) + R_i(L_i)\right)\right) > 0$$

Therefore $ITC(Q,L_i,m)$ is convex in Q, for a fixed m and $L_i \epsilon [L_{i,l}\ , L_{i,l-1}]\ \forall i$

$$\frac{\partial ITC(Q,L_i,m)}{\partial L_i} = \frac{D}{Q}y - \frac{D}{Q}c_i + \frac{1}{2}rc_{b_i}k_i\sigma_i\sqrt{L_i}$$

Similarly

$$\frac{\partial^2 ITC(Q,L_i,m)}{\partial L_i^2} = -\frac{1}{4}rc_{b_i}k_i\sigma_i L_i^{-\frac{3}{2}} < 0$$

Hence for a fixed (Q,m), the minimum total integrated cost per unit time will occurs at the end points of the interval $L_i \epsilon [L_{i,l}\ , L_{i,l-1}]\ \ \forall\, i$

$$\frac{\partial ITC(Q,L_i,m)}{\partial m} = -\frac{D}{Q}\frac{S}{m^2} + \frac{Q}{2}r\left(1 - \frac{D}{P}\right)c_v$$

Similarly

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$$\frac{\partial^2 ITC(Q,L_i,m)}{\partial m^2} = \frac{2DS}{Qm^3} > 0$$

Therefore, $\partial ITC(Q, L_i, m)$ is convex in m, for fixed Q and $L_i \epsilon [L_{i,l}\ , L_{i,l-1}]\quad \forall\, i$

5. Numerical example

To illustrate the above solution procedure, let us consider a following data used in Pan and Yang (2002)
P = 3200 units/year, $c_v = 20$/units, r = 0.2, A_0 = \$25/order, S = \$400/setup, $\omega = 5.00$. The data of the buyers are given in Table 1 and the lead time of every buyer has three components with the data shown in table 2.

Table 1: Data for the buyers

Buyer i	D_i (units per year)	c_{b_i} (\$ per order)	σ_i (units per week)	k_i
1	1000	25	7	2.33
2	5000	20	7	2.33
3	800	22	7	2.33

Table 2: The buyers' lead time data

Buyer i	Lead time component l	Normal duration $b_{i,l}$ (days)	Minimum duration $a_{i,l}$(days)	Unit crashing cost $c_{i,l}$ (\$ per day)
1	1	20	6	0.1
	2	20	6	1.2
	3	16	9	5.0
2	1	20	6	0.5
	2	16	9	1.3
	3	13	6	5.1
3	1	25	11	0.4
	2	20	6	2.5
	3	18	11	5.0

Table 3: Results of the solution procedure for the illustrative example (lead time in weeks).

Case:1 $m = 3, \omega = 5$						
	$\boldsymbol{l}$	$\boldsymbol{L_{i,l}}$	$\boldsymbol{A_i(L_i)}$	$\boldsymbol{Q}$	$\boldsymbol{ITC}$	$\boldsymbol{R_i(L_i)}$
$\boldsymbol{i = 1, L_2 = 7, L_3 = 9}$	0	8	25	873.8	3918	0
	1	6	23.7	874.10	3886.87	1.4
	2	4	22.5	906.2	3973.66	18.2
	3	3	21.8	973.12	4195	53.2
$\boldsymbol{i = 2, L_1 = 6, L_3 = 9}$	0	7	24.4	874	3886.87	0
	1	5	23.1	885.97	3898.76	7
	2	4	22.5	903.3	3946.96	16.1
	3	3	21.8	971.97	4177.3	51.8
$\boldsymbol{i = 3, L_1 = 6, L_2 = 5}$	0	9	25.6	885.9	3898.7	0
	1	7	24.4	895	3899.6	5.6

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	2	5	23.1	961.5	4112	40.6
	3	4	22.5	1025	4333.6	75.6
Case:2 $m = 4, \omega = 5$						
	$\boldsymbol{l}$	$\boldsymbol{L_{i,l}}$	$\boldsymbol{A_i(L_i)}$	$\boldsymbol{Q}$	$\boldsymbol{ITC}$	$\boldsymbol{R_i(L_i)}$
$\boldsymbol{i = 1, L_2 = 7, L_3 = 9}$	0	8	25	1408	2365	0
	1	6	23.7	1408.7	2333.8	1.4
	2	4	22.5	1470	2375.8	18.2
	3	3	21.8	1596.8	2503.85	53.2
$\boldsymbol{i = 2, L_1 = 6, L_3 = 9}$	0	7	24.4	1408.7	2333.9	0
	1	5	23.1	1431.4	2329.5	7
	2	4	22.5	1464.7	2353.3	16.1
	3	3	21.8	1594	2484.9	51.8
$\boldsymbol{i = 3, L_1 = 6, L_2 = 5}$	0	9	25.6	1431	2329.4	0
	1	7	24.4	1448.7	2318.1	5.6
	2	5	23.1	1575	2437	40.6
	3	4	22.5	1694	2563.2	75.6

Case:1

For $m = 3$, the optimal lead time of the buyers are $L_1 = 6$, $L_2 = 7$, $L_3 = 9$ weeks and the corresponding optimal shipment lot size 874 units and $ITC^* = 3886.87$

Case:2

For $m = 4$, the optimal lead time of the buyers are $L_1 = 6$, $L_2 = 5$, $L_3 = 7$ weeks and the corresponding optimal shipment lot size 1448 units and $ITC^* = 2318.1$

6. Conclusion

In this paper deals with single-vendor multiple-buyer integrated inventory model based on equal shipment cycle time, ordering cost reduction and lead time reduction. In this case of multiple buyers, a vendor may supply a single item to more buyers to fulfill their requirements. It has been proven that integration is more beneficial to reduces integrated total cost of the vendor-buyers system. The benefits of a properly managed supply include reduced cost, faster product delivery, greater efficiency and lower costs for both the buyers and vendor.

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