Effective Computations and Foundations of Science

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Abstract — The problem of effective computation is discussed. Short historical analysis of this problem and basic ways of its development in culture and science, including computer science, are represented. Place the problem of the creation effective calculation in modern science is discussed. Necessity of creation "computation metascience" as system with variable hierarchy (open system) is observed. Polymetric analysis as example of this metascience is represented and discussed.

Keywords *- Plato numbers; Turing machine; metamathematics; table of God Thot; Gedel numbers; open systems; thermodynamics of isolated points; informative calculations; polymetric analysis; hybrid theory of systems.*

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I. Introduction

The problem of creation effective computation systems has long history. This problem is connected with development of science, culture and civilizations [1 – 5].

For the research this problem we must analyze basic ways of the development this problem and its role in modern science and culture.

This problem has three aspects: methodical, mathematical and generally scientifically. Therefore we research this problem as complex problem of modern science.

Differentiation of science is necessary process of its development but this development must be optimal and controlling. Therefore we must observe and represent this problem with system point of view.

The mathematics is Greek word "precision knowledge" [5]. But science is ordered knowledge. According B. Russel "Mathematics is doctrine, which is precisely represented and substantiated" [6]. This definition included two concepts, which is developed beginning from antiquity, logistics (in antiquity sense – art of computation) and logic (formal logic). Later W. Leubniz was founded the basis of mathematical logic [7]. Mathematical logic was developed in XIX – XX centuries by Boole, Morgan, Frege, Peano, Russel, Whihead and other scientists [8]. But only B. Russel and A. N. Whitehead created the foundation of mathematics on the basis of mathematical logic [8, 9]. But this way didn't allow receiving consistent theory of foundation of mathematics. D. Gilbert formal (axiomatic) concept of foundations of mathematics [10] isn't resolve of this problem. L. Brauer intuitionistic (inductive) doctrine can't be foundation of mathematics too [11].

But mathematics is move to science and roughly speaking it is the theoretical science. We must represent S. Banach phrase: "The good mathematicians see analogies between theorems and theories; the very best see analogies between analogies." [12]

Science of XX century cybernetics is result of synthesis of many science: mathematics, physics, linguistics, biology and other. The basic problem of this science may be represented with help S. Beer phrase [13]: "Apparently, the complexity becomes the problem of the century, just as the ability to process natural materials has been a problem of life and death for our forefathers. Our tool must be computers, and their efficiency should be provided by science, able to handle large and complex systems of probabilistic nature. This science may be cybernetics – the science of management processes and communication. The basic thesis of cybernetics can be set forth as follows: there are natural laws behavior of the large multibonds systems of any character submits that - biological, technical, special and economic." But nature of some problems may be nonprobalistic too. But this problem must be expanded on problem of formalization of all science and knowledge, including mathematics. Therefore this problem was formulated as problem of simplicity – complexity [1, 14 – 18]. It is corresponded to the basis of Descartian concept may be represented as "ordering his thoughts to analyse the complex by disserting it into its simpler parts" [1, 12, 19].

The role of mathematics in modern science may be represented with help B. Russel phrase "Mathematics, rightly viewed, possesses not only truth but supreme beauty…" [6]. But according Yu. Mitropolskiy "Mathematics is empress and servant of science" [20]. It is basic rule of correlation mathematics and other science.

II. Computation and Science

The art of computation (calculation) had large role in all ancient civilizations: esoteric type (ancient Egyptian table of God Thot) and open type (Sumerian). These two systems were synthesized by Pythagor [1]. The basic motto of his school is Pythagorean phrase "Numbers are rued of the World" [1].

In the time of antiquity all possible knowledge were represented as three types of numbers according to Plato. First type, mathematical numbers, is the pure mathematics in modern sense of this word; second type, sensitive numbers, is the applied mathematics; and third type, ideal numbers, is represented other chapters of knowledge and culture. In this time was created Euclid's classification the basic chapters of mathematics and Aristotle's classification of science [1].

Computation science was developed in astronomy (Sumerian civilization) in building (all ancient civilizations) and irrigation (Egypt and other "wilderness" civilizations).

Development of modern science may be represented as realization of R. Backon – Descartes thesis "Everything science is so science how much it has mathematics" [1].

But each science must be optimal system. This problem must be beginning from the Newtonian four rules of conclusions in the physics [1, 21]:

Rule 1. Do not require natural reasons than those that are true and sufficient to explain the phenomena.

Rule 2. Therefore, as far as possible, the same reason we should attribute displays the same kind of nature.

Rule 3. Such properties of bodies which can not be either amplified or weaken and are all bodies over which you can do the test, must properties considered for all bodies in general case.

Rule 4. In experimental philosophy, propositions derived from phenomena through a common induction must be considered for accurate or approximately correct, despite the possibility of opposing their hypothesis until there are phenomena that are more précised or are found to be invalid.

Determination of mathematics with numerical (measured) point of view was formulated by Ch. Volf [1]: “Mathematics is the science to measure everything that can be measured. Of course it is described as the science of numbers, the science of value, ie the things that can increase or decrease. Since all finite things can be measured in all that they have a finite, that is what they are, then nothing is, what can not be applied math, and because you can not have any more precise knowledge than when the properties of things can be measured, the math leads us to the most perfect knowledge of all possible things in the world”.

This same concept of measured value was formulated by "king" of mathematics of XVIII-th century, L.Euler [1]: “First value called everything that can increase or decrease, or something that you can add anything or subtract anything which can be ... There are many different kinds of values that can not be account, and from them come the various branches of mathematics, each of which has to do with their native values. However, it is impossible to determine whether a measured value except as known to take as another value of the same type and specify the ratio in which it is to her.”

Search of ways of foundations of mathematics has long history. It is the Leubnizian attempt the creation the universal calculus on basis of formal logic. This concept is basis for the creation of mathematical logics.

But attempts to create foundations of mathematics on basis of closed systems (mathematical Logic, arithmetic, geometry or other finite system of axioms) give negative results.

Therefore we must create theory of foundations of mathematics, which is based on nature of mathematics: analysis, synthesis and formalization of all possible information and knowledge. In more narrow sense, mathematics is the system of formalized ordering, measurement and estimation of information.

In determining whether the measurement values of any kind so we come to that first established some known value of the same kind, called the measure or unit and depends solely on our choice. Then determined in what respect the value is corresponding to the extent that it is always expressed in numbers. Thus measurement is no more than an attitude, which is one size to another, taken as a unit. Mathematics in general case is nothing but the science of the quantities involved finding ways to measure past.

But in XX century the three concepts of foundation of mathematics were observed. Logical concept (B. Russel and A. N. Whitehead) is based on logical formalization of knowledge; formal concept is based on axiomatic aspects of formalization; and third, intuitionist conceppt, is based on constructive (analytic) aspects of mathematics [1, 8, 16, 17]. First two concepts are based on closed theories and third concept is based only in analysis and therefore it isn’t optimal. One of founder of logic concept A. N. Whitehead refused from his concept and express an opinion about “organism” concept of mathematics [22].

One of the founders of modern mathematical analysis (integral-differential calculus) W.Leubniz working on the problem of creating a universal calculus, came to the conclusion to synthesis of logic and mathematics [1, 7]. In XVIII-th century this program was developed further by E. B. de Condillac. However, only in the nineteenth century Morgan, Boole, Frege and Peano [1, 8] created mathematical logic. A.N. Whitehead and B. Russel on the basis of logic created the foundations of mathematics [1, 8]. This was the beginning of a structural approach to the fundamentals of mathematics. "Credo" logical concept is as follows [1, 8]: mathematics – is part of the logic [8]. However, the program encountered paradoxes inter-hierarchy type (Russell's paradox) and A.Puankare showed that axiom reduction relates not only to mathematics, what was the opinion to B. Russel [1, 8].

In a further development of modern mathematics are strongly affected by the following results of logistic direction [1, 8]:

1. Logical concept had a crucial role in the development and justification of mathematical logic.
2. This concept helped establish the fact that the axiomatic method can accurately apply only within mathematical logic.
3. Principia Mathematica was the first attempt to create a unified mathematics.

Formal (formalist) approach in the foundations of mathematics (school of proof theory) was developed by D.Hilbert P.Bernayse [1, 8, 10], but it also took paradoxes inter-hierachy type. This program is the first place any mathematical theory that has some practical value, such as arithmetic. Then there is a formal system that meets the specific theory and expresses it. At the last stage of the chain ends metatheory, that deals with the formal system. The first who showed the futility of this trend for the foundations of mathematics was K. Gedel [8, 23]. His incompleteness theorem showed the impossibility of describing a mathematical theory of permanent measure (mathematical logic and Kantor set theory) all the basic laws of mathematics. Later American mathematician P. Kohan showed independence axiom of selection (constructive axiom) from other axioms of mathematics [8].

The structural (intuitive) concept in the fundamentals of mathematics (in the Brauer-Heyting) [8, 11] do not give universal theory too. The main thesis of Heiting constructivism is the following [11]: "The goal, which poses a mathematician, is next. He wants to do mathematics as a natural function of the intellect as free live active thinking. Mathematics for he is a product of the human mind. The language of both conventional and formal, it applies only to messages that interest to others or himself to his mathematical analysis of opinion... At it is a core mathematical subject... due to human thinking. They exist only to the degree to which they can be identified thinking, and have only the properties, as they can be known of thinking." Not all agree with intuitionist Heyting [8, 11]. Overall intuitionism based on the following principles: the subject of mathematics can be only that consciousness can be built on the basis of certain well-defined basic intuitive representations. This implies that the number of math in constructive mathematics practically infinite and nothing to "to take up" to infinity this could work. That is a constructive approach in this sense also applies to the structural approach in the foundations of mathematics.

The theory of foundation of mathematics must be based on its nature: analysis, synthesis and formalization of whatever chapter of knowledge. All three concept of foundation of mathematics aren't corresponded to its nature in toto [16, 17].

Therefore we must again to address to Pythagor, Newton, Volf and Euler. This problem now is one of central in modern cybernetics and computer science [1, 24, 25].

But it connected with problem of complexity, which is one of central problem in modern mathematics and cybernetics [1, 24 – 27]. This problem is caused in synthetically sciences. Roughly speaking it has two aspects: system (problem of century in cybernetics according S. Beer [1, 13]) and computational (problem of computational complexity [1, 26, 27]. Last problem is included in basic problems of modern mathematics (Smale problems) [1, 28].

This direction was and is basic for the development of science in XVII – XXI centuries [1].

But some nuances appear in the process of creation and development of computer science.

Basic peculiarities of computation in computer science have few aspects: complexity problems of computer mathematics and specific computation for finite and infinite machines [2].

Now we analyze second concept, which is created by M. L. Minsky [4]. He introduced two notions effective procedure and effective computation.

Effective procedure (EP) or algorithm is computation of value of some quantity or search of resolution of some mathematical problem [4]. Other definition is "Effective procedure is set of rules, which are regulated our behavior precisely" [4]. This definition isn't precision and may be changed for various system. Concrete EP may be characterized proper system.

So, effective procedures may be action and entropy principles in modern physics and theory of information.

Roughly speaking realization EP for computation may be called effective computation (EC). But this notion may be used for Turing machine, von Neumann automata or other computation systems [3, 4].

These two notions (EP and PC) are connected with optimality and reliability of proper system with point of view of its functionality.

But PE and PC haven't mathematical form because these notions are more rules of action as principles. Mathematical forms have proper concrete scientific lows.

One of basic lows of information, Shennon theorem [29], is the analogous to entropy principle in thermodynamics. But in information theory entropy increasing is decreasing the step of information order. Therefore L. Brillouin introduced the notion "negentropy" (negative entropy) as measure of information [30]. Thermodynamically entropy is increased of noise level in system and decrease the information quantity.

The information quantity may be represented in next form according by Shennon [31]

$$I(X) = -\sum_i p(x_i)\log_a p(x_i) = S_e. \qquad (1)$$

Where x_i – event, $p(x_i)$ – probability of event, S_e – entropy.

Further unification of entropy and action may be made with help de Broglie formula [32]:

$$\frac{S_e}{k_B} = \frac{S_a}{\hbar}, \qquad (2)$$

which was obtained from the analysis of thermodynamics point [1], a measure of disordered physical information (number of photons) equally structured information (where S_a – action; $\hbar$ – Planck constant, S_e – entropy, k_B – Boltzmann constant).

Through (2) the ratio of the increase of entropy for nonequilibrium processes (open systems) can be expanded at the action, that is, in other words, the physics of open systems can be built and action functional .

Roughly speaking the formula (2) is the mathematical form of next law: quantities of ordered and disorder information is equaled in each closed system.

We can introduce of dimensionless quantity S_g, which was called generalizing measure.

$$\frac{S_e}{k_B} = \frac{S_a}{\hbar} = S_g, \qquad (2a)$$

For this case basic optimal principles of physics and information theory may be represented in the next form

$$dS_g \geq 0. \qquad (3)$$

A sign equal (=) in (3)`is corresponded to case of closed systems, A sign greater-than (>)is corresponded to case of open systems,

In [15] was shown that Yu. Klimontovich "thermodynamically" theory of optimal systems may be expanded on ordering part of physics too:

$$dS_a \geq 0. \qquad (4)$$

In this case we have for the equals sign, roughly speaking, the action principle, to mark the event more – physics of open systems.

But formula (3) allows to transit to other nonphysical systems. Quntity S_g may be represented number of proper mathematical operations, including transformations.

Modern science is complex system. The evolution of each science is realization the transition from complex to simple system. The change of structure of science and appearance of new chapters of science are caused the change of notions, including axiomatic, which must explain new chapters of knowledge.

But modern science is the realization of the R. Bacon – Descartes concept "Science is so science, how many mathematics is in her" [1].

Modern realization of this concept is the development of computer science. Therefore complexity and unresolved problems of modern science are transited in computer science. But computer science has own unresolved problems and complexity. Development of modern science practically isn't possible without computers. Thus we must observe this problem together with other science.

Computational complexity theory [27, 28] is a branch of the theory of computation in theoretical computer science that focuses on classifying of computational problems according to their inherent difficulty, and relating those classes to each other. A computational problem is understood to be a task that is in principle amenable to being solved by a computer, which is equivalent to stating that the problem may be solved by mechanical application of mathematical steps, such as an algorithm.

Search of new concepts the creation of optimal system of knowledge is connected with development of computing [1]. Basic thesis of computing science according A. Ershov may be formulated as formalization of Canadian philosopher L. Hall "Everything, that goes from head, is reasonable" [14]. Therefore problem of united rational and irrational (or verbal and nonverbal) knowledge, including science, art, religious and other, is one of central problem of modern computing. For resolution this problem we must included in one system philosophy and science. So, mathematical logic is differed from formal logic one rule (Leubnizian rule of sufficient proof) [1].

III. FOUNDATIONS OF SCIENCE

The creation of proper science may be represented with help Fig.1 [1].

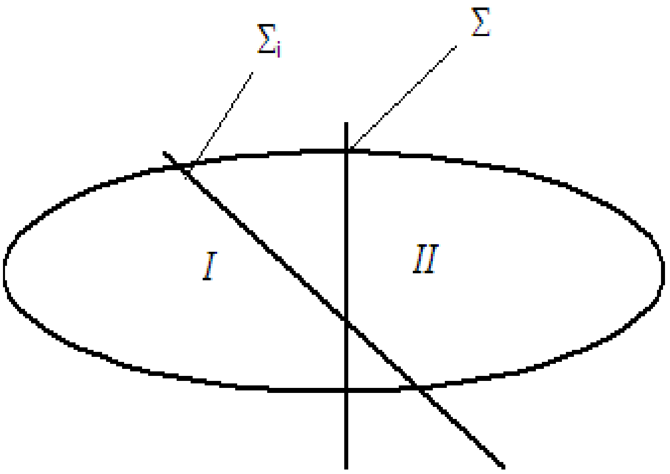

Figure 1. Rough schema of science creation [1].

Part I of Figure 1 is called Logic and characterizes the well-known order knowledge. Part II of Fig. 1 is called Chaos and characterizes the unknown or little-known disorder knowledge [1]. Section Σ (Fig. 1) is characterized relative border between Logic and Chaos. Section Σ_i (Fig. 1) is characterized concrete science. Roughly speaking selection of proper section Σ_i is one variant of famous Archimedes phrase: "Give me fulcrum and I reverse World" [1]. The way of search section Σ_i may be represented as M. Minsky effective procedure [2] and creation new theory or science may be represented as search new effective procedure. Schema of Figure 1 is very approximate approach, but it allows seeing basic peculiarities, which are necessary for creation new theory or science.

Modern science is basically hierarchical system. All science and knowledge have hierarchical levels and create polyphasic system [1].

The problem selection of proper section Σ_i was resolved by I. Newton (four rules of conclusions in physics). He created second deductive science – Newtonian mechanics [1] and synthesized in one system terrestrial and celestial mechanics.

But each science is characterized of set proper concepts, principles and laws. Therefore we must select concept and numbers and types of basic principles and laws for creation new metascience theory, which is corresponded of present state of modern science and may be used for its development.

The basic conditions of this metascience must be next:

1. It must be open theory or theory with variable hierarchy.
2. This theory must be having minimal number of principles.
3. We must create sign structure, which unite verbal and nonverbal knowledge (mathematical and other) in one system.
4. We must have system, which is expert system of existing system of knowledge and may be use for the creation new systems of knowledge.
5. Principle of continuity must be true for all science.

Large role in modern science has a problem of completeness the proper system or theory. In epistemological sense the notion is equivalence the concept of closing in theory of system.

The concept of completeness is basic in mathematical Logic and was development by K. Gödel.

His two incompleteness theorems may be represented in next forms[8, 33].

First Incompleteness Theorem: That if the arithmetic is consistent, then these exists an irreducible and irrefutable formula in it.

This theorem may be represented in more general form as "Any consistent formal system *F* within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of *F* which can neither be proved nor disproved in *F*."

The improvable statement G_F referred to by the theorem is often referred to as "the Gödel sentence" for the system *F*. The proof constructs a particular Gödel sentence for the system *F*, but there is infinitely many statements in the language of the system that share the same properties, such as the conjunction of the Gödel sentence and any logically valid sentence.

For each formal system *F* containing basic arithmetic, it is possible to canonically define a formula Cons(*F*) expressing the consistency of *F*. This formula expresses the property that "there does not exist a natural number coding a formal derivation within the system *F*whose conclusion is a syntactic contradiction." The syntactic contradiction is often taken to be "0=1", in which case Cons(*F*) states "there is no natural number that codes a derivation of '0=1' from the axioms of *F*."

Gödel's second incompleteness theorem shows that, under general assumptions, this canonical consistency statement Cons(*F*) will not be provable in *F*. The theorem first appeared as "Theorem XI" in Gödel's paper [33]. In the following statement, the term "formalized system" also includes an assumption that *F* is effectively axiomatized theory.

Second Incompleteness Theorem: States that if formal arithmetic is consistent the there is an irreducible formula that substantiates the contradiction of the arithmetic.

This theorem is stronger than the first incompleteness theorem because the statement constructed in the first incompleteness theorem does not directly express the consistency of the system. The proof of the second incompleteness theorem is obtained by formalizing the proof of the first incompleteness theorem within the system *F* itself.

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From system point of view these theorems may be represented as particularly formalization of rule 3 from Newtonians rules of conclusions in physics.

The large value in modern science has concepts of reduction and resolutions of various problems of modern science. Problem of resolution of proper problems may be represented as reduction or transformation this problem in form, which is necessary for using.

In mathematical physics many complex problems of nonlinear Hamiltonian dynamics were reduced to systems of proper linear equations by famous Japanese mathematician M. Sato with colleagues [1]. Therefore we are formulated general theorem of reduction as

General Sato theorem [1]. Any nonlinear system of nonlinear integral-differential equations, that represents correctly problem of mathematical physics, may be reduced to system of linear algebraic equations. Moreover groups of monodromy of these two systems are coincided.

Presently we'll go to classifications. We'll ask question that we must make, if we have little information even for the construction of system integral-differential equations, to say nothing of systems of linear algebraic equations? In modern theoretical and mathematical physics chapter of physical and chemical kinetics is least "regularized" on present day. Therefore generalizing Sato theorem must be expanded on classification too, another words on correlations between basic physical characteristics, which are caused proper physical phenomena. That is to say, that such old way of creation science as classification is correctly coordinated with problems of modern theoretical and mathematical physics.

IV. Polymetric Analysis as Metascience

Polymetric analysis (PA) was created as alternative optimal concept to logical, formal and constructive conceptions of modern mathematics and mor general theory of information [1, 34] and is corresponded the basic notions of the universal theory of open systems. This concept is based on the idea of triple minimum: mathematical, methodological and concrete scientific [1, 34].

However, one of the main tasks of polymetric analysis is the problem of simplicity-complexity that arises when creating or solving a particular problem or science.

In methodological sense, PA is the synthesis of Archimedes thesis: "Give me a fulcrum and I will move the world", and S.Beer idea about what complexity is a problem of century in cybernetics, in one system. And as cybernetics is a synthetic science, the problem should be transferred and for all of modern science. Basic elements of this theory and their bonds with other science are represented in Figure 2 [1, 34].

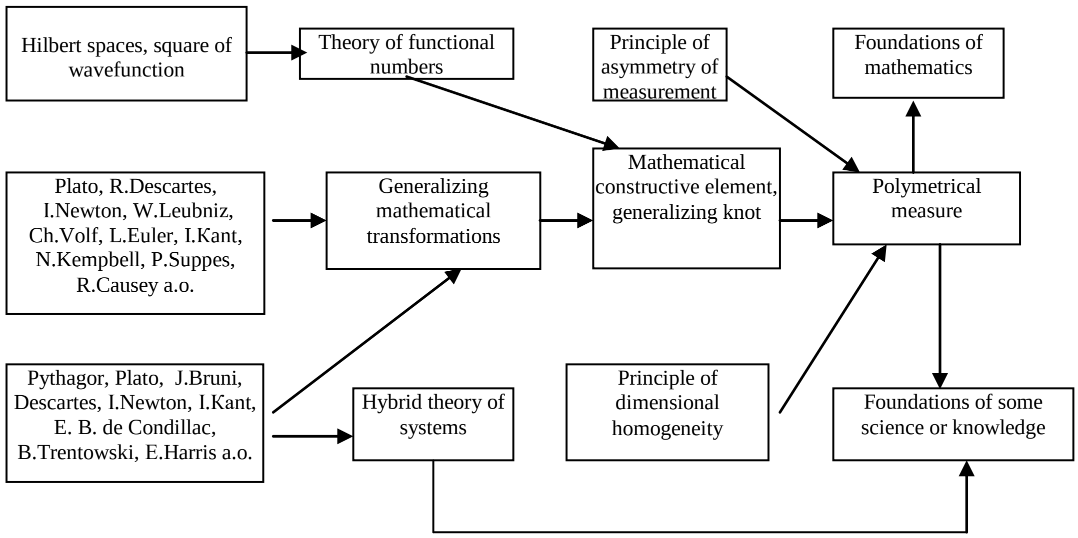

Figure 2. Schema of polymetric method and its place in modern science

The universal simple value is unit symbol, but this symbol must be connected with calculation. Therefore it must be number. For the compositions of these symbols (numbers) in one system we must use system control and operations (mathematical operations or transformations). After this procedure we received the proper measure, which is corresponding system of knowledge and science.

Roughly speaking the basic peculiarity of polymetric analysis is the realization of Plato concepts of three types of numbers with computational point of view [1].

The basic questions, which must be resolve polymetric analysis are:

1) creation united system of optimal formalization the knowledge;

2) creation natural concept of foundations of mathematics, which is based on nature of mathematics,

3) creation universal theory of open systems.

Therefore the basic axiomatic of the polymetric analysis was selected in the next form [1, 34]. This form is corresponded to schema of Figure 2.

Definition 1. **Mathematical construction** is called set all possible elements, operations and transformations for resolution corresponding problem. The basic functional elements of this construction are called constructive elements.

Definition 2. The mathematical constructive elements $N_{x_{ij}}$ are called **the functional parameters**

$$N_{x_{ij}} = x_i \cdot \bar{x}_j, \quad (5$$

where $x_i, \bar{x}_j$ – the straight and opposite parameters, respectively; $\cdot$ – respective mathematical operation.

Definition 3. The mathematical constructive elements $N_{\varphi_{ij}}$ are called the **functional numbers**

$$N_{\varphi_{ij}} = \varphi_i \circ \bar{\varphi}_j. \quad (6)$$

Where $\varphi_i\left(x_1,\ldots x_n, \bar{x}_1,\ldots \bar{x}_m,\ldots N_{x_{ij}},\ldots\right)$, $\bar{\varphi}_j\left(x_1,\ldots x_n, \bar{x}_1,\ldots \bar{x}_m,\ldots N_{x_{ij}},\ldots\right)$ are the straight and opposite functions, respectively; $\circ$ – respective mathematical operation.

Remark 1. Functions $\varphi_i, \bar{\varphi}_j$ may be have different nature: mathematical, linguistic and other.

Definition 4. The mathematical constructive elements $N^d_{x_{ij}}$ are called the **diagonal functional parameters**

$$N^d_{x_{ij}} = \delta_{ij} N_{x_{ij}}. \quad (7)$$

Where δ_{ij} is Cronecker symbol.

Definition 5. The mathematical constructive elements $N^d_{\varphi_{ij}}$ are called the **diagonal functional numbers**

$$N^d_{\phi_{ij}} = \delta_{ij} N_{\varphi_{ij}}. \quad (8)$$

Example 1. If $x_i = x^i$, $\bar{x}_j = x_j^{-1}$ and max{*i*} = max{*j*} = =*m*, then $\left\{N^d_{\varphi_{ij}}\right\}$ is diagonal single matrix.

Another example may be the orthogonal eigenfunctions of the Hermitian operator.

Remark 2. This example illustrate why quantities (5) – (8) are called the parameters and numbers. Practically it is the simple formalization the measurable procedure in Fig.1. The straight functions correspond the "straight" observation and measurement and opposite functions correspond the "opposite" observation and measurement. This procedure is included in quantum mechanics the Hilbert's spaces and Hermitian operators.

The theory of generalizing mathematical transformations is created for "work" on functional numbers [1, 34].

Definition 6. **Qualitative transformations** on functional numbers $N_{\varphi_{ij}}$ (straight A_i and opposite $\bar{A}_j$) are called the next transformations. The straight qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on *i* units for straight parameters, and the opposite qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on *j* units for opposite parameters.

Definition 7. **Quantitative (calculative) transformations** on functional numbers $N_{\varphi_{ij}}$ (straight O_k and opposite $\bar{O}_p$) are called the next transformations. The straight calculative transformations are reduced $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on *k* units its measure. The opposite quantitative transformations are increased $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on *l* units its measure, i.e.

$$O_k O_p N_{\varphi_{ij}} = N_{\varphi_{ij}} - k \oplus p. \quad (9)$$

Definition 8. **Left and right transformations** are called transformations which act on left or right part of functional number respectively.

Definition 9. The maximal possible number corresponding transformations is called **the rang of this transformation**

$$rang(A_i \bar{A}_j N_{\phi_{ij}}) = \max(i, j), \ (10)$$

$$rang(O_k \bar{O}_p N_{\phi_{ij}}) = \max(k, p). \ (11)$$

Remark 3. The indexes *i, j, k, p* are called **the steps of the corresponding transformations.**

For this case we have finite number of generalizing transformations.

The basic types of generalizing mathematical transformations are represented in Table 1 [1, 34].

Table 1. *The basic types of generalizing mathematical transformations.*

№	Tronsfor-mation	1	2	3	4	5	6	7	8	9	10	11	12	Representation		
														S	O	M
1	full straight	+	+	+	+	+	+	+	+	+	+	+	+	+	–	–
2	full opposite	+	+	+	+	+	+	+	+	+	+	+	+	-	+	–
3	full mixed	+	+	+	+	+	+	+	+	+	+	+	+	-	–	+
4	left full straight	+	-	+	–	+	-	+	–	+	–	+	–	–	+	–
5	right full opposite	–	+	–	+	–	+	–	+	–	+	–	+	+	–	–
6	left straight	+	–	+	–	+	–	+	–	+	–	+	–	+	–	–
7	right opposite	–	+	–	+	–	+	–	+	–	+	–	+	–	+	–
8	mixed full straight	+	-	+	–	+	–	+	–	+	–	+	–	–	–	+
9	mixed full opposite	–	+	–	+	–	+	–	+	–	+	–	+	–	–	+
10	left half-straight	+	–	–	+	–	+	+	–	–	+	–	+	–	+	–
11	mixed half-straight	+	–	–	+	–	+	+	–	–	–	–	–	–	–	+
12	right semi-opposite	–	+	+	–	+	–	–	+	+	–	+	–	+	–	–
13	mixed semi-opposite	–	+	+	-	+	–	–	+	+	–	+	–	–	–	+
14	mixed straight	+	–	+	–	+	–	+	+	+	+	+	+	–	–	+
15	mixed opposite	–	+	–	+	–	+	+	+	+	+	+	+	–	–	+

Remarks to Table 1. S – straight; O – opposite; M – мixed; 1 – A_i; 2 – $\bar{A}_j$; 3 – A^r; 4 – $\bar{A}^r$; 5 – A^l; 6 – $\bar{A}^l$; 7 – O_k; 8 – $\bar{O}_p$; 9 – O^r; 10 – $\bar{O}^r$; 11 – O^l; 12 – $\bar{O}^l$.

In Table 1 sigh + (plus) is defined that action of corresponding transformation on $N_{\varphi_{ij}}$ is fully or particularly; sign (minus) – is absented.

Basic element of PA is the generalizing mathematical elements or its various presentations – informative knots [1, 34]. Generalizing mathematical element is the composition of functional numbers (generalizing quadratic forms, including complex numbers and functions) and generalizing mathematical transformations, which are acted on these functional numbers in whole or its elements [1]. Roughly speaking these elements are elements of functional matrixes.

This element ${}^{stqo}_{nmab}M_{ijkp}$ may be represented in next form

$$ {}^{stqo}_{nmab}M_{ijkp} = A_i \bar{A}_j O_k \bar{O}_p A_s^r \bar{A}_t^r O_q^r \bar{O}_o^r A_n^l \bar{A}_m^l O_a^l \bar{O}_b^l N_{\varphi_{ij}} . \quad (12) $$

Where $N_{\varphi_{ij}}$ – functional number; $O_k, O_q^r, O_a^l, \bar{O}_p, \bar{O}_o^r, \bar{O}_b^l$; $A_i, A_s^r, A_n^l, \bar{A}_j, \bar{A}_t^r, \bar{A}_m^l$ are quantitative and qualitative transformations, straight and opposite (inverse, with tilde), (r) – right and (l) – left.

Polyfunctional matrix, which is constructed on elements (12) is called **informative lattice.** For this case generalizing mathematical element was called knot of informative lattice [1, 18, 34]. Informative lattice is basic set of theory of informative calculations. This theory was constructed analogously to the analytical mechanics [1].

Basic elements of this theory are [1, 18, 34]:

1. **Informative computability** C is number of possible mathematical operations, which are required for the resolution of proper problem.
2. **Technical informative computability** $C_t = C\sum t_i$, where t_i – realization time of proper computation.
3. **Generalizing technical informative computability** $C_{t0} = k_{ac}C_t$, where k_{ac} – a coefficient of algorithmic complexity [1, 18, 34].

Basic principle of this theory is **the principle of optimal informative calculations** [1, 18, 34]: any algebraic, including constructive, informative problem has optimal resolution for minimum informative computability C, technical informative computability C_t or generalizing technical informative computability C_{to}.

The principle of optimal informative calculations is analogous to action and entropy (second law of thermodynamics) principles in physics. This fact is caused of formula (2). where S_g may be represented as dimensionalless system function of information. Roughly speaking it may be informative calculations too.

The principle of optimal informative calculation is more general than **negentropic principle the theory of the information** and **Shennon theorem** [1, 18, 34]. This principle is law of the open systems or systems with variable hierarchy. The negenthropic principle and Shennon theorem are the principles of systems with constant hierarchy.

Idea of this principle of optimal informative calculation may be explained on the basis de Broglie formula (2) (equivalence of quantity of ordered and disorder information) [1]. Therefore we can go from dimensional quantities (action and entropy) to undimensional quantity – number of proper quanta of information or after generalization to number of mathematical operations. Thus, theory of informative calculations may be represented as numerical generalization of classical theory of information and analytical mechanics according to computational point of view [1].

For classification the computations on informative lattices hybrid theory of systems was created [1, 18, 34]. This theory allow to analyze proper system with point of view of its complexity,

The basic principles of hybrid theory of systems are next: 1) **the criterion of reciprocity;** 2) **the criterion of simplicity**.

The criterion of reciprocity is the principle of the creation the corresponding mathematical constructive system (informative lattice). The criterion of simplicity is the principle the optimization of this creation.

The basic axiomatic of hybrid theory of systems is represented below.

Definition 10. The set of functional numbers and generalizing transformations together with principles reciprocity and simplicity (informative lattice) is called **the hybrid theory of systems** (in more narrow sense the criterion of the reciprocity and principle of optimal informative calculations).

Criterion of the reciprocity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;
2) the equilibrium;
3) the equality of the number epistemological equivalent known and unknown notions.

Criterion of the simplicity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;
2) the equilibrium;
3) the principle of the optimal calculative transformations.

Criterion of reciprocity is the principle of creation of proper informative lattice. Basic elements of principle reciprocity are various nuances of completeness. Criterion of the simplicity is the principle of the optimality of this creation.

For more full formalization the all famous regions of knowledge and science the **parameter of connectedness** $\boldsymbol{\sigma_t}$ was introduced. This parameter is meant the number of different bounds the one element of mathematical construction with other elements of this construction. For example, in classic mathematics σ_t = 1, in linguistics and semiotics $\sigma_t > 1$. The parameter of connectedness is the basic element for synthesis in one system of formalization the all famous regions of knowledge and science. It is one of the basic elements for creation the theory of functional logical automata too.

At help the criteria of reciprocity and simplicity and parameter of connectedness the basic famous parts of knowledge and science may be represent as next 10 types of hybrid systems [1, 18, 34]:

1. The system with conservation all positions the criteria of reciprocity and simplicity for all elements of mathematical construction ($N_{\varphi_{ij}}$ and transformations) is called the *simple system.*
2. The system with conservation the criterion of simplicity only for $N_{\varphi_{ij}}$ is called the *parametric simple system.*

Remark. Further in this classification reminder of criteria of reciprocity and simplicity is absented. It means that these criteria for next types of hybrid systems are true.

3. The system with conservation the criterion of simplicity only for general mathematical transformations is called *functional simple system.*

4. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t = 1$ is called the *semisimple system.*

5. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ and with $\sigma_t = 1$ is called the *parametric semisimple system.*

6. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t = 1$ is called the *functional semisimple system.*

7. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t \neq 1$ is called *complicated system.*

8. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ is called *parametric complicated system.*

9. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t \neq 1$ is called *functional complicated system*.

10. The system with nonconservation the criteriums of reciprocity and simplicity and with $\sigma_t \neq 1$ is called *absolute complicated system*.

With taking into account 15 basic types of generalized mathematical transformations we have 150 types of hybrid systems; practically 150 types of the formalization and modeling of knowledge and science.

Only first six types of hybrid systems may be considered as mathematical, last four types are not mathematically in classical sense. Therefore HTS may be describing all possible system of knowledge. Problem of verbal and nonverbal systems of knowledge is controlled with help of types the mathematical transformations and parameter connectedness [1,34].

V. Polymetric Analysis and Problems of Modern Science

Polymetric anaslysis may be used for the resolution many problems of modern science in whole and with using theories. These problems are included in its structure.

So, HTS may be used for the classification and creation old and new chapters of all science, including computing science.

HTS may be used for the represented of evolution of systems in two directions: 1) from simple system to complex system (example, from classic to quantum mechanics) and 2) conversely, from complex system to simple system (example, from formal logic to mathematical logic) [1].

Hybrid theory of systems is open theory. Parameters of openness are number of generalizing mathematical transformations and parameter of connectedness. Thereby we have finite number of types of systems, but number of systems may be infinite. Hybrid theory of systems allows considering verbal and nonverbal knowledge with one point of view [1, 34]. Therefore this theory may be represented as variant of resolution S. Beer centurial problem in cybernetics (problem of complexity) [1, 34].

HTS may be represented as application PA (HTS) to the problem of calculation [1, 34]. This theory was used for the problem of matrix computation and problem of arrays sorting [1, 18].

HTS may be connected with problem of computational complexity. This problem was appeared in modern cybernetics for resolution of problem the transition from infinite (analytical) to discrete representation of computing procedures [1,18]. In may be connected with 4 and 5 Smale problems [1, 28].

HTS may be used for the classification of knowledge and science with point of view of their complexity. These results may be represented as theorems [1].

Theorem 1. The classical mechanics is the simple system.

Proof. The classical mechanics is closed system therefore criteria of the reciprocity and simplicity are true. The action principle is the analogous the principle of optimal informative calculations. Parameter of connectedness is equal 1. But its definition of simple system and theorem is proved.

Theorem 2. The quantum mechanics is the semisimple system : 1) in Heisenberg's representation – parametric simple; 2) in Shrödinger's representation – functional simple; 3) in the representation of interaction – semisimple system.

Proof. The quantum mechanics is closed system as classical mechanics. But the criterion of the simplicity isn't true for operators (in Heisenberg's representation), for wave functions (in Shrödinger's representation) and for operators and wave functions (in representation of the interaction). Parameter of connectedness is equal 1 for all three representations. But it is the definitions of proper systems and theorem is proved.

Theorem 3. Logic is a simple system.

Proof. Logic is a closed system. Criteria for reciprocity and simplicity of the system implemented. Parameter connectivity is equaled one. Thus mathematical logic is a simple system.

Theorem 4. Linguistics is a complex system.

Proof. Linguistics is semi-closed or open system. Criteria reciprocity and simplicity may not come true. Parameter connectivity, typically, is greater than one. But it is the definition of a complex system.

These theorems practically are represented the system character of the theoretical (classic) and quantum mechanics in modern science.

Once again we return to the foundations of mathematics. Classical mathematics is characterized by parameter of connectedness that is equal to one. It means that quite complex and sophisticated mathematical system is not mathematical in the classical sense. But the foundations of mathematics we have a theory with a broader subject base as classical mathematics (including mathematical logic and set theory). This theory is in our view should include formalizing the procedure (functional numbers and criteria of reciprocity and simplicity), process analysis and synthesis (qualitative and quantitative transformation) and the problem of uniqueness (parameter of connectedness). This theory is also essential to have provisions that take into account its opening from the system point of view. In polymetric analysis meet this requirement parameter of connectedness and possible failure of certain provisions of criteria of reciprocity and simplicity. The theories of «structural lines» in the foundations of mathematics do not meet these requirements. This provision can be formulated as the following theorem.

Theorem 5. The theory of "structural lines" in the foundations of mathematics (logical, formal and intuitive) can not be extended to all mathematics.

Proof. The theory of «structural lines» is permanent measures, while mathematics because of its development has a variable structure, and each structural element has its own measure. But metamathematycal theory must to be theory with variable measure. We got a contradiction and thus proved the theorem.

If we consider polymetric conceptin terms of H. Kantor expression "The essence of mathematics lies in its freedom", this freedom is included in the variable measure.

Polymetric analysis may be represented as generalization of basic problem of cybernetics in Wiener sense "Cybernetics is the science of the Control and Communication in the Animal and the Machine" [35].

But PA is more general as cybernetics. It may be used as metascience and expert system for real systems and theory of formation of new scientific systems [1].

PA may be represented as "dynamical" expanding formalization of Errol E. Harris polyphasic concept of modern science [1, 36]. But Harris method is philosophical and "static", polymetric method is "dynamic". PA allows to select and change measure in inside of proper system and select and change the hierarchy of this system.

PA is universal system of synthesis of knowledge. But this synthesis is realized through measure (number). Each science or knowledge has own treasures and measure. Therefore problem of division science on philosophy and other sciences (see N. R. Campbell [37]) is very relative. The classification science and knowledge with help polymetric concept, according to simplicity-complexity of optimal formalization, is fuller and more corresponded of present state and development of science and knowledge.

Selection of quadratic forms as basic elements of PA is further development of direction of observation many scientists: Pythagor (Pythagorean theorem), Plato (three types of numbers), Descartes (using Pythagorean theorems for creation analytical geometry), B. Riemann (creation Euclidean space as generalizing of analytical geometry), D. Hilbert (Hilbertian spaces) etc [1].

PA may be used as theoretical foundations of computer science too. It describes this science in your standing and development more simple, optimal and sufficient as logical or constructive concepts [1], and may be used as expert system for existing sciences and instrument for creation of new sciences.

PA may be represented as more full formalization E. B. de Condillac "A treatise about systems" [38]. But in this case we have open systems, systems with variable measure and variable hierarchy. These both conditions are interconnected and therefore it represents the one universal system.

Polymetric analysis may be used for more full formalization of neuronets [1, 25, 39]. Basic concept of creation PA is concept of triple optimum (minimum): mathematical, methodological and concrete scientific [1, 25-27].

The polymetric analysis may be represented as universal theory of synthesis in Descartian sense. For resolution of this problem we must select basic notions and concepts, which are corresponded to optimal basic three directions of Figure 2.

Thus, we show that Polymertric Analysis is general theory of open systems and may be used for the resolutions various problems of system type for many sciences. It may be represented as metamathematics is more widely sense as "logical" Kleene metamathematics [40]. PA is as metaknowledge, metascience and metamathematics together in one system.

If see to development of science with system point of view we have to general classifications. First is Euclid's Elements as classification of basic chapters of mathematics [1]. Second is Aristotle's classification of science. Roughly speaking, modern mathematics is the expansion of Euclid's Elements, and modern science is expansion of Aristotle's classification.

But modern science and mathematics is more complex systems as in Euclides and Aristotle times. It is polyphasic system. But it must be open system. This metascience must include elements of integration (synthesis) and differentiation of knowledge and science. It must be theory with variable measure and hierarchy.

Therefore we must include the elements of simplicity-complexity these systems for its comparative analysis. S. Beer problem of century in cybernetics is one of central problem of modern knowledge and science, including mathematics (two Smale problems) too. This problem has two aspects: firs – system (global) and second – local scientifically.

The concept of PA is third way of classification of science. But it is "dynamical" concept, which is based on variable measure, problem of simplicity-complexity and two "open" parameters number and types of generalizing mathematical transformation and parameters of connectedness.

From mathematical point of view PA is expansion of mathematics on all possible systems of knowledge and science with point of view the simplicity-complexity Mathematics must be no instrument for the resolve pure mathematical problems. It must be expert system for old systems of knowledge and science and scientific prognostication system for new systems of knowledge and science.

Such expansion of mathematics is corresponded to basic thesis of computer science and allow to bond mathematics (précised knowledge), science (ordered knowledge) and other knowledge, including culture and religious, in one system.

PA may be represented as expansion the Newtonian method (four rules of conclusions in physics) on all science and knowledge. But Newton had problem of ended second Descartian synthesis (classical mechanics) and transit from complex system to more simple [1]. Basic problem of PA is creation universal system of formalization of knowledge and basically the transit from local (more simple) to global (more complex) problems.

PA may be represented as answer on E. Wigner problem of creation the more universal system science or metascience [41]. He said that necessity in more strong of science integration is caused the reciprocal of mathematics, physics and psychology for natural sciences. But development of computing science allows transiting this problem on all science and knowledge. Therefore, only generalizing computing science as PA may be represented the universal system of foundation of science.

VI. CONCLUSIONS

1. Short historical analysis about influence computation to processes of awakening and development of science is represented.
2. System aspects of correlation computation, knowledge and science are discussed.
3. Basic concepts of foundation of mathematics are represented and analyzed.
4. Conditions of created metascience are formulated.
5. Polymetric analysis as system example of metascience is analysed.
6. Role of polymetric analysis for resolution next problems of modern science (natural approach of modern mathematics; S. Beer problem of century and modern cybernetics and classification science by step of complexity its chapters) are represented.
7. Comparative analysis of basic optimal systems of science and knowledge is represented too.

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