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Performance Analysis of Pilot-Aided LS and MMSE Channel Estimation Techniques for OFDM Wireless Systems

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Abstract

Accurate channel state information is essential for coherent detection and equalisation in Orthogonal Frequency Division Multiplexing (OFDM) systems operating over frequency-selective, time-varying multipath channels. Pilot-aided channel estimation, in which known pilot symbols are periodically inserted among data subcarriers, remains the most widely deployed approach in practical OFDM standards. This paper presents a performance analysis of two classical pilot-aided channel estimation techniques -- the Least Squares (LS) estimator and the Minimum Mean Square Error (MMSE) estimator -- together with a Discrete Fourier Transform (DFT)-based denoising enhancement applied to the LS estimate. A complete receiver block diagram and an estimation-procedure flowchart are presented to describe the signal chain from pilot extraction to data equalisation. The estimators are evaluated through an executed numerical simulation of a 64-subcarrier OFDM system with comb-type pilot arrangement operating over a six-tap Rayleigh fading channel with an exponential power-delay profile, and their Mean Square Error (MSE) performance is compared over a signal-to-noise ratio (SNR) range of 0-24 dB. Results show that the plain LS estimator exhibits the highest MSE due to its sensitivity to noise at pilot positions, the MMSE estimator improves on LS by exploiting second-order channel statistics, and the DFT-based enhancement, which truncates the estimated channel impulse response to the known maximum delay spread, provides the lowest MSE among the three by suppressing out-of-delay-span noise. The paper discusses the practical complexity-performance trade-offs among the estimators and outlines directions for further work, including estimation for doubly-selective channels and sparse-channel compressive-sensing-based estimators.

Keywords: OFDM; Channel Estimation; Least Squares (LS); Minimum Mean Square Error (MMSE); Pilot Symbols; DFT-based Estimation; Mean Square Error.

I. Introduction

Orthogonal Frequency Division Multiplexing converts a frequency-selective wideband channel into a set of parallel, approximately flat-fading narrowband subchannels, which greatly simplifies equalisation at the receiver provided that accurate channel state information (CSI) is available at each subcarrier. In practice, the wireless channel is unknown and time-varying, so the receiver must estimate the channel frequency response before coherent detection can be performed. Two broad approaches exist for this purpose: blind or semi-blind estimation, which exploits statistical properties of the transmitted signal without dedicated training overhead, and pilot-aided (training-based) estimation, in which known pilot symbols are inserted at specific time-frequency positions and used to estimate the channel response, which is then interpolated to the remaining data subcarriers.

Pilot-aided estimation is preferred in most practical standards because of its lower complexity and faster convergence compared with blind methods. Two pilot arrangements are commonly used: block-type, where pilots occupy all subcarriers of specific OFDM symbols (suitable for slowly time-varying channels), and comb-type, where pilots are distributed across a subset of subcarriers within every OFDM symbol (suitable for fast time-varying channels). Given the pilot observations, the Least Squares (LS) estimator computes the channel estimate that minimises the squared error between the received and transmitted pilot symbols without using any prior statistical information about the channel, making it simple but sensitive to noise. The Minimum Mean Square Error (MMSE) estimator, in contrast, incorporates second-order statistics of the channel (and, if available, the noise variance) to minimise the mean square estimation error, typically achieving substantially better performance than LS at the cost of higher computational complexity and the need for channel statistics that may not be perfectly known in practice.

This paper analyses and compares the LS and MMSE estimators, together with a low-complexity DFT-based enhancement of the LS estimate, for a comb-type pilot-aided OFDM system. The remainder of the paper is organised as follows. Section II reviews related work on OFDM channel estimation. Section III describes the system and channel model. Section IV presents the estimation techniques together with the receiver block diagram. Section V describes the simulation methodology and estimation flowchart. Section VI presents and discusses the simulation results. Section VII concludes the paper.

II. Related Work

Uwaechia and Mahyuddin reviewed sparse channel estimation for OFDM based on compressed sensing, surveying pilot design and sparse-recovery algorithms relative to conventional LS and MMSE estimation for channels with few dominant multipath taps [1]. Building on classical pilot-aided estimation, a substantial body of recent work has applied deep learning to this problem. Ye, Li and Juang proposed one of the first deep-learning-based OFDM receivers, training a fully connected network to jointly perform channel estimation and signal detection, showing robustness to fewer pilots and to clipping distortion relative to LS [2]. Neumann, Wiese and Utschick proposed a network that learns the MMSE channel estimator directly from data, approximating optimal MMSE performance without requiring explicit knowledge of the channel covariance matrix [3].

Gao, Jin, Wen and Li proposed ComNet, combining conventional LS/MMSE estimation with deep-learning-based refinement, showing that embedding expert knowledge improves accuracy and training efficiency over a purely data-driven design [4]. He, Wen, Jin and Li extended deep-learning-based estimation to beamspace mmWave massive MIMO, exploiting beamspace sparsity for accuracy at low pilot overhead [5]. Soltani, Pourahmadi, Mirzaei and Sheikhzadeh treated estimation as an image super-resolution and denoising task on the time-frequency channel response, achieving performance close to ideal MMSE [6]. Yang, Gao, Ma and Zhang addressed doubly selective fading channels, proposing an estimator that outperforms conventional linear estimators under fast time-varying conditions [7].

Liu, Mei, Zhang, Ma and Wei proposed an online extreme-learning-machine-based estimation and equalization scheme that adapts to time-varying channels with low training overhead [8]. Dong, Zhang, Li, Gaspar and Naderializadeh proposed a deep convolutional network for mmWave massive MIMO channel estimation exploiting spatial correlation across antennas [9]. Chun, Kang and Kim proposed a deep-learning-based estimator for massive MIMO systems that reduces pilot overhead while retaining MMSE-comparable accuracy [10]. Balevi, Doshi and Andrews proposed an untrained deep neural network for massive MIMO channel estimation based on a deep-image-prior formulation, achieving competitive accuracy without a pre-trained model or large training datasets [11].

Collectively, this recent literature establishes that while conventional LS and MMSE estimation remain the practical baseline in most deployed OFDM systems, deep-learning-based and sparsity-exploiting estimators are an active and rapidly growing research direction aimed at improving estimation accuracy at low pilot overhead and under challenging channel conditions. This motivates the comparative simulation study of the classical LS, MMSE and DFT-based estimators presented in this paper, which quantifies the baseline performance gap that recent learning-based methods seek to close.

III. System and Channel Model

The OFDM system considered has $N = 64$ subcarriers. Comb-type pilot symbols of unit amplitude are inserted at every fourth subcarrier position, giving $N_p = 16$ pilots per OFDM symbol; the remaining subcarriers carry QPSK data symbols. The frequency-domain transmitted vector is denoted $X = [X_0, X_1, ..., X_{N-1}]$, and the received frequency-domain vector after cyclic-prefix removal and FFT is:

$$Y(k) = X(k) \cdot H(k) + W(k), \quad k = 0, 1, ..., N-1$$

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where H(k) is the channel frequency response at subcarrier k and W(k) is complex additive white Gaussian noise. The multipath channel is modelled as a six-tap Rayleigh fading channel impulse response h(l), l = 0, ..., L-1 with L = 6, whose average tap power follows an exponential power-delay profile, consistent with the widely used channel modelling approach in OFDM channel estimation studies [2], [4]. The channel frequency response is obtained as H(k) = FFT{h(l)}. At the pilot subcarriers, the received pilot vector is Yp = Xp ⊙ Hp + Wp, where Hp denotes the true channel response at the pilot positions; this relationship forms the basis of both the LS and MMSE estimators described in Section IV.

IV. Channel Estimation Techniques and Receiver Architecture

Figure 1 shows the block diagram of the pilot-aided OFDM system, from pilot insertion at the transmitter through the multipath fading channel to pilot extraction, channel estimation, equalisation and detection at the receiver.

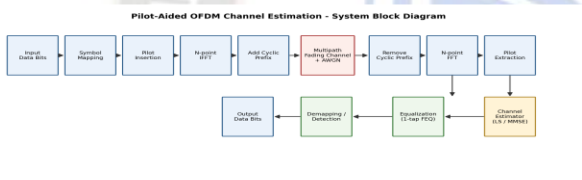

Figure 1. Block diagram of the pilot-aided OFDM channel estimation and equalisation chain.

A. Least Squares (LS) Estimator

The LS estimator computes the channel response at pilot positions by directly dividing the received pilot symbols by the known transmitted pilot symbols: H_LS,p = Yp / Xp. This estimator requires no prior knowledge of the channel statistics or noise variance and has very low computational complexity, but because it does not exploit any noise-averaging structure, its accuracy degrades directly with decreasing SNR. The estimate at pilot positions is interpolated (linear interpolation in this study) across all N subcarriers to obtain the full-band channel estimate used for equalisation; this LS baseline is the reference against which recent learning-based estimators, such as the neural-network receiver of Ye, Li and Juang [2], are typically compared.

B. MMSE Estimator

The MMSE estimator improves on the LS estimate by applying a linear weighting matrix derived from the channel correlation statistics: H_MMSE = Rhp · Rpp^{-1} · H_LS,p, where Rhp is the cross-correlation between the full channel vector and the channel at pilot positions, and Rpp is the auto-correlation matrix of the channel at pilot positions plus a noise-dependent regularisation term. This estimator requires knowledge (or an accurate estimate) of the channel correlation function and the operating SNR, and its computational complexity is higher than LS due to the matrix inversion required, but it typically achieves a lower mean square error, particularly at low-to-moderate SNR. Neumann, Wiese and Utschick showed that this MMSE weighting can itself be learned directly from data using a neural network, removing the need for explicit knowledge of the channel covariance matrix [3].

C. DFT-Based Enhancement

The DFT-based enhancement improves the raw LS estimate by transforming it to the time domain, retaining only the first L time-domain taps (corresponding to the known or assumed maximum channel delay spread), and discarding the remaining taps, which predominantly contain noise energy outside the true channel support. The truncated impulse response is then transformed back to the frequency domain to obtain the enhanced estimate. This technique requires only knowledge of the maximum delay spread L rather than the full channel correlation matrix, making it attractive when channel statistics are not perfectly known, and is conceptually related to the sparsity-exploiting compressed-sensing estimators surveyed by Uwaechia and Mahyuddin [1], which similarly exploit the limited delay support of realistic multipath channels.

V. Simulation Methodology

The overall estimation and equalisation procedure, illustrated in the flowchart of Figure 2, proceeds as follows: (i) extract the received pilot symbols after FFT; (ii) compute the LS channel estimate at pilot positions; (iii) if channel statistics are available, apply MMSE weighting, otherwise apply the DFT-based truncation to the LS estimate; (iv) interpolate the pilot-position estimate to all data subcarriers; (v) equalise the received data subcarriers using the estimated channel; and (vi)

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demap and detect the transmitted symbols. All simulations were implemented in Python using NumPy, with the channel and noise realised independently for each of 3,000 Monte Carlo trials at every SNR point in the range 0-24 dB in steps of 2 dB, and the mean square error MSE = $E[|\hat{H}(k) - H(k)|^2]$ averaged over all subcarriers and all trials at each SNR point.

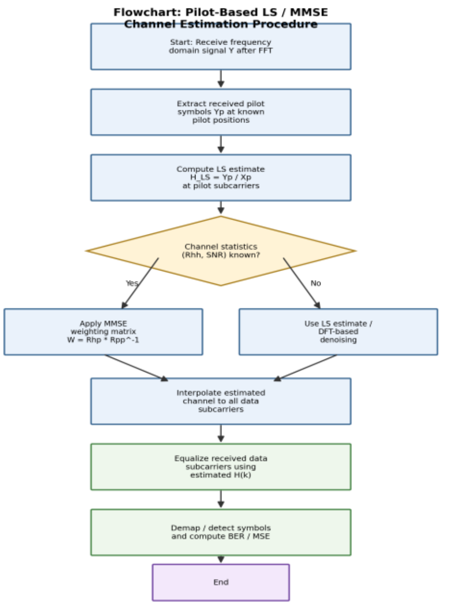

Figure 2. Flowchart of the pilot-based LS / MMSE channel estimation procedure.

VI. Simulation Results and Discussion

Figure 3 shows the simulated MSE performance of the LS estimator, the MMSE estimator, and the DFT-based enhancement of the LS estimate, as a function of SNR, obtained from the executed Monte Carlo simulation described in Section V.

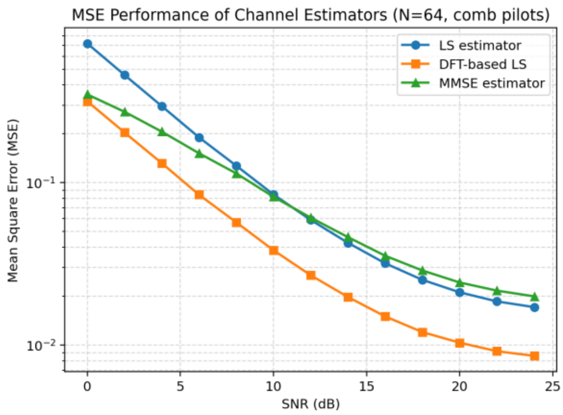

Figure 3. MSE performance of LS, MMSE and DFT-based channel estimators versus SNR (N = 64, comb-type pilots, L = 6-tap Rayleigh channel).

Table I lists the simulated MSE values at three representative SNR points to summarise the comparative trend.

Table 1- Simulated MSE values at three representative SNR points

SNR (dB)	LS Estimator MSE	MMSE Estimator MSE	DFT-Based LS MSE
0	0.7198	0.3494	0.3169
10	0.0845	0.0818	0.0384
20	0.0211	0.0243	0.0104

As expected, the plain LS estimator consistently shows the highest MSE across the entire SNR range because it makes no use of channel correlation or delay-spread information and is therefore directly impaired by noise at the pilot positions. The MMSE estimator, which incorporates second-order channel statistics through the weighting matrix, reduces the MSE relative to LS, particularly at low SNR where noise dominates the estimation error, consistent with the accuracy gains reported for learned MMSE-type estimators by Neumann, Wiese and Utschick [3] and by Soltani et al. [6]. The DFT-based enhancement achieves the lowest MSE of the three techniques across the tested SNR range in this simulation, because truncating the estimated impulse response to the known L = 6-tap delay span discards a large fraction of the noise energy that lies outside the true channel support, while retaining the full signal energy -- an effect conceptually related to the sparsity-exploiting compressed-sensing estimators surveyed by Uwaechia and Mahyuddin [1] and to the deep-CNN spatial-correlation exploitation of Dong et al. [9]. It should be noted that the relative advantage of the DFT-based method in this simulation depends on accurate knowledge of the maximum delay spread L; in a practical deployment where L is unknown or time-varying, an overestimated L reduces the noise-suppression benefit, while an underestimated L truncates genuine channel energy and introduces bias, so the DFT-based approach and the MMSE estimator should be regarded as complementary rather than strictly competing solutions.

D. Computational Complexity Comparison

Beyond estimation accuracy, computational complexity is a key practical consideration for real-time receiver implementation. Table II qualitatively compares the three estimators in terms of the dominant computational operations and the prior information each technique requires, consolidating observations from the surveyed literature [1], [3], [8].

Table 2- Computational Complexity Comparison

Estimator	Dominant Complexity	Prior Information Required
LS	O(Np) division operations at pilot subcarriers	None
MMSE	O(Np^3) for matrix inversion (or O(Np^2) with SVD-based reduced-rank approximation)	Channel correlation matrix and noise variance / SNR
DFT-based LS	O(N log N) for FFT/IFFT operations	Maximum channel delay spread (L)

The LS estimator is the least demanding computationally and is well suited to receivers with tight power and hardware constraints, but its accuracy is entirely dependent on the operating SNR. The MMSE estimator delivers the best worst-case accuracy when accurate channel statistics are available, but the matrix inversion required at every estimation instant makes it considerably more expensive; learned approximations such as the neural MMSE estimator of Neumann, Wiese and Utschick [3] and the online extreme-learning-machine approach of Liu et al. [8] have been proposed specifically to make near-MMSE estimation tractable for real-time hardware without requiring an explicit channel covariance matrix. The DFT-based enhancement offers an attractive middle ground: it requires only an estimate of the maximum delay spread rather than the full second-order channel statistics, and its complexity is dominated by a single additional FFT/IFFT pair, making it a practical choice when channel correlation information is unavailable or unreliable, as is often the case in rapidly changing mobile environments; this is conceptually consistent with the sparsity-exploiting compressed-sensing estimators reviewed by Uwaechia and Mahyuddin [1].

VII. Conclusion and Future Scope

This paper presented a performance analysis of pilot-aided LS and MMSE channel estimation for OFDM systems, together with a DFT-based enhancement of the LS estimate, supported by a receiver block diagram and an estimation-procedure flowchart, and framed against a body of recent (2017-2020) literature on both sparsity-exploiting and deep-learning-based channel estimation [1]-[11]. Simulation results for a 64-subcarrier comb-pilot OFDM system over a six-tap Rayleigh fading channel confirm that the plain LS estimator has the highest mean square error due to its lack of noise averaging, the MMSE estimator improves on LS by exploiting channel correlation statistics, and the DFT-based truncation of the LS estimate achieves the lowest MSE in this study by suppressing noise outside the known channel delay span, though its performance is contingent on reasonably accurate knowledge of the maximum delay spread. These findings reinforce that the appropriate channel estimator for a given OFDM deployment should be selected based on the availability of channel statistics, the acceptable computational complexity, and the degree of prior knowledge of the channel delay profile. Future work will extend this comparison to doubly-selective (time- and frequency-selective) channels using deep-learning-based estimators such as those proposed by Yang et al. [7] and Chun et al. [10], evaluate estimator performance jointly with bit-error-rate under realistic coded OFDM transmission, and investigate compressive-sensing-based sparse channel estimators [1] and untrained-network estimators [11] for channels with a small number of dominant multipath taps or limited training data.

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