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Analysis of Coded Space-Time Block Transmission

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Abstract

In this article, the analysis for a special case, that of concatenated channel codes and orthogonal space-time block codes are provided. The equivalent SISO channel model and recognize that it is a block fading channel are used to derive PEPs for spatially and temporally correlated and Independent and Identically Distributed cases. The analysis for Rayleigh and Rician fading are also provided.

1.1 Introduction

1.2 System model

We consider a coding-diversity scheme where a channel code and a STBC are used as shown in Figure. The channel code can be a single or concatenated code. The channel encoder maps a sequence of k information bits to n coded bits. Each coded bit is modulated by a signal with unit energy. This is further encoded by the space time block encoder with T transmit antennas. The receiver employs R receive antennas and combines their output optimally. We consider a frequency non-selective fading channel. The output of the channel is given by

$$y = \mathrm{H\,s + n}$$

where y is $n_R \times 1$ received signal vector, s is the modulated $n_T \times 1$ vector transmitted over T transmit antennas and n is $n_R \times 1$ i.i.d. Gaussian noise at the input of the antennas. The channel matrix is represented by H whose elements h, are the complex Gaussian channel coefficients for the pair of transmit antenna i and receive antenna j.

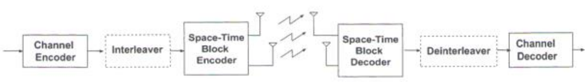

Figure.2.1. Concatenated channel code and space-time block code

In a STBC with n_T transmit antennas, it is assumed that the channel coefficients h_{ij} remain fixed through n_T consecutive intervals, and the receiver has either perfect or partial knowledge about them [1, 32]. Hence, the channel is block fading with block length n_T. Also, in the decoded sequence, fly' consecutive symbols are affected by the same set of fading coefficients h_{ij} 's.

The multiple-input multiple output channel, driven by an orthogonal STBC, can be represented by an equivalent single-input single-output (SISO) channel. Assuming the receiver combines the received signals from R antennas optimally, the MIMO channel can be represented as a 5150 block fading channel with fading coefficient for each block of n_T symbols equal to:

$$h_{eq} = \sqrt{\frac{1}{n_T}\sum_{i=1}^{n_T}\sum_{j=1}^{n_R}\left|h_{ij}\right|^2}.$$

(2.1)

Alternately, we can write the equivalent SNR

$$\gamma = \bar{\gamma}\left\|H\right\|_F^2, \tag{2.2}$$

where $\left\|\cdot\right\|_F$ denotes the Frobenius norm, $\bar{\gamma} = \frac{1}{n_T}\frac{R_c E_b}{N_0}$ is the average SNR per information bit per transmit antenna, and R is the code rate.

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If the noise components of the actual channel are independent, so are the noise components of the equivalent channel [3, 8]. The transmitted power is scaled by the number of transmit antennas to keep the total transmitted power constant. The equivalent fading coefficient follows a generalized Rayleigh. distribution [25]. The resultant instantaneous SNR per bit, -y, follows chi-square distribution with degree of freedom $2n_T n_R$ [4].

The problem is now reduced to the analysis of a block fading SISO channel which is no longer Rayleigh, but rather follows a generalized Rayleigh distribution. Spatially correlated and temporally correlated channels, which we also consider in this work, further modify the probability distribution.

Here it is appropriate to make a note on interleaving. Some coded space-time transmission systems, e.g. [14], have been proposed that do not include interleaving between the outer and inner codes. However, our simulations show that the codes of [14] can be improved by 1.7dB with an interleaver'. Bauch and Hagenauer [3] also do not employ interleaving between inner and outer codes, where potentially similar gains in performance would be possible. In view of these gains and the relatively low cost of interleaving, it is important to include interleaving in the analysis of coded space-time systems.

Interleaving, however, requires a complicated and cumbersome book-keeping for calculating pairwise error probabilities. To manage this complexity and to avoid interleaver-dependent probabilities, we use the concept of a uniform interleaver. To demonstrate the efficacy of this approach, Figure 5.2 shows the pairwise error probability of the dominant error event (Hamming distance d = 5) of a convolutional code concatenated with Alamouti signaling. The (averaged) uniform interleaver gives a good approximation to the best interleaver in realistic signal-to-noise ratios. [2] The usage of random uniform interleaving was first proposed by Benedetto and Montorsi [6] for the analysis of turbo codes and has also been used by Zummo and Stark [38] to explore the effects of channel interleavers.

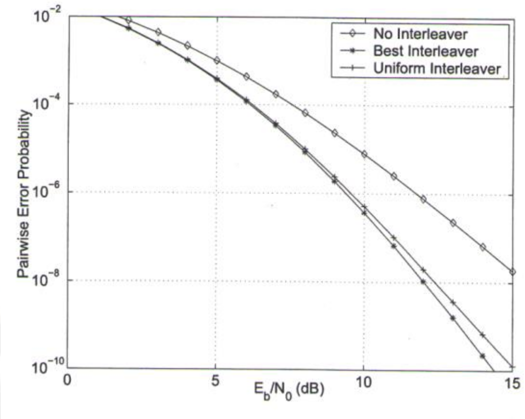

Figure 2.2. Convolutional Code, 2-Tx and 1-Rx antennas, d = 6, Block by block i.i.d. Rayleigh fading

1.3 Analysis of block fading channel

The performance of channel codes in block fading environments is studied in lot of articles. The original analysis in requires a generalized weight enumerating function of the channel code (or generalized transfer function for convolutional codes), which depends on the order of transmitted bits of a codeword. Therefore, the existence of interleaver complicates the analysis. We use the concept of uniform interleaver to address this problem in a manner closely following Zummo and Stark.

If the fading coefficient remains constant over a period of l symbols, the channel is called a block fading channel with block length l. Such a channel may arise in practice if the coherence time of channel is greater than symbols. However, block fading channels are only an approximation of time correlated channels. The channel coefficient is assumed to change independently from one block to another.

Assume that a frame of signals $\{s_l\}_{l=1}^{N}$ is transmitted over block fading channel with block length l. The number of blocks F The received signal is given by,

$$y_{f,1} = h_f s_{f,1} \qquad f = 1,.....,F \;\; l = 1,....,\ell$$

where $y_{f,1}$ and $s_{f,1}$ are the l-th received and transmitted values in block f respectively. h_f is the channel coefficient in the corresponding block.

A maximum likelihood decoder will maximize the metric,

$$m\left(y,s\right)=\sum_{f}\sum_{l}\left|y_{f,1}-h_{f}s_{f,1}\right|^{2} \qquad (3.1)$$

From Equation 4.4 it is clear that the analysis required knowledge of distribution of coded symbols in the block f. This distribution is interleaver dependent, which makes the analysis harder. There have been several efforts to solve this problem.

The length of the coded sequence (frame length) is n. The length of a fading block is l, thus the number of fading blocks in each coded frame is $F=\left[n/\ell\right]$. We now need to determine how the error bits are distributed among different blocks, i.e., how much error weight is present in each fading block. To characterize that, we build a histogram of weights as follows: assume the number of blocks that have weight m is f_m, and consider the vector $f=\left(f_0,...,f_w\right)$ where $w=\min\left(l,d\right)$. A given vector f is a valid histogram if $\sum f_m=F$ and $\sum mf_m=d$.

For example, let the frame length be n = 5 and fading block length be l = 2. If an error event with weight d = 4 is interleaved, the following histograms are possible: (3,0,2), (2, 2, 1), (1,4,0). It can be easily seen that the total number of blocks is F 5, and the total weight in each case is d = 4. The pattern (2, 2, 1) is shown in the Figure 5.3. Note that all the fading coefficients shown in the figure are independent.

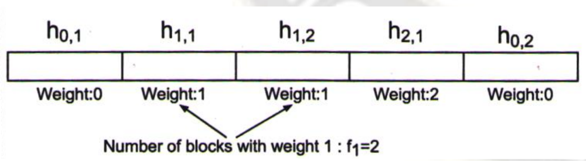

Figure 4.3. One possible block pattern for the case N = 5, d = 4, l = 2

Now, using the uniform interleaving concept, one may average the PEP over all valid error patterns (histograms).

$$P\left(d\right)=\mathrm{E}_{\mathrm{f}}\left[P\left(d\,|\,\mathrm{f}\right)\right]=\sum_{f_1=1}^{F}\sum_{f_2=1}^{F/2}\cdots\sum_{f_w=1}^{F/w}P\left(d\,|\,\mathrm{f}\right)p\left(\mathrm{f}\right)$$

(3.2)

where is the expectation operator and, p(f) is the weight of occurrence of the pattern f which can be found by combinatorics.

1.4 PEP based on moment generating functions

For a given channel code C, assuming all-zero codeword is transmitted, the PEP of a codeword with weight d given the pattern f of the fading blocks, is

$$P\left(d\,|\,\mathrm{f},\gamma\right)=Q\left(\sqrt{2\sum_{m=1}^{w}m\sum_{i=1}^{f_m}\gamma_{m,i}}\right) \qquad (4.1)$$

Here we have collected terms corresponding to blocks with equal weight patterns. Thus $\gamma_{m,i}$ is the SNR for the i-th block that has weight m (there are a total of f_m such blocks).[3]

In the case of T transmit and R receive antennas the resultant SNR per bit, from (4.2), is

$$\gamma=\bar{\gamma}\left\|H\right\|_F^2 \qquad (4.2)$$

where $\bar{\gamma}=\frac{1}{n_R}\frac{R_cE_b}{N_0}$ is the average SNR per information bit, and R_c is the code rate. Representing Q-function in its alternative form [29], the PEP conditioned on the block fading pattern f is

$$P\left(d\,|\,\mathrm{f},\gamma\right)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{1}{\sin^2\theta}\sum_{m=1}^{w}m\sum_{i=1}^{f_m}\gamma_{m,i}\right)d\theta.$$

Averaging the above conditional PEP over the instantaneous SNR γ we find $P\left(d\,|\,\mathrm{f}\right)=\mathrm{E}_{\gamma}\left[P\left(d\,|\,\mathrm{f},\gamma\right)\right]$. Assuming $\gamma_{m,i}$ are independent,

$$P\left(d\,|\,\mathrm{f}\right)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{m=1}^{w}\prod_{i=1}^{f_m}\int_0^{\infty}\left(-\frac{m\gamma_{m,i}}{\sin^2\theta}\right)p_{\gamma}\left(\gamma_{m,i}\right)d\gamma_{m,i}d\theta$$

The inner integral is the moment generating function (MGF) of $\gamma,\Phi\left(s\right)=\mathrm{E}\left[e^{s\gamma}\right]$, evaluated at $s=-m/\sin^2\theta$, hence

$$P\left(d\,|\,\mathrm{f}\right)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{m=1}^{w}\left[\left(-\frac{m}{\sin^2\theta}\right)\right]^{f_m}d\theta.$$

(4.3)

The expression (4.3) is general for all the channels. In the sequel, we use moment generating function of different channels with expression (4.3) to derive pairwise error probabilities.

We start our analysis with spatially and temporally independent fading.

1.5 Independent fading

If the entries of the channel matrix H are independent, the resulting SNR is the sum of $n_T n_R$ independent exponential variables and hence has a chi-square distribution with the pdf [29]

$$p_\gamma(\gamma) = \frac{1}{(D-1)!\bar{\gamma}D}\gamma^{D-1}\exp(-\gamma/\bar{\gamma}),$$

where $D = n_T n_R$. The MGF of this pdf is given by [29]

$$\Phi_\gamma(s) = (1-s\bar{\gamma})^{-D}. \quad (5.1)$$

Using this MGF in (4.8) we obtain the following bound for $P(d|\mathrm{f})$

$$P(d|\mathrm{f}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{m=1}^{w}\left(1+\frac{m\bar{\gamma}}{\sin^2\theta}\right)^{-f_m D} d\theta$$

(5.2)

where the last inequality is the Chernoff bound. One may also obtain the corresponding result for quasi-static Rayleigh fading by the setting F = 1 which is equivalent to m = d, fm 1.

4.5 Spatially correlated fading

Theorem 1

The moment generating function of γ is given by

$$\Phi_\gamma(s) = \prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\left(1-s\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}\right)^{-1},$$

(5.3)

where $\lambda^{(t)}$ and $\lambda^{(r)}$ are eigenvalues of RT and RR respectively.

Proof:

$$\|H\|^2 = vec(H)^H vec(H) = vec(\tilde{H})^H \Lambda vec(\tilde{H})$$

$$= \sum_{i=1}^{n_T}\sum_{j=1}^{n_R}\lambda_i^{(t)}\lambda_j^{(r)}\left|\tilde{h}_{ij}\right|^2. \quad (5.4)$$

From (4.7) and (4.12),

$$\gamma = \bar{\gamma}\|H\|^2 = \bar{\gamma}\sum_{i=1}^{n_T}\sum_{j=1}^{n_R}\lambda_i^{(t)}\lambda_j^{(r)}\left|\bar{h}_{ij}\right|^2.$$

The MGF of γ is

$$\Phi_\gamma(s) = \mathrm{E}\left\{\exp\left(-s\bar{\gamma}\|H\|^2\right)\right\} = \prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\mathrm{E}\left\{\exp\left(-s\bar{\gamma}\lambda_i^{(t)}\lambda_j^{(r)}\left|\bar{h}_{ij}\right|^2\right)\right\}.$$

Each term in the last expression is the moment generating function of an exponential random variable. Substitution gives (4.11).

We can now substitute in (4.8) to obtain

$$P(d|\mathrm{f}) = \frac{1}{\pi}\prod_{m=1}^{w}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\left(1+\frac{s\mu_k\lambda_i^{(t)}\lambda_j^{(r)}}{\sin^2\theta}\right)^{-f_m} d\theta,$$

(5.4)

$$\leq \frac{1}{2}\prod_{m=1}^{w}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\left(1+m\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}\right)^{-f_m}. \quad (5.5)$$

Using this formula, it is instructive to consider two extreme cases: uncorrelated and fully correlated channels. In the case of uncorrelated channel, $\lambda_i^{(t)} = \lambda_j^{(r)} = 1$ for all i, j, and the formula reduces to (5.10), as expected. In the case of fully correlated channel, the correlation matrix is rank deficient and we have, $\lambda_i^{(t)} = n_T, \lambda_j^{(r)} = n_R$, and all other $\lambda_i^{(t)} = \lambda_j^{(r)} = 0$. Thus the above moment generating function reduces to

$$\Phi_\gamma(s) = (1-sD\bar{\gamma})^{-1}, \quad (5.6)$$

which shows no diversity, but a receive gain of $D = n_T n_R$ (recall that $\bar{\gamma} = \frac{1}{n_T}\frac{R_c E_b}{N_0}$)

1.6 Temporal and spatial correlation

For various reasons such as long data blocks or long fading periods, it may not be practical to use interleavers to remove the channel memory. In such cases, we need to analyze the system with channel memory, a task which we undertake in this section. We assume that the coherence time is much greater than n_T symbols, so that the channel remains effectively constant over each STBC block and linear decoding is possible.

Assuming a given error event has weight d, we must concentrate on the channel matrix at time instances $\{k_1,...,k_d\}$ where the error event has nonzero value. Let the channel matrix at time k_j, be denoted as H and define $\mathrm{H} = \left[vec(H_1)vec(H_2)...vec(H_d)\right]$. Each H may be spatially correlated; the spatial correlations are modeled by a matrix R_s as before. We assume the statistics to be stationary (time-invariant), therefore only one spatial correlation matrix suffices. We model the temporal correlation of the channel by R_t, that is, $R_t(i,j) = \mathrm{E}\left[vec(H_1)^H vec(H_1)\right]$. Therefore, H can be modeled as

$$\mathrm{H} = R_s^{1/2}\bar{\mathrm{H}}R_t^{1/2}, \qquad (4.16)$$

where, H is a $(n_T n_R)\times d$ matrix with i.i.d. elements.

Using Lemmas 1 and 1, we can write the MGF of $dn_T n_R$ correlated exponential variables as

$$\Phi_\gamma(s) = \prod_{k=1}^{d}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\left(1 - s\mu_k\lambda_i^{(t)}\lambda_j^{(r)}\right)^{-1},$$

(4.17)

where are eigenvalues of R_t. Using this, we can calculate the pairwise error probability

$$\Phi_\gamma(s) = \prod_{k=1}^{d}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\left(1 + \frac{\bar{\gamma}\mu_k\lambda_i^{(t)}\lambda_j^{(r)}}{\sin^2\theta}\right)^{-1} d\theta$$

(4.18)

$$\le \frac{1}{2}\prod_{k=1}^{d}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\left(1 + \mu_k\lambda_i^{(t)}\lambda_j^{(r)}\right)^{-1}. \qquad (4.19)$$

It is easy to see that for the special case of quasi-static fading, $(R_t)_{ij} = 1$ for all i and j, therefore $\mu_1 = d$ and all other $\mu_k = 0$, and the equation reduces to the familiar PEP for the quasi-static fading, where there is no time diversity but there is a coding gain of d.

4.7 Performance analysis with multilevel modulation

We now proceed to the analysis of a concatenation of TCM or MTCM with space-time block codes. The design of TCM and MTCM for space-time block codes has been addressed in [14] and [8].

Following the same steps as before, we need to consider error patterns f (histograms) in a manner similar to Section 5.2. Because the errors can assume multiple values (more than two), the construction of the patterns is comp1icated. In a 2^m-ary modulation, an error event can be represented by $d = \left(d_0,...,d_{2^m-1}\right)$, where d is the number of times i-th symbol repeats in the error event. Obviously, $\sum_{i=0}^{2^m-1} = N$ the codeword length. As in Section 5.2, we denote number of fading blocks with F and block length with l. Then fading pattern in block j can be given by $v_j = \left(v_j^{(0)},...,v_j^{(2^m-1)}\right)$, where $v_j^{(i)}$ is the number of times symbol i repeats in that particular block. A histogram $\mathrm{f} = \{f_j : v_j \text{ repeats } f_j \text{ times}\}$ is a valid histogram if:

$$\sum_j f_j = F, \quad \sum_j f_j v_j^{(i)} = d_i, \quad i = 0,...,2^m - 1$$

For example, let the error event (compared to all zero codeword) in a TCM with QPSK modulation and codeword length $N = 8$ be

$\{s_2, s_1, s_0, s_3, s_1, s_2, s_0, s_1\}$. Obviously $d=(2,3,2,1)$. If the block length is 2, the distribution of errors can be given as follows,

	$j=1$	$j=2$	$j=3$	$j=4$
	$s_2 s_1$	$s_0 s_3$	$s_1 s_2$	$s_0 s_1$
$v_j^{(0)}$	0	1	0	1
$v_j^{(1)}$	1	0	1	1
$v_j^{(2)}$	1	0	1	0
$v_j^{(3)}$	0	1	0	0

Hence, In the first block, $v_j=(0,1,1,0)$, in the second block $v_j=(1,0,0,1)$ etc. The above block distribution can be represented by the histogram:

v_j	f_j
$(0,1,1,0)$	2
$(1,0,0,1)$	1
$(1,1,0,0)$	1
$(2,3,2,1)$	$=\sum_j f_j v_j^{(i)}$

The pairwise conditional probability of error between the all-zero codeword and a codeword **e** is given by,

$$P(0\to e|\mathrm{f},\gamma)=Q\left(\sqrt{2\sum_j\sum_{i=1}^{f_j}\gamma_{j,i}\alpha_i}\right),$$

(4.20)

where j is the index of block patterns and $\gamma_{j,i}$ is the instantaneous SNR per bit for i-th block in fading pattern j. We have defined an aggregate distance metric α_j for each block pattern j, calculated by

$$\alpha_j=\sum_{k=0}^{2^m}v_j^{(k)}d^2(s_k,s_0),$$

where $v_j^{(k)}$ is the multiplicity of symbol s_k in the block pattern indexed by j. Averaging over γ, we find the PEP expression

$$P(0\to e|\mathrm{f})=\frac{1}{\pi}\int_0^{\pi/2}\prod_j\left[\Phi_\gamma\left(-\frac{\alpha_j}{2\sin^2\theta}\right)\right]^{f_j}d\theta$$

. (4.21)

For the useful class of uniform error probability (UEP) codes, where the reference codeword can always be chosen as the all-zero codeword [29, 19], the union bound on frame error probability is

$$P_e\le\sum_{e\neq 0}\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_j\left(2^{-(m-1)}\sum_{cl}\Phi_\gamma\left(-\frac{\sum_{i=1}^{2m}v_j^i(cl\oplus s_i,s_i)}{2\sin^2\theta}\right)^{-f_j}\right)d\theta,$$

(4.22)

where c is a symbol that belongs to the first level of set-partitioning of the 2tm-ary modulation [19]. Note that and f_j depend on the error word e, but the dependence has been suppressed in the formula above for notational simplicity.

To calculate the union bound in the case of spatially and temporally i.i.d. fading, the moment generating function (4.9) is substituted in (4.22). To calculate the union bound in the case of spatially correlated fading, we insert the moment generating function (4.11) into (4.21).

The union bound in the case of temporally correlated channel requires a little twist. In the previous cases, the equivalent SNR was a function of $\|H\|$ only, therefore decorrelating H simplified the MGF expressions. However, in the case of temporal correlations, the effective SNR is expressed as

$$\gamma = \sum_{k=1}^{d}\sum_{i=1}^{n_T}\sum_{j=1}^{n_R}\left|h_{ij}(k)\right|^2 \delta_k^2$$

(4.23)

where δ_k is the Euclidean distance of the error event at k-th error position. Obviously decorrelating H no longer works. Define $D = diag(\delta_1,...,\delta_d)$ and note that $\gamma = \|\mathrm{H}D\|^2$, where $\mathrm{H} = [vec(H_1)vec(H_2)....vec(H_d)]$. To obtain a sum of independent SNR components, we must diagonalize the autocorrelation of $\mathrm{H}D$.

$$\gamma = \|\mathrm{H}D\|^2 = \sum_{k=1}^{d}\sum_{i=1}^{n_T}\sum_{j=1}^{n_R}\left|h_{ij}(k)\right|^2 \delta_k^2$$

$$= \sum_{k=1}^{d}\sum_{i=1}^{n_T}\sum_{j=1}^{n_R}\left|\tilde{h}_{ij}(k)\right|^2 \delta_k^2 \lambda_i^{(t)}\lambda_i^{(r)}\hat{\mu}_k .$$

Recall that the spatial and temporally correlated H is modeled as

$$\mathrm{H} \equiv R_s^{1/2}\tilde{\mathrm{H}}R_t^{1/2},$$

where $\tilde{\mathrm{H}}$ has i.i.d. entries and R_s and R_t are the spatial and temporal correlation matrices, respectively. It follows that $\hat{\mu}_k$ are the eigenvalues of DRD. Therefore we can still use equations (5.17) and (5.21) except we should substitute $\hat{\mu}_k$ for μ_k.[4]

4.8 Performance under Rician fading

In this section we consider the Rician fading channels with parameter K describing the ratio of the energy of the line-of-sight component to the multipath component. For the uncorrelated Rician channel, the moment generating function of $\gamma = \bar{\gamma}\|H\|^2$ is given by [29]

$$\Phi_\gamma(s) = \left(\frac{(1+K)}{1+K-s\bar{\gamma}}\right)^D \exp\left(\frac{(Ks\bar{\gamma})}{1+K-s\bar{\gamma}}\right)^D$$

By using this MGF with equation (4.21), the PEP for the fast fading Rician channel and multilevel modulation is given by

$$P(0\to e) = \frac{1}{\pi}\int_0^{\pi/2}\prod_j\left[\frac{(1+K)}{1+K+\frac{\alpha_j\bar{\gamma}}{2\sin^2\theta}}\exp\frac{K\frac{\bar{\gamma}\alpha_j}{2\sin^2\theta}}{1+K+\frac{\bar{\gamma}\alpha_j}{2\sin^2\theta}}\right]^D d\theta$$

(4.24)

For the case of spatially correlated fading, the MGF can be once again derived using Lemmas 1 of Section 3.1 and Lemma 2 of Section 4.5,

$$\Phi_\gamma(s) = \prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\frac{(1+K)}{1+K-s\mu_k\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}}\left(\frac{Ks\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}}{1+K-s\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}}\right),$$

(4.25)

where $\lambda^{(t)}$ and $\lambda^{(r)}$ are the eigenvalues of transmit and receive correlation matrices R_{Tx} and R_{Rx} respectively. Expressions (5.25) and (5.22) directly yield the desired bounds on error probability.

When temporal as well as spatial correlation is present, it is straightforward to show that the moment generating function is expressed as follows

$$\Phi_\gamma(s) = \prod_{k=1}^{d}\prod_{i=1}^{n_T}\prod_{j=1}^{n_R}\frac{(1+K)}{1+K-s\mu_k\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}}\left(\frac{Ks\mu_k\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}}{1+K-s\mu_k\lambda_i^{(t)}\lambda_j^{(r)}\bar{\gamma}}\right),$$

(4.26)

where μ_k are the eigenvalues of temporal correlation matrix R_t. Once again, in combination with (5.22), the desired bounds are obtained.

4.9 Conclusion

We derived the PEP expression for concatenated channel codes and space-time block codes. This setup provides full diversity and coding gain with simple design, at the cost of loosing some rate. This article analyzed the system for spatially and/or temporally correlated channels as well as i.i.d. channels and also provided analysis for Rayleigh and Rician fading.

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