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Metric Fuzziness and Eigenvalue Theory

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Abstract:
This article is dedicated to the exploration of fuzzy eigenvalues and fuzzy eigenvectors within the context of a fuzzy metric space. To facilitate this discussion, we introduce a specific metric for this space. Furthermore, we provide comprehensive definitions for fuzzy eigenvalues and fuzzy eigenvectors, focusing on their application to fuzzy square matrices. In the course of our exploration, we establish a series of theorems pertaining to fuzzy eigenvalues and eigenvectors within a fuzzy metric space. To enhance understanding, we illustrate these theorems with practical examples.

Keywords: Fuzzy eigenvalue, Fuzzy eigenvector, Fuzzy eigen space, metric space in fuzzy.

1. INTRODUCTION

In 1975, the concept of a fuzzy metric space was initially presented by Kramosil and Michálek in their seminal work [1]. This innovative concept extends beyond the traditional statistical (probabilistic) metric space, offering a more flexible framework for distance measurement. Subsequently, Grabiec's work [2] focused on establishing the notion of completeness within the realm of fuzzy metric spaces, a pivotal property for studying convergence and continuity. This field has since evolved, with researchers exploring two distinct approaches to address the challenges and applications of fuzzy metric spaces. There are two primary approaches in the study of fuzzy metric spaces. One approach, as pioneered by Kaleva in 1984 [6], involves the use of fuzzy numbers to define metrics in conventional spaces. Following Kaleva's work, several researchers have delved into this approach, exploring topics such as fuzzy normed spaces, fuzzy topology derived from fuzzy metric spaces, fixed point theorems, and other properties of fuzzy metric spaces. Notable contributions include the research of Felbin in 1992 [7], George in 1994 [8], George in 1997 [9], Gregori in 2000 [10], and Hadzic in 2002 [11], among others.

The second approach focuses on utilizing real numbers to quantify the distances between fuzzy sets.

The sources related to this particular approach can be found in works by Dia in 1990 [5], Chaudhuri in 1996 [4], Boxer in 1997 [3], Fan in 1998 [12], Voxmam in 1998 [15], Przemyslaw in 1998 [14], and Brass in 2002 [13]. The findings of these studies have found practical applications in various real-world challenges within fuzzy environments. However, it's worth noting that different problems often require distinct measurement approaches. In other words, there is no universal measure that can be applied across all types of fuzzy environments.

2. PRELIMINARIES

Definition:1

If $A \in M_x$ and $x \in \mathbb{R}^n$, then $\exists$ a constant $\bar{\lambda} \in [0,1]$ such that $Ax \in \bar{\lambda}x$. Then $\bar{\lambda}$ is called an eigenvalue of matrix A, along with its associated eigenvector x.

Definition:2

For $A \in M_x$, $\bar{\sigma}(A) = \{\bar{\lambda} : Ax = \bar{\lambda}x \text{ has a solution for a non} - \text{zero vector } x\}$, is referred to as the spectrum of A.

$\bar{\rho}(A) = \sup_{\lambda \in \bar{\sigma}(A)} |\lambda|$; is called the spectral radius.

The characteristic polynomial:

The polynomial $(A - \bar{\lambda}I).x = 0$ has a non-trivial solution.

i.e. $det(A - \bar{\lambda}I) = 0$

Thus $det(A - \bar{\lambda}I)$ must be a polynomial in $\bar{\lambda}$.

Definition:3

Let D represent the collection of all closed and bounded intervals $X = [a_1, a_2]$ on real numbers. For $\bar{X}, \bar{Y} \in D$, we define $\bar{X} \leq \bar{Y}$, if $p_1 \leq q_1$ and $p_2 \leq q_2$.

$d(\bar{X}, \bar{Y}) = max(|p_1 - q_1|, |p_2 - q_2|)$, when $\bar{X} = [p_1, p_2]$ and $\bar{Y} = [q_1, q_2]$. It is a well-established fact that the metric space (D, d) is complete.

Definition [16]:4

Assume that X is a nonempty set, and d_S: P_S (X) × P_S (X) → $S_s^+(R)$
is a function. (PS (X), dS) is termed a fuzzy metric space provided that, for every
{(x_s, λ_s), (y_s, γ_s), (z_s, ρs)} ⊂ PS (X), dS meet the following set of three conditions,
(1) Non-negative: dS ((x_s, λ_s), ^{S}(y_s, γ_s)) = 0 iff $x_s = y_s$ and $\lambda_s = \gamma_s = 1$;
(2) Symmetric: dS ((x_s, λ_s), (y_s, γ_s)) = dS ((y_s, γ_s), (xs,λs));
(3) Triangular inequality: dS ((x_s, λ_s), (z_s, ρ_s)) < dS ((x_s, λ_s), (y_s, γ_s)) + dS ((y_s, γ_s), (z_s, ρ_s)).
dS is termed as fuzzy metric defined in PS (X) and dS ((x_s, λs), (y_s, γ_s)) is referred to as a fuzzy distance between the two fuzzy points.

Definition: 5

Let $\bar{A} \in M_x$, then $\bar{\rho}_{\bar{A}}(\bar{\lambda}) = |(\bar{\lambda}I - \bar{A})|$ is called the characteristic polynomial of $\bar{A}$. Its roots are called the latent value of $\bar{A}$.

Theorem - 1:

Let $\bar{A} \in M_x$. The collection of fuzzy eigenvectors corresponding to a specific fuzzy eigenvalue forms a subspace within the vector space $\mathbb{C}_n$.

Proof:

Let $\bar{\lambda} \in \bar{\sigma}(\bar{A})$ be a fuzzy eigenvalue of a fuzzy square matrix $\bar{A}$.

The set of fuzzy eigenvectors associated with $\bar{\lambda}$ is the kernel of the matrix (A - $\bar{\lambda}$).
Once more, the kernel of any fuzzy matrix constitutes a subspace within the fundamental vector space.
Hence the result.
(Thus, we get the following definition)

Definition: 6

Let $\bar{A} \in M_x$ and let $\bar{\lambda} \in \sigma(\bar{A})$. The kernel of (A - $\bar{\lambda}$) is referred to as the fuzzy eigenspace of A corresponding to λ.

Theorem – 2:

Let $\bar{A} \in M_x$, then fuzzy eigenvectors corresponding to distinct fuzzy eigenvalues exhibit linear independence. Suppose $\bar{\lambda} \in \bar{\sigma}(\bar{A})$ with fuzzy eigenvectors x is different from the set of fuzzy eigenvalues $\{\overline{\mu_1}, \overline{\mu_2}, \ldots, \overline{\mu_k}\} \in \bar{\sigma}(\bar{A})$ and V_μ is the subspace generated by the associated fuzzy eigenspace. Hence $x \notin V_{\bar{\mu}}$.

Proof:

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Let us consider $\overline{\mu}, \overline{\lambda} \in \overline{\sigma}(\bar{A})$ with $\overline{\mu} \neq \overline{\lambda}$ be the fuzzy eigen value corresponding to the fuzzy eigen vectors x and y respectively.

Then vectors are L.D,

i.e. $y = \overline{c}x$

Then $\overline{\mu}y = \overline{\mu c}x = \overline{c\mu}x$

$= \overline{c}Ax$

$= A(\overline{c}x)$

$= Ay$

$= \overline{\lambda}y$

This statement presents an inconsistency. And hence (i) is said to be true.

Consider the opposite scenario, where it is assumed that $x \in V_{\overline{\mu}}$. Thus

$x = \overline{a}_1 y_1 + \cdots + \overline{a}_m y_m;$

where $\{y_1, \dots, y_m\}$ are L.I in $V_{\overline{\mu}}$. Every one of the vectors y_i is eigen vector.

Let's assume the relevant fuzzy eigenvalues, represented as $\overline{\mu}_j$.

i.e. $Ay_i = \overline{\mu}_{ii} y_i$, for each $i = 1,2, \dots, m$

Then $\overline{\lambda}x = Ax$

$= A(\overline{a}_1 y_1 + \cdots + \overline{a}_m y_m)$

$= \overline{a}_1 \overline{\mu}_{j1} y_1 + \cdots + \overline{a}_m \overline{\mu}_{jm} y_m$

If $\overline{\lambda} \neq 0$, we have

$$x = \overline{a}_1 y_1 + \cdots + \overline{a}_m y_m$$

$$= \overline{a}_1 \frac{\overline{\mu}_{j1}}{\lambda} y_1 + \cdots + \overline{a}_m \frac{\overline{\mu}_{jm}}{\lambda} y_m$$

`

We know from the above that at least two of the co-efficient $\overline{a}_i$ must be non-zero.

Consequently, we encounter two distinct representations of a single vector through linearly independent vectors, which is an infeasible scenario.

On the other hand, if $\overline{\lambda} = 0$, Then

$$\overline{a}_1 \overline{\mu}_{j1} y_1 + \cdots + \overline{a}_m \overline{\mu}_{jm} y_m = 0$$

Which is also impossible. Thus (ii) is proved.

Illustration-1 of theorem-2:

Let $\overline{A} = \begin{bmatrix} \bar{a} & \bar{b} \\ \bar{c} & \bar{d} \end{bmatrix}$, where $a, b, c, d \in [0,1]$ be a fuzzy square matrix.

Then $det(\overline{\lambda}I - \overline{A}) = det\begin{pmatrix} \overline{\lambda} - \bar{a} & -\bar{b} \\ -\bar{c} & \overline{\lambda} - \bar{d} \end{pmatrix}$

$= (\overline{\lambda} - \bar{a})(\overline{\lambda} - \bar{d}) - \bar{b}\bar{c}$

$= \overline{\lambda}^2 - (\bar{a} + \bar{d})\overline{\lambda} + \bar{a}\bar{d} - \bar{b}\bar{c}$

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∴ the fuzzy eigen values will be the root of the equation (1).

$$=\bar{\lambda}^2-(\bar{a}+\bar{d})\bar{\lambda}+\bar{a}\bar{d}-\bar{b}\bar{c} \quad (1)$$

∴ the fuzzy eigen values will be the root of the equation (1).

$$\therefore\ \bar{\lambda}=\frac{\bar{a}+\bar{d}\pm\sqrt{(\bar{a}+\bar{d})^2-4(\bar{a}\bar{d}-\bar{b}\bar{c})}}{2}$$

$$=\frac{\bar{a}+\bar{d}\pm\sqrt{(\bar{a}-\bar{d})^2+4\bar{b}\bar{c}}}{2}$$

Regarding this quadratic, three potential outcomes are under consideration:

i) A pair of genuine solutions → Distinct values / → Identical values

ii) A pair of complex solutions

Illustration-2:
Consider a fuzzy matrix denoted as $\bar{A}$ with a rank unity.

In such a case, there exist two vectors $w, z \in \mathbb{C}_n$ In this context $\bar{A}=wz^T$.

To determine the spectrum of $\bar{A}$, we examine the fuzzy ch. Polynomial as

$$\bar{A}x=\bar{\lambda}x$$

$$\Rightarrow \langle z,x\rangle w=\bar{\lambda}x$$

For this, we see that $x = w$ is one fuzzy eigen vector with fuzzy eigen value $\langle z, x\rangle$. If $z \perp x$, x is a fuzzy eigen vector pertaining to the eigen value 0.

$$\therefore \sigma_{\bar{A}}=\{\langle z,x\rangle, 0\}$$

Hence, the ch. Polynomial is

$$\bar{p}(\bar{\lambda})=(\bar{\lambda}-\langle z,x\rangle)\bar{\lambda}^{n-1}$$

In the case of orthogonal vectors w and z, then,

$$p(\bar{\lambda})=\bar{\lambda}^n$$

If w and z are not orthogonal, even though there are only two fuzzy eigen values, it can be stated that the fuzzy eigenvalue 0 has a multiplicity of $(n-1)$.

Theorem – 3:
Each polynomial of degree n with a leading coefficient of 1, i.e.

$$q(\bar{\lambda})=\bar{\lambda}^n+b_1\bar{\lambda}^{n-1}+\cdots+b_{n-1}\bar{\lambda}+b_n$$

It serves as the characteristic polynomial for a certain fuzzy matrix. Proof:
Let us suppose a $n \times n$ fuzzy square matrix

$$\bar{B}=\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -\bar{b}_n & -\bar{b}_{n-1} & -\bar{b}_{n-2} & \cdots & -\bar{b}_1 \end{pmatrix}$$

Then $\bar{\lambda}I - \bar{B}$ has the form

$$\bar{\lambda}I - \bar{B} = \begin{pmatrix} \bar{\lambda} & -1 & 0 & \cdots & 0 \\ 0 & \bar{\lambda} & -1 & \cdots & 0 \\ \cdots & \cdots & \bar{\lambda} & \cdots & \cdots \\ \bar{b}_n & \bar{b}_{n-1} & \bar{b}_{n-2} & \cdots & \bar{\lambda} + \bar{b}_1 \end{pmatrix}$$

Next, perform a expansion in minor along the bottom row to obtain

$$det(\bar{\lambda}I - \bar{B}) = b_n(-1)^{n+1}det\begin{pmatrix} -1 & 0 & \cdots & 0 \\ \bar{\lambda} & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \bar{\lambda} & \cdots & \cdots \end{pmatrix}$$

$$+ b_{n-1}(-1)^{n+2}det\begin{pmatrix} \bar{\lambda} & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \bar{\lambda} & \cdots & \cdots \end{pmatrix} + \cdots$$

$$+ b_1(-1)^{n+n}det\begin{pmatrix} \bar{\lambda} & -1 & 0 & \cdots \\ 0 & \bar{\lambda} & -1 & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \bar{\lambda} & \cdots \end{pmatrix}$$

$$= b_n(-1)^{n+1}(-1)^{n-1} + b_{n-1}(-1)^{n+2}\bar{\lambda}(-1)^{n-2} + \cdots + (\bar{\lambda} + b_1)(-1)^{n+1}\bar{\lambda}^{n-1}$$

$$= b_n + b_{n-1}\bar{\lambda} + \cdots + b_1\bar{\lambda}^{n-1} + \bar{\lambda}^n$$

Which is what we set out to prove.

Multiplicity of fuzzy matrix:

Let $A \in M_n(C)$ be a fuzzy matrix. Since $p(\bar{\lambda})$ represents a polynomial with a precise degree of n, therefore, it necessitates a precise count of n fuzzy eigenvectors $\bar{\lambda}_1, \bar{\lambda}_2, \dots, \lambda_n$. The multiplicity of a fuzzy eigenvalue is determined by how often the monomial $(\bar{\lambda} - \bar{\lambda}_i)$ appears in the factorization of $p(\bar{\lambda})$.

Suppose $\bar{\mu}_1, \bar{\mu}_2, \dots, \bar{\mu}_k$ are the distinct fuzzy eigen values with multiplicities $\bar{m}_1, \bar{m}_2, \dots, \bar{m}_k$ respectively. Then the ch. polynomial can be written as

$$p(\bar{\lambda}) = det(\bar{\lambda}I - \bar{A}) = \prod_{i=1}^{n}(\bar{\lambda} - \bar{\lambda}_i) = \prod_{i=1}^{k}(\bar{\lambda} - \bar{\mu}_i)^{m_i}$$

The multiplicities $\bar{m}_1, \bar{m}_2, \dots, \bar{m}_k$ are referred to as the fuzzy algebraic multiplicities associated with their corresponding fuzzy eigenvalues.

Algebraic multiplicity: The algebraic multiplicity of a fuzzy eigenvalue is the number of times it appears as a root of the characteristic polynomial (i.e., the polynomial whose roots are the fuzzy eigenvalues of a matrix).

Geometric multiplicity: The geometric multiplicity of a fuzzy eigenvalue is the dimension of the linear space of its associated fuzzy eigenvectors (i.e., its eigenspace).

Proposition: Let A be a $k \times k$ fuzzy matrix, $k \in [0,1]$. Let be one of the fuzzy eigenvalues of A. Then, the fuzzy geometric multiplicity of λ_k is less than or equal to its fuzzy algebraic multiplicity.

Proof: Suppose that the fuzzy geometric multiplicity of λ_k is equal to r, so that there

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is r linearly independent fuzzy eigenvectors x1, x2, ……., xr associated to λ_k. Arbitrarily choose k-r vectors xr+1, xr+2, ……., xk, all having dimension $k \times 1$ and such that the n column vectors x1, x2, ……, xk are linearly independent. Define the $k \times k$ fuzzy square matrix

$X = [x_1 \cdots\ x_k]$

For any r, denoted by br the vector that solves

Xbr=Axr

which is guaranteed to exist because X is full-rank (its columns are linearly independent). Define the $k \times (k-r)$ fuzzy matrix

$B = [b_{r+1} \cdots b_k]$

and denote by C its upper $r \times (k-r)$ block and by D its lower $(k-r) \times (k-r)$ block

$$B = \begin{pmatrix} C \\ D \end{pmatrix}$$

Denote by I_k the $k \times k$ fuzzy identity matrix. For any scalar λ, we have that

$$\begin{aligned}
&(\lambda I_K - A)X \\
&= \begin{bmatrix} (\lambda I_K - A)x_1 & \ldots & (\lambda I_K - A)x_r & (\lambda I_K - A)x_{r+1} & \ldots & (\lambda I_K - A)x_K \end{bmatrix} \\
&= \begin{bmatrix} (\lambda - \lambda_k)x_1 & \ldots & (\lambda - \lambda_k)x_r & \lambda x_{r+1} - Ax_{r+1} & \ldots & \lambda x_K - Ax_K \end{bmatrix} \\
&= \begin{bmatrix} (\lambda - \lambda_k)x_1 & \ldots & (\lambda - \lambda_k)x_r & \lambda x_{r+1} - Xb_{r+1} & \ldots & \lambda x_K - Xb_K \end{bmatrix} \\
&= X\begin{bmatrix} (\lambda - \lambda_k)I_r & -C \\ 0 & \lambda I_{K-r} - D \end{bmatrix}
\end{aligned}$$

Since X is full-rank and, as a consequence its determinant is non-zero, we can write

$$\begin{aligned}
&|\lambda I_K - A| \\
&= |\lambda I_K - A| \cdot |X| \cdot \frac{1}{|X|} \\
&= |(\lambda I_K - A)X| \cdot \frac{1}{|X|} \\
&= \left| X\begin{bmatrix} (\lambda - \lambda_k)I_r & -C \\ 0 & \lambda I_{K-r} - D \end{bmatrix} \right| \cdot \frac{1}{|X|} \\
&= |X| \left| \begin{bmatrix} (\lambda - \lambda_k)I_r & -C \\ 0 & \lambda I_{K-r} - D \end{bmatrix} \right| \cdot \frac{1}{|X|} \\
&= |(\lambda - \lambda_k)I_r| \cdot |\lambda I_{K-r} - D| \\
&= (\lambda - \lambda_k)^r |\lambda I_{K-r} - D|
\end{aligned}$$

where in the last step we have used a result about the determinant of block-matrices. The fuzzy eigenvalues of A solve the characteristic equation

$$|\lambda I_k - A| = 0$$

or, equivalently, the equation

$$(\lambda - \lambda_k)^r |\lambda I_{k-r} - D| = 0$$

This equation has a root

$$\lambda = \lambda_k$$

that is repeated at least r times. Therefore, the fuzzy algebraic multiplicity of λ_k is at least equal to its fuzzy geometric multiplicity r. It can be larger if λ_k is also a root of
$|\lambda I_{k-r} - D| = 0$

3. RESULTS

This paper provides an insight into the concept of fuzzy metric spaces with a focus on fuzzy eigenvalues and fuzzy eigenvectors. We present several theorems concerning the eigenvalues and eigenvectors within the context of a fuzzy metric space. Notably, we find that many propositions pertaining to metric spaces are applicable to fuzzy eigenvalues and eigenvectors. Additionally, we attempt to offer some insights into the multiplicity of fuzzy matrices. This field presents significant opportunities for further exploration by researchers, indicating the vast potential for future study and development.

CONFLICT OF INTEREST

The authors state that they do not have any conflicts of interest.

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