

Load Ratio to Line Flows to Estimate Congestion Costs for Customers in a Restructured Power Grid

Yashvant Bhavsar

Electrical Engineering Department
Vishwakarma Government Engineering College
Ahmedabad, India
yashvantbhavsar@gmail.com

Abstract—During the last decade of the 20th century, there was a global trend to deregulate the power sector. However, with the deregulation of the power system, operations of the power system tend to become more complex. In a deregulated power system, due to the operation and thermal limits of the line, cheaper generators are not utilized to dispatch the load at all times. Hence there is an increase in the cost of the energy produced. This increased cost is identified as congestion cost. Determination of congestion cost and distribution among all the participants is a crucial issue in the deregulated power system. For proper allocation of congestion cost, it is required to evaluate the share of load to the line flows. Here in this paper load share to the line flow is determined by using Bialek's algorithm. Using Bialek's downstream algorithm a method is developed to identify load share to the line flows. Standard test systems are used to get the results of this method.

Keywords- Transmission congestion cost, Congestion cost allocation, Bialek' down-stream theorem

I. INTRODUCTION

The electrical power sector had a monopolized structure until 1990. After that, power system engineers and operators found that, because of the absence of healthy competition in the electrical power sector, it was confined to certain limitations only. Hence, a major shift in the electrical power system structure was proposed and widely adopted globally. In this restructured power system, all operations regarding the power system were unbundled, i. e. deregulated. Restructuring results in the power industry becoming turbulently competitive and going through technological and regulatory changes, which affect its planning, operation, control, and services to customers. It is important to spot the consequences and impacts of those changes on planning, operation, control, and the cost of the power system. Open access to the power transmission line is an important aspect of the deregulated power system. All market players in the power system must have open access to the power transmission line without any discrimination. Transmission line corridors have always operational and thermal limits which prevent cheaper generators from using them. This limitations leads to an increase in the generation cost of the energy. The system operator has to identify the responsible entities for the increase in the cost. This increase in the cost is known as the congestion cost. Evaluation of the congestion cost and assigning it to all the system entities effectively is the crucial issue for the successful operation of the power grid.

Dispatch methods, generator rescheduling, load management or nodal pricing based methods are market based methods for evaluating congestion cost. A priority of load curtailment has been applied using social welfare and contracts between generators and load [1,2] for pool and contract dispatch models. Optimal bus price based simple methods were proposed [3] to calculate congestion cost supported by the very fact that additional flow increases congestion. Various dispatch methodologies introduce [4] for open access to the transmission lines by representing a conceptual model of pool dispatch, bilateral dispatch and multilateral dispatch alongside the need of dispatch coordination between these models. The effectiveness of the congestion clusters method for congestion management was discussed for cost and loss minimization [5] with the definition of transmission congestion distribution factors (TCDFs) which were used to evaluate change in the flow of a rise in the injection of power at any bus. Real Power transmission Congestion distribution Factors (PTCDFs) and Reactive Power transmission Congestion Distribution Factors (QTCDFs) used as sensitivity indices for congestion management [6] where the selection and participation of generators depend upon both their reactive sensitivity and bid price for up/down regulation. A simplified approach proposed [7] for security-oriented power system operation in which the first contribution of each generator for a particular overloaded line is identified and then based on the relative electric distance (RED) concept of the specified proportion of generation for the specified overload relieving obtained. Combinations of the

generation cost function and congestion management cost function incorporating additional penalty factors [8] have been used to form an objective function which was further solved by a bacterial foraging algorithm. A multi-objective congestion management framework with optimization of congestion management cost, voltage security and dynamic security called augmented ϵ -constraint technique discussed with three objective functions: cost of congestion management, voltage stability margin (VSM), Corrected Transient Energy Margin (CTEM) [9]. Because load is sensitive to a congested branch, simple indices were introduced [10] to effectively and agreeably curtail load in congested lines. These indices provide economic incentives to reduce consumption and increase customer willingness to curtail load during congestion. Mathematical formulation for maximizing social welfare, the overall index for possible load management and generator re-dispatch have been used to obtain these indices. Using real and reactive power sensitivity index as well as balancing energy up/down service settlement and other similar factors Zonal congestion management approach introduced in [11,12]. For a given number of buses, total congestion cost index (TCCI) is minimized [13] based on the calculation of the congestion cost index (CCI). CCI is obtained by the difference in bidding rate and power during the pre-dispatch and re-dispatch era. To determine total congestion cost which further and line-wise congestion cost, security constraint optimal power flow (SCOPF) with and without line flow constraints [14] was used. Whereas the same SCOPF with inputs from an energy management system (EMS) and state estimator applied iteratively [15] with contingency analysis used to estimate the actual cost of congestion. A modified z-bus matrix [16] was applied to evaluate the participation of generators and loads within the use of the transmission network. This modified z-bus matrix formed with the model of generators and loads as constant admittances independently. A novel topological approach to MW-Mile proposed based on the electricity tracing method [17]. The equation for topological generation distribution factor almost like that of generalized generation distribution factor defined, which further used for formation of supplement transmission charge of any generator in the network. For alleviating transmission congestion cost Distributed energy sources (DERs) plays an important role. The usage based transmission cost division method is introduced in which transmission capacity is divided in four capacity to gauge contribution of DERs to transmission cost [18]. A method [19] suggested to allocate the transmission fixed charge based on the modification of the impedance matrix in which real power flow through the line expressed in terms of load current also as electrical distance and voltage injection at the bus. Iteratively one by one impact of every generator selected on the network using the z-bus of the network [20] and transmission congestion

cost determined based on the current decomposition method. A DC power flow based problem [21] using a generation shift factor is defined as a lossless system to work out contribution-based congestion cost allocation methods in a bilateral market.

A fair allocation of transmission congestion cost to the consumer i. e. loads is required in poolco market where generator dispatch pattern changes due to congestion. Congested transmission lines be identified by the dual variable associated with the line flow constraints [14]. Generator re-dispatch method with and without constraints applied. The dual variable associated with the congested line is utilized to find the line allocation factor. With the help of the line allocation factor Line-wise congestion cost (LWCC) of each congested line is evaluated. Further using Bialek's upstream algorithm [17, 22] this TCC is allocated to all generators as well as generators' contribution to supplying the load of the system.

In this paper, TCC and LWCC are derived using the algorithm developed in [23]. Loads share the line flow of congested lines identified by tracking power flow using Bialek's downstream algorithm [17, 22]. LWCC is assigned to the responsible loads according to their proportionate contribution to the congested line.

II. PROBLEM FORMULATION

Here for problem formulation for line wise congestion cost is done by evaluating total congestion cost (TCC). Then after an algorithm is developed to determine load share to the line flows and accordingly load share to the congestion cost.

A. Evaluation of line wise congestion cost (LWCC)

TCC is evaluated by a simple method of generator re-dispatch with and without line flow constraints. In this method, optimal power flow (OPF) is applied with and without line flow constraints. The difference in evaluated costs of both cases is identified as TCC. This TCC is allocated to the lines under congestion. Generalized optimal power flow problem with constraints is formulated as:

$$C(P_{Gi}) = \sum_{i \in N_{gi}} (a_i P_{Gi}^2 + b_i P_{Gi} + C_i) \quad (1)$$

Where,

N_{Gi} = Number of generators in the system

The above optimized problem is with several constraints as follows:

$$P_{Gi} - P_{Di} - P_{loss} = 0 \quad (2)$$

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad (3)$$

$$Q_{Gimin} \leq Q_{Gi} \leq Q_{Gimax} \quad (4)$$

$$V_{imin} \leq V_i \leq V_{imax} \quad (5)$$

$$P_{ij} \leq P_{ijmax} \quad (6)$$

Where,

P_{Gi} = Active power generation in MW at bus i .

P_{Di} = Load in MW at bus i .

P_{Gimin} = Minimum limit of active power generation at bus i .

P_{Gimax} = Maximum limit of active power generation at bus i .

Q_{Gimin} = Minimum limit of reactive power generation at bus i .

Q_{Gimax} = Maximum limit of reactive power generation at bus i .

V_{Gi} = Voltage at bus i

V_{Gimax} = Maximum voltage limit at bus i

V_{Gimin} = Minimum voltage limit at bus i

P_{ij} = Power flow in MW between bus i and j

P_{ijmax} = Power flow limit in MW between bus i and j

In the above constraints (2) is power balance constraint whereas (3) to (6) are inequality constraints.

Following are steps to evaluate TCC and its allocation to the congested line:

1. To obtain total generation cost (TGC) run optimal line flow without and line flow constraints (1).
2. Repeat optimal power flow with line flow constraints, for obtaining a revised total generation cost (TGC').
3. Evaluation of Total congestion cost (TCC) is as:

$$TCC = TGC' - TGC \quad (7)$$

4. To obtain line allocation factor L_{ij} The dual variable associated with the constraint of (6) is utilized [14],

$$L_{ij} = \frac{\mu_{ij}(S_{ijmax} - S_{ij})}{\sum \mu_{ij}(S_{ijmax} - S_{ij})} \quad (8)$$

Where,

μ_{ij} = Dual variable associated with constrained line

S_{ij} = Actual line flow between bus i and j

S_{ijmax} = Maximum line flow limit between bus i and j

5. Line wise congested cost allocated to the constrained line is determined as:

$$LWCC_{ij} = TCC * L_{ij} \quad (9)$$

Equation (9) determines the line-wise congestion cost allocated to the constrained line from that of TCC.

B. Load share to the Line flows and to LWCC

In poolco model of the deregulated power system, it requires to identify loads (customers) liable for line congestion as well as the allocation of congestion cost to those generators is equally important. For fair allocation of congestion cost to the loads, generator share to line flow is to be determined. Here Bialek's down-stream algorithm [17,22] is employed to spot loads participation for the corresponding line flows from congested lines. The subsequent steps demonstrate to determine the sharing of line congestion cost to loads:

1. Convert net real power into lossless line flows $|P_{i-j}|$, after carrying out OPF.

2. Load injection power determined as,

$$P_i = \sum_j |P_{i-j}| + P_{Li} \quad (10)$$

Where j is the load directly supplying to the node i

For $i = 1, 2 \dots n$

n = number of buses

3. The load injection power to line flow ratio is calculated as follows.

$$C_{ija} = \frac{|P_{i-j}|}{P_i} \quad (11)$$

4. An down-stream distribution matrix A_d evaluated whose elements are determined as,

$$[A_d]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -C_{ij} & \text{for } i \text{ supplying to node } j \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

5. Take the inverse of the down-stream distribution matrix evaluated with help of (12).

6. Determine the topological distribution factor of load by,

$$D_{i-l,k}^{d \text{ fact}} = \frac{|P_{i-l}| [A_d^{-1}]_{lk}}{P_i} \quad (13)$$

7. Load share to line flow is determined as,

$$|P_{i-l}| = \sum_{k=1}^n (D_{i-l,k}^{d \text{ fact}}) P_{LK} \quad (14)$$

8. Line congestion cost allocated to loads as,

$$P_{lcc} = \frac{|P_{i-l}|}{P_{ij}} LWCC_{ij} \quad (15)$$

Equation (15) is for distribution of line congestion cost to the loads on the proportional sharing.

III. RESULT AND ANALYSIS

The standard bus test system is used to apply the problem formulation completed in section II for the evaluation of LWCC and load share to LWCC. Results on six-bus and nine-bus systems are first obtained. Results were then obtained by using the same algorithm on the IEEE-14 and IEEE-30 bus systems. This section solely discusses the findings from the IEEE-14 and IEEE-30 bus systems. The results are obtained for TCC, congestion costs for lines and proportion of the loads to TCC as well as proportion of loads to line flows using MATLAB programming.

A. Test results on IEEE-14 bus system

The standard IEEE 14 bus test system is shown in Fig. 1. It consists of twenty lines with five generators connected to different buses and 11 loads in different buses. As explained in the previous section, optimal power flow is performed in this test system without line flow restrictions. During this OPF, all generators will be dispatched with minimal fuel costs as there are no restrictions on line flows. After that, OPF is performed again, with the line flow restrictions of line number 1 (bus 1 and 2) and line number 4 (bus 2 and 4). Due to line flow limitations, the generator dispatch pattern changed and all cheaper generators were not used to dispatch the maximum load in its

capacity. Hence there will be increase in generation cost during the OPF with line flow constraints. This difference of generation costs with and without line flow constraints is TCC.

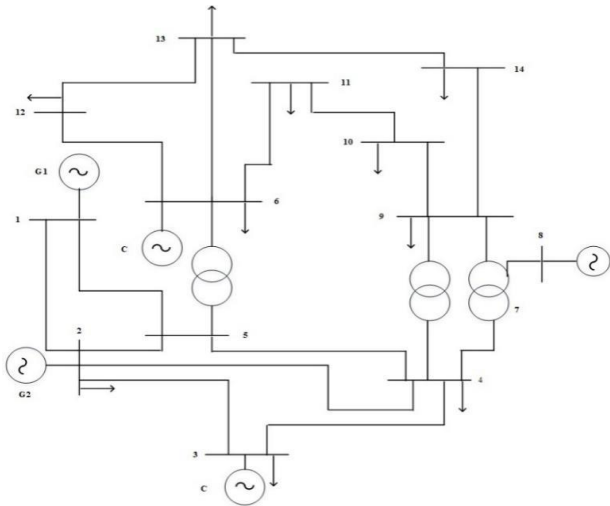


Figure 1. IEEE-14 bus test system

Table 1 shows the line-wise congestion cost assessed with line flow constraints. Lines 1 and 4 are subject to line flow limitations of 110 MW and 40 MW, respectively. As a result of line flow restrictions, the generation cost has increased by 57.88 \$/hr. Equation (9) further allocates this cost to the congested line. The congestion cost allocated to Line 1 is 28.53 \$/hr, and Line 2 is 29.34 \$/hr of total TCC.

TABLE 1. IEEE 14 BUS RESULTS FOR LINE CONGESTION COST

L in e n o	Fb us	T bu s	Pflow w/o constrai nts in MW	Pflow with constra ints in MW	Max line flow in MW	Line constr aints alloca tion factor	Line conge stion cost in \$/hr	Total conge stion cost in \$/hr
1	1	2	129.667	109.94	110	0.493	28.53	57.88
4	2	4	55.3137	38.392	40	0.507	29.34	

Next, identify the share of those loads which are responsible for these congested lines. For this equation (14) is applied in the algorithm discussed in the previous section. Table 2 reflects the results of load share to line flow particularly for line 1 and line 2.

TABLE 2A. IEEE 14 BUS LOAD SHARE TO LINEFLOW UP TO LOAD L10

Fbu s	Tbu s	L2 in MW	L3 in MW	L4 in MW	L5 in MW	L6 in MW	L9 in MW	L10 in MW
1	2	16.6	46.4	25.1	2.14	2.86	5.18	1.60
2	4	0.00	7.38	23.8	0.00	0.00	4.91	1.47

TABLE 2B. IEEE 14 BUS LOAD SHARE TO LINEFLOW FROM LOAD L10 TO L14

Fbus	Tbus	L11 in MW	L12 in MW	L13 in MW	L14 in MW	total of share in MW	Line flow in MW

							by OPF
1	2	0.90	1.57	3.47	2.91	108.90	108.90
2	4	0.00	0.00	0.00	1.94	39.55	39.55

This combined table in 2A and 2B shows the load share in MW of different system loads to line flows 1 and 4. The line flow assessed by the OPF is equal to the total load share by all loads, as can be seen. Bus 3 load is in charge of a maximum load share of 46.4 MW and 7.38 MW. The load share for the load connected to bus 11 is at least 0.9 MW.

TABLE 3A. IEEE 14 BUS LOAD SHARE TO TCC UP TO LOAD L9

fbus	tbus	L2 in \$/hr	L3 in \$/hr	L4 in \$/hr	L5 in \$/hr	L6 in \$/hr	L9 in \$/hr
1	2	4.36	12.17	6.59	0.56	0.75	1.36
2	4	0.00	5.48	17.70	0.00	0.00	3.64

TABLE 3B. IEEE 14 BUS LOAD SHARE TO TCC FROM LOAD L9 TO L14

fbus	tbus	L10 in \$/hr	L11 in \$/hr	L12 in \$/hr	L13 in \$/hr	L14 in \$/hr	Total in \$/hr
1	2	0.42	0.24	0.41	0.91	0.76	28.53
2	4	1.09	0.00	0.00	0.00	1.44	29.35

According to their load share as determined by equation (15) in the preceding section, the line-wise congestion costs assigned to the concern loads are shown in Tables 3A and 3B. According to the generators' fuel costs, the load connected to bus 4 is accountable for the maximum TCC cost, while the load connected to bus 3 is accountable for the maximum load share. The TCC shares the minimum cost with the load connected at bus 11. Line-wise congestion cost, as previously assessed, equals 28.53 \$/hr, the total congestion cost share by all loads to line 1. Similar is that for congestion cost of load share to the line 4. The TCC previously assessed is equal to the sum of the congestion cost share by all loads, as indicated by the final column in table 3B.

B. Test results on IEEE-30 bus system

The standard IEEE-30 bus system is shown in Fig. 2.

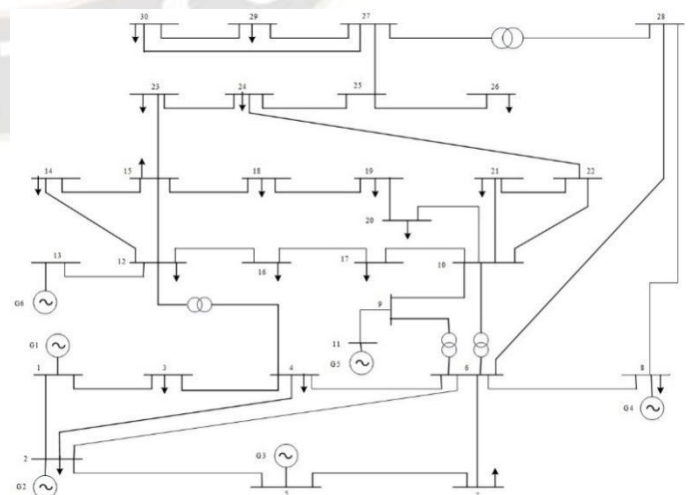


Figure 2. IEEE-30 bus test system

This IEEE-30 bus test system has 21 loads connected to different buses, 41 lines, and six generators. Similar results to those obtained in the IEEE-14 bus system are obtained.

The IEEE-30 bus system's TCC and line congestion costs are assessed, and the findings are shown in Table 3. In order to identify TCC, line flow constraints are applied to lines one (buses 1 and 2) and seven (buses 4 and 6) after OPF is completed without them. There are 100 MW and 40 MW line flow restrictions placed on lines one and seven, respectively. Table 3 displays the TCC allocation to line congestion cost.

This shows that after the introduction of line flow limitations, the power flow through lines one and seven fell to 99.99 MW and 38.39 MW.

TABLE 4. IEEE 30 BUS RESULTS FOR LINE CONGESTION COST

Line no	Fbus	Tbus	Pflow w/o constraints in MW	Pflow with constraints in MW	Max line flow in MW	Line constraints allocation factor	Line congestion cost in \$/hr	Total congestion cost in \$/hr
1	1	2	139.115	99.997	100	0.051	8.16	159.4
7	4	6	55.314	38.393	40	0.948	151.32	

Due to congestion, the calculated TCC is \$159.48/hr. Using the line allocation factors of each congested line, this TCC was further distributed to 8.16 /hr for line 1 and 151.32 \$/hr for line 7. TCC for the IEEE-30 bus system is 159.4 \$/hr

TABLE 5A. IEEE 14 BUS LOAD SHARE TO LINEFLOW UP TO LOAD L10

fbus	tbus	L2	L3	L4	L5	L7	L8	L10
1	2	15.66	0.00	2.08	29.09	10.65	4.34	2.20
4	6	0.00	0.00	0.00	0.00	8.74	4.16	2.11

TABLE 5B. IEEE 14 BUS LOAD SHARE TO LINEFLOW FROM LOAD L12 TO L19

fbus	tbus	L12	L14	L15	L16	L17	L18	L19
1	2	3.08	1.71	2.27	0.97	3.28	0.89	3.48
4	6	0.00	0.00	0.00	0.00	2.78	0.00	2.92

TABLE 5C. IEEE 14 BUS LOAD SHARE TO LINEFLOW FROM LOAD L20 TO L30

fbus	tbus	L20	L21	L23	L24	L26	L29	L30	total
1	2	0.84	6.68	0.89	3.38	1.63	1.11	4.92	99.14
4	6	0.81	6.39	0.00	3.06	1.56	1.06	4.71	38.30

This combined table in Figures 5A, 5B and 5C shows the load share in MW of various system loads on Line Flows 1 and 7. The total line flow share of all loads on Line 1 and 7 corresponds to the line flow achieved by the OPF. The bus connected to load 5 shares the maximum load of 29.09 MW, but only comes from line 1. The load on bus 3 does not share any load with the line flow from lines 1 and 7.

TABLE 6A. IEEE 30 BUS LOAD SHARE TO TCC UP TO LOAD L10

fbus	tbus	L2	L3	L4	L5	L7	L8	L10
1	2	1.29	0.00	0.17	2.39	0.88	0.36	0.18
4	6	0.00	0.00	0.00	0.00	34.53	16.42	8.34

TABLE 6B. IEEE 30 BUS LOAD SHARE TO TCC FROM LOAD L12 TO L20

fbus	tbus	L12	L14	L15	L16	L17	L18	L19	L20
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1	2	0.25	0.14	0.19	0.08	0.27	0.07	0.29	0.07
4	6	0.00	0.00	0.00	0.00	10.99	0.00	11.56	3.18

TABLE 6C. IEEE 30 BUS LOAD SHARE TO TCC FROM LOAD L21 TO L30

fbus	tbus	L21	L23	L24	L26	L29	L30	Total
1	2	0.55	0.07	0.28	0.13	0.09	0.40	8.15
4	6	25.26	0.00	12.09	6.15	4.20	18.60	151.32

Similar outcomes to those of the IEEE-14 bus system are obtained when allocating loads to line-wise congestion costs for the IEEE-30 bus system. Tables 6A, 6B, and 6C tabulate these findings collectively. While the load connected to bus 3 does not share any congestion costs, the load connected to bus 7 shares the maximum cost of 34.53 \$/hr. Because line 7's line-wise congestion cost is significantly higher than line 1's, loads on line 7 share higher congestion costs.

IV. CONCLUSION

This paper suggests a way to determine the total congestion cost (TCC). A dual variable linked to these lines aids in allocating this TCC to congested lines. Bialek's downstream algorithm for tracking the flow of electricity is also used to assign these line congestion costs to loads. For the IEEE-14 and IEEE-30 bus test systems, results are obtained. According to the self-validated results, TCC is equal to the sum of the individual congestion costs for each line and the individual congestion costs shared by all loads.

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