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A Study on Fuzzy Neighbourly Edge Irregular Graphs

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Abstract— A induced sub graph $< H >$ of a intuitionistic fuzzy graph $G(V,E)$ is said to be a matching if every vertex of $G(V,E)$ is incident with at most a vertex in $< H >$.A induced fuzzy sub graph $< H >$ of a intuitionistic fuzzy graph $G(V,E)$ is said to be a perfect matching if every vertex of $G(V,E)$ is incident with exactly a vertex in $< H >$. We shall define a perfect matching in intuitionistic fuzzy graphs in this paper. We also look into the matching number's bounds and characteristics in a variety of intuitionistic fuzzy graphs

Keywords- Fuzzy Neighbourly Edge Irregular fuzzy graphs, support neighbourly irregular graphs, Support Fuzzy Neighbourly Edge Irregular graphs.

I. INTRODUCTION

We consider finite, simple connected graphs throughout this paper. Consider a graph G that has m edges and n vertices. The Gare vertex and edge sets are represented by the notations V (G) and E(G), respectively. The degree of a vertex v ∈ V (G) is represented by dG(v) or simply d(v), which is the number of vertices that are adjacent to v. S. Gnaana Bhragsam and S.K. Ayyaswamy introduced and researched the idea of neighboringly irregular graphs[2]. Fuzzy Neighbourly Edge Irregular fuzzy graphs were first conceptualized by N.R. Santhi Mahaheswari and C. Sekar[9]. The notion of vertex support and neighboring irregular graphs' support has been presented and examined by Selvam Avadayappan and M. and R.Sinthu[11].

The degree of an edge e = (u,v) as the number of edges which have a common vertex with the edge e.(i.e)deg(e) = deg(u) + deg(v) − 2[5]. The distance between two edges e1 = (u1,v1) and e2 = (u2,v2) is defined as ed(e1,e2)=0 min{d(u1,u2),d(u1,v2),d(v1,u2),d(v1,v2)}. If ed(e1,e2) = 0, these edges are neighbour edges[4]. The purpose of this paper is to introduce a new family of irregular graphs based on distance property in edge sense.

This is the background to introduce support Fuzzy Neighbourly Edge Irregular graphs and we have discussed some of its properties.

PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in the paper.

Definition 2.1. A graph G is said to be neighbourly irregular if no two adjacent vertices of G have the same degree.

Definition 2.2. Let G : (σ,μ) be a connected fuzzy graph on G∗(V,E). Then G is said to be Fuzzy Neighbourly Edge Irregular fuzzy graph if every pair of adjacent edges having distinct degrees.

Definition 2.3. The support sG(v) or simply s(v) of a vertex v is the sum of degrees of its neighbours. That is, s(v) = Pu∈N(v) d(u).

Definition 2.4. *A* connected graph is said to be support neighbourly irregular (or simply SNI), if no two vertices having same support are adjacent.

Definition 2.5. Let be G be a graph. For any two distinct vertices u and v in G, u is pairable with v if N[u] = N[v] in G. A vertex in G is called a pairable vertex if it is pairable with a vertex in G.

Definition 2.6. Let G be a graph. A full vertex of G is a vertex in G which is adjacent to all other vertices of G.

Definition 2.7. A simple graph G(V,E) is Fuzzy Neighbourly Edge Irregular if no two adjacent edges of G have the same edge degree.

REVIEW CRITERIA

In this section, we introduce Support Fuzzy Neighbourly Edge Irregular graphs and study some properties of these graphs.

Definition 3.1. *The support* $s_G(e)$ *or simply* $s(e)$ *of an edge e is the sum of edge degrees of its neighbour edges. That is,* $s(e)$ = P$_{e_i \in N(e)}$ $d(e_i)$.

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Definition 3.2. *A simple graph G(V,E) is Support Fuzzy Neighbourly Edge Irregular graph(or simply SNEI) if no two edges of G having same support are adjacent. A graph H proving the existence of SNEI graphs is shown in Figure* 1.

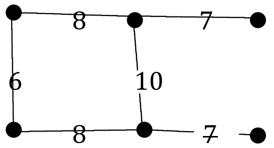


Figure 1

Fact 3.3. Regular graphs, edge regular graphs, pairable graphs and paths (Pn,n≠ 6) are not SNEI graphs.

Fact 3.4. *Not all Support Fuzzy Neighbourly Edge Irregular graphs are NEI graphs. Not all NEI graphs are SNEI. For example, P_3 is NEI but not SNEI and P_6 is SNEI but not NEI.*

Fact 3.5. *Let G be a SNEI graph. Then for any two vertices u and v in V (G), N(u) ≠N(v).*

Fact 3.6. *Any graph with more than one full vertex is not a SNEI graph.*

Fact 3.7. *If G is a SNEI graph, then the set of all extreme vertices is independent in G.*

Fact 3.8. *If G is a SNEI graph, there is no P_4 such that external vertices of degree 1 and one internal vertex of degree 2, For if there is a P_4 (say uvwx) such that d(u) = d(x) = 1 and d(w) = 2, then s(uv) = s(vw).*

Result 3.9. *If G is a SNEI graph, then there is no P_5 with external vertices of same degree and internal vertices of degree 2.*

Note 3.10. *For any edge e ∈ E(G), support of* $e=s(e)=s(uv)=d(u)^2 + d(v)^2 + s(u) + s(v) - 4d(u) - 4d(v) + 4$.

The following theorem proves a necessary and sufficient condition for a graph to be SNEI graph.

Theorem 3.11. *A graph G is a SNEI graph if and only if for any two adjacent edges uv and vw , then* $d(u)^2 - d(w)^2 - 4(d(u) - d(w)) \neq s(w) - s(u)$.

Proof. Let *G* be a SNEI graph. Then no two adjacent edges have same support. If possible $d(u)^2 - d(w)^2 - 4(d(u) - d(w)) = s(w) - s(u)$ for some adjacent edges *uv*and *vw*, then $d(u)^2 - 4d(u) + s(u) = d(w)^2 - 4d(w) + s(w) \Longrightarrow s(uv) = s(vw)$, which is a contradiction.

Conversely, suppose,

$d(u)^2 - d(w)^2 - 4(d(u) - d(w)) \neq s(w) - s(u$
and vw. Then $d(u)^2$ $4d(u) + s(u) \neq d($)for any two adjacent edges *uv*$-w)^2 -$ $4d(w) + s(w) \Longrightarrow s(uv) \neq s(vw)$, for any two adjacent edges *uv*and *vw*. Hence *G* is a SNEI graph.

Corollary 3.12. *Let G be a SNEI graph. If there are two adjacentedges(say uv and vw) with same edge degree, then s(u) ≠s(w).*

Proof. Let *G* be a SNEI graph. Let *uv*and *vw*be two adjacent edges with *ed*(*uv*) = *ed*(*vw*). Then *d*(*u*) = *d*(*w*). By above theo 3.10, $d(u)^2 - 4d(u) + s(u) \neq d(w)^2 - 4d(w) + s(w)$, which implies *s*(*u*) ≠*s*(*w*). □

Corollary 3.13. *Let G be a SNEI graph. If there are two adjacent edges (say uv and vw) with s(u) = s(w) in G, then* $d(u)^2 - d(w)^2 = 4(d(u) - d(w))$.

Corollary 3.14. *Let G be a SNEI graph. If there are no two adjacent edges (say uv and vw) with s(u) = s(w) and d(u) = d(w) in G.*

Corollary 3.15. *Let G be a SNEI graph. Then there is no P_3 (say uvw) such that d(u) = d(w) and s(u) = s(w).*

Corollary 3.16. *Let G be a SNEI graph. Let v be any vertex. Then for any two vertices v_i and $v_j$such that $d(v_i) + k = d(v_j)$, $s(v_i) + m \neq s(v_j)$ where*

$$m = \begin{cases} 2kd(vi) - 3k + k2 \text{ if } vi \text{ and } vj \text{are not adjacent} \\ 2kd(vi) - 4k + k2 \text{ if } vi \text{ and } vj \text{ are adjacent} \end{cases}$$

Yousef alavi[1] *proved that for every positive integer n = 36 ,5,7, there exists a highly irregular graph of order n.*

Theorem 3.17. *For every positive even integer n = 2k,k≥ 3, there exists a SNEI graph of order n and it is denoted by $SNEI_{(n)}$.*

Theorem 3.18. *There exist SNEI graph of order n, except 3,5 and 7.*

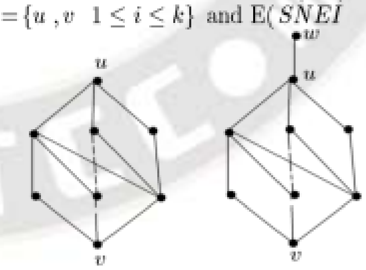
Proof. It is enough to prove there is a SNEI graph of order *n* where *n* ≥ 8.

$(n-2)$ i i For $(n-2)$ *n*V(*SNEI,i j,* $j \leq k\}$. In $SNEI_{(n-2)}$, u and v introduce two new verticesand join the edges $uu_j$and $vv_j$for 1 ≤ *j* ≤ *k*. We obtain a *SNEI* graph of order *n*. Note further that introduce a pendent edge at either *u* or *v*, we will get a SNEI graph of order *n*+1 = 2*k*+2+1 ≥ 9. Therefore there exists a SNEI graph of order *n* ≥ 8.

$1 \leq j \leq k$ 1, p and q , where $1 \leq i \leq k-(j-1)$
and p q , $1 \leq j \leq k$ i k j

Figure 2 illustrates theorem 3.18 for *n* = 8 and *n* = 9

is even, let us first construct $SNEI_{(n-2)}$, $n-2 = 2k, k \geq 3$. Let
)={u ,v $1 \leq i \leq k$} and E($SNEI$)={u v $1 \leq i \leq k, i \leq$



Result 3.19. *We can construct a SNEI graph G_1^* from $SNEI_{(n)}$, Suppose n = 2d. Let V($SNEI_{(n)}$)= U ∪ V where U = $u_i$and V = v_i,1 ≤ i≤ d.*

By introducing a new vertex u and joining the edges uu_1,uu_2,...,uu_d. The resulting graph G_1^* is also SNEI graph. Further we can construct a SNEI graph G_2^* from G_1^* by introducing a pendant edge at u.

Result 3.20. *We can construct a SNEI graph $G_3^*$from two copies of $SNEI_{(n)}$, say G_1 and G_2. Let V(G_1)= U_1∪V_1 where U_1 = u_{1i} and V_1 = v_{1i},1 ≤ i≤ d and V(G_2)= U_2∪V_2 where U_2 = u_{2i} and*

$V_2 = v_{2i}, 1 \le i \le d$. By introducing two new vertices u and v and joining the edges $u_{1j}u, u_{2j}v$ and uv. The resulting graph G_3^* is also SNEI graph.

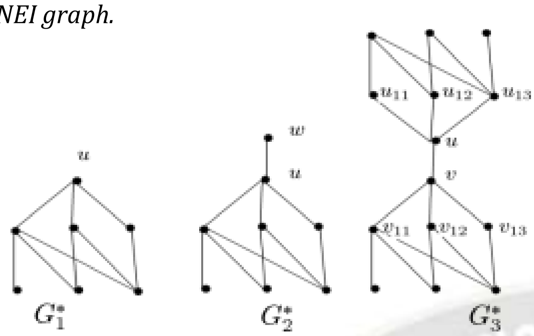


Figure 3

Result 3.21. *We can construct a SNEI graph G^*_4 from $SNEI_{(n_1)}$ and $SNEI_{(n_2)}$ where $n_1 \neq n_2$ of order $n_1 + n_2 + 3$. Suppose $n_1 = 2d$ and $n_2 = 2k$. Let $V(SNEI_{(n_1)}) = U_1 \cup V_1$ where $U_1 = u_{1i}$ and $V_1 = v_{1i}, 1 \le i \le d$ and $V(SNEI_{(n_2)}) = U_2 \cup V_2$ where $U_2 = u_{2j}$ and $V_2 = v_{2j}, 1 \le j \le k$. By introducing three new vertices u,v and w and joining the edges $u_{1i}u$, $u_{2j}w$, uv and vw, $1 \le i \le d$ and $1 \le j \le k$. The resulting graph G_4^* is also a SNEI graph.*

Result 3.22. *We can construct a SNEI graph G_5^* from $SNEI_{(n)}$ by introducing x_j and y_j for each u_j and v_j, $1 \le j \le k$ respectively and for each x_j, $1 \le j \le k -$ 1, s_{ji} and t_{ji} where $1 \le i \le k-(j-1)$ and for each y_j, and joining the edges u_jx_j, v_jy_j, $x_js_{ji}, y_jp_{ji}, s_{ji}t_{ji}$ and $i \le$, $1 \le \; \le \; -(\;-1)$ $d \le k$ and attach pendant vertices at x_d and y_d. The resulting graph G_5^* is also a SNEI graph.*

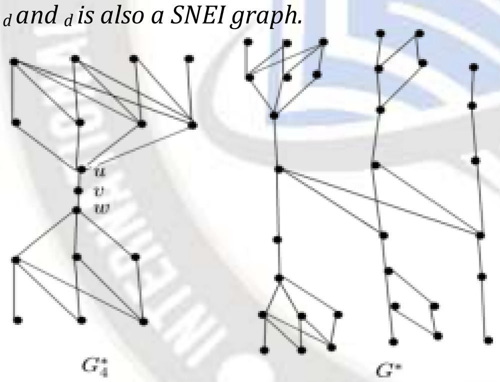


Figure 4 5

Result 3.23. *Let G be a triangle free SNEI graph with diameter 3. Let v be any vertex of the graph G having degree d, adjacent to the vertices $v_1, v_2, \ldots, v_d$ and u_{ji} be the vertices adjacent to each v_j, $1 \le j \le d$. If (a) $s(v_ju_{ij}) \neq s(vv_j)+2(d(v)-1)$ and (b) $Ped(v_ju_{ij}) \neq ed(vv_j)-(d(v)-1)$ for each i and j, then we can construct a SNEI graph G^* from G by introducing a pendant edge at v. For, $s(v_ju_{ij})$ in $G^* = s(v_ju_{ij})$ in G+1 and $s(vv_j)$ in $G^* = s(vv_j)$ in G $+2d(v)-1$ and by (a) $\Longrightarrow s(v_ju_{ij})$ … $G^* = \sum ed(v\,u\;)$ in $G + \sum ed(vv_k) +$ … G $+d(v) = ed(vv_j) + \sum_{k \neq j}$ $ed(vv_k)+d(v)-1$ and by (b), $s(vv_j) \neq s(vw)$.*

Theorem 3.24. *Every cycle C_n of order $n \ge 4$ is an induced subgraph of a SNEI graph of order at most $n + 2\lfloor \frac{n}{4} \rfloor + 2$*

Proof. Let C_n be a cycle of order n, $n \ge 4$. Let $v_1, v_2, \ldots, v_n$ be the vertices of C_n. Suppose n is even, introduce new pendant vertices v_{i1}, v_{i2} at u_{4i+1}, u_{4i+2} respectively, $0 \le i \le \lceil \frac{n}{4} \rceil - 1$. The resulting graph is a SNEI graph of order $n + 2\lceil \frac{n}{4} \rceil$. Suppose n is odd, introduce new pendent vertices v_{i1}, v_{i2} at u_1, u_2 and P_3 at u_3 and pendant vertices $v_{(i+1)1}, v_{(i+2)2}$ at u_{4i+2}, u_{4i+3} respectively, 1 1. The resulting graph is also a SNEI graph of order $n + 2 \;_{-4} + 2$.

Figure 5 illustrates theorem 3.24 for n = 12 and n = 11

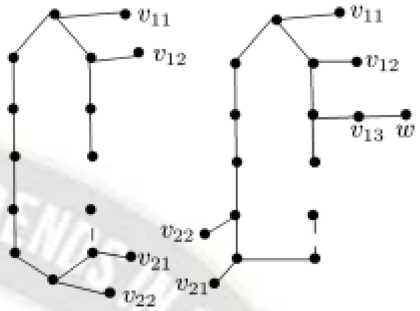


Figure 5

Theorem 3.25. *Every complete graph of order $n \ge 3$ is an induced subgraph of SNEI graph of order $n^2 + 2n$.*

Proof. Let G be a complete graph of order $n \ge 3$. Let $u_1, u_2, \ldots, u_n$ be the vertices of G. For each u_i, $1 \le i \le n$, we introduce new vertices v_{ij} and w_{ij}, $1 \le i \le n, 1 \le j \le i$. The vertices u_i, v_{ij}, w_{ij}, $1 \le i \le n, 1 \le j \le i$ constitute the vertex set for the desire graph H. For the edge set, along with the edges of G, join the edges (a) u_iv_{ij}, $1 \le i \le n, 1 \le j \le i$ and (b) $w_{ij}v_{ik}$, $1 \le i \le n, 1 \le j \le i, 1 \le k \le j$. The resulting graph H is a SNEI graph and it contains G as an induced subgraph.

Figure 6 illustrates theorem 3.25 for n = 3

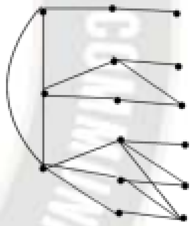


Figure 6

Theorem 3.26. *Every complete bipartite graph $K_{r,r}$ is an induced subgraph of a SNEI graph of order 4r.*

Proof. Let $u_1, u_2, \ldots, u_r$ and $v_1, v_2, \ldots, v_r$ be two partites of $K_{r,r}$. Let $u_1', u_2', \ldots, u_r'$ and $v_1', v_2', \ldots, v_r'$ be the newly added vertices. Construct the graph with vertex set $V(H) = \{u_i, v_i, u^0_i, v_i^0\}$ and edge set $E(H) = E(K_{r,r}) \cup \{u_iu^0_j$ and $v_iv_j^0$, $1 \le i \le r$, $i \le j \le r\}$. It is obvious that G is an induced subgraph of H. From our construction of H it is clear that H is a SNEI graph of order 4r.

Figure 7 illustrates theorem 3.26 for r = 3

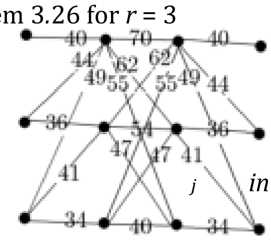


Figure 7

Theorem 3.27. *Any graph G of order $n \ge 3$ is an induced subgraph of a SNEI graph.*

Proof. Let G be a graph of order $n \geq 3$. Let G^0 be another copy of G where $V(G)=\{u_1,u_2,...,u_n\}$ and $V(G^0)=\{v_1,v_2,...,v_n\}$, u_i corresponds to v_i, $1 \leq i \leq n$. Let $u_1,u_2,...,u_n$ and $v'_1, v'_2, \ldots, v'_n$ be the newly added vertices. Construct a graph H with the vertex set $V(H)=\{u_i, v_i, u'_i, v'_i\}(1 \leq i \leq n)$ and $E(G)=E(G) \cup E(G^0) \cup \{u_iv_j: u_iu_j \notin E(G), 1 \leq i \leq n, 1 \leq i \leq n$ and $u_iv_i, 1 \leq i \leq n\} \cup \{u_iu'_j, v_iv'_j, 1 \leq i \leq n, 1 \leq i \leq n\}$. It is obvious that G is an induced subgraph of H. From our construction of H it is clear that H is a SNEI graph of order $4n$.

Figure 8 illustrates theorem 3.27 for $n = 3$

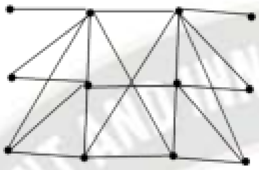


Figure 8

Result 3.28. *For any positive integer $d \geq 3$, we can construct a SNEI graph with maximum degree $2d$ and order $3d + 4$.*

Proof. Let $d \geq 3$ be any positive integer. Let $V(G)=\{v_i,u_i,w_i,u,v,w,x, 1 \leq i \leq d\}$, be the vertex set of the required graph G and the edge set, $E(G) = \{v_iw_j\} \cup$ $u_iw_j, 1 \leq i \leq d, \leq j \leq d\} \cup \{v_iv, 1 \leq i \leq d\} \cup \{u_iu, 1 \leq i \leq d - 1$
$\{ux, vx, wx, u_dw\}$. The resulting graph is a SNEI graph of order $3d + 4$ and maximum degree d.

Figure 9 illustrates theorem 3.28 for $d = 6$

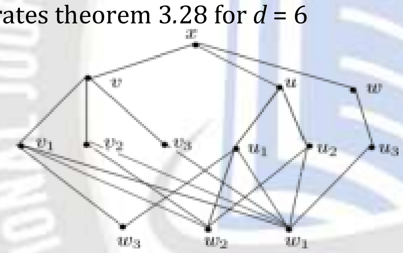


Figure 9

CONCLUSION

Based on the degree of edge sense, we deal with irregular graphs in this paper, which we refer to as support fuzzy neighborly edge irregular graphs. A number of techniques for creating SNEI graphs from other graphs have been explored, along with the necessary and sufficient conditions for a graph to be SNEI.

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