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DISCOVERY OF DIFFERENT GRAPHS INTO PRIME GRAPHS

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Abstract— A collection that is edge disjoint and all of its edges belong to is referred to as a decomposition of. It is referred to as a prime decomposition of if every graph is a prime graph. . The minimum cardinality among the minimal ID set is called an ID number of $G(V,E)$ and it is denoted by $\gamma'(G)$. Further study the inverse domination number $\gamma'(G)$. of complete and complete bipartite IFG some results and also identify some bounds of the ID-number are investigated. The ID-number of some standard operations join of two IFGs and Cartesian product of two IFG are investigated. Some results like $G(V,E)$ is an IFG without isolated vertices, then $\gamma(G) \leq \gamma'(G)$. The IFG $G(V,E)$ be a complete IFG, then $\gamma_{if}'(G) = |u|$, here u is the vertex having the second minimum cardinality in $G(V,E)$ .AnIFG $G(A,B)$ be a complete bipartite IFG, then $\gamma_{if}'(G) = |u| + |v|$, here u and v are the vertex having the second minimum cardinality in vertices set V_1 *and* V_2 in $G(V,E)$.

Keywords- prime graphs, decomposition, prime number, and prime decomposition

I. INTRODUCTION

A graph is a set of unordered pairs (2-element pairs) with a non-empty finite set (referred to as the set of vertices or nodes of G) and a well-ordered pair. The full n-vertice graph, represented by, is an n-vertice graph in which each pair of vertices is connected by an edge. The empty graph on n vertices is a graph on n vertices without any edges, and it is represented as.If and only if and is a subgraph of a graph.The number of vertices in a graph determines its order. The number of edges on G represents its size. The number of edges that coincide with a node, denoted by, is its degree.A graph such that and is a subgraph H of G. The subgraph of G induced by W, represented as, is the graph such that for a graph and a subset.

In this paper, we presented the idea of an inverse dominating set (ID-Set) and inverse domination number (ID-number) of aIFG. Further investigate the inverse domination number of complete and complete bipartite IFG some results and also some bounds of the ID-number

If a path connects each pair of vertices in a graph, the graph is said to be linked. We declare G to be a tree if every pair of vertices has exactly one path linking them. To put it another way, a tree is a n − 1 connected network. A connected graph with n vertices that has a maximum degree of 2 for each vertex is called a pathgraph. A connected graph with n vertices in which each vertex has degree 2 is called a cycle graph.A graph with n vertices that has every vertex next to every other vertex is said to be complete. Conversely, an independent set is a collection of graph vertices where no two vertices are next to each other.

A graph is said to be bipartite if there is a partition of V such that each edge of G joins a vertex in A with a vertex in B. Alternatively said, if A and B are independent sets, then G is said to be bipartite. Another notation for the bipartite graph is.

By connecting the star graph at each path vertex, the brush graph may be created using the path graph. for example.

The prime decomposition and prime decomposition number of a graph are defined in this study. Additionally, look into some product graph bounds, such as composition, Cartesian product, etc.

1. BASIC DEFINITIONS

An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where $V = \{v_1, v_2, \ldots, v_n\}$ suchthat $\mu_1 : V \to [0,1],\ \gamma_1 : V \to [0,1]$ denotethedegreeofmembershipand non-member ship of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V, (i = 1,2,..n)$. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1],\ and\ \gamma_2 : V \times V \to [0,1]$ aresuchthat

$$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j) ------(2.1)$$

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$\gamma_2(v_i, v_j) \le \gamma_1(v_i) \vee \gamma_1(v_j) - - - - - (2.2)$ and
$0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1 - - - - - (2.3)$

An arc (v_i, v_j) of an IFG G is called an strong arc if
$\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$
$\gamma_2(v_i, v_j) = \gamma_1(v_i) \wedge \gamma_1(v_j) - - - - (2.4)$.

Let G =(V,E) be an IFG. The vertex cardinality of Gisdefinedtobe $|v_i| = \left| \sum_{v_i \in V} \left[\frac{(1 + \mu_1(v_i) - \gamma_1(v_i))}{2} \right] \right|$ for all $v_i \in V, (i = 1,2,..n)$.

Let G=(V, E) beanIFG. A set $D \subseteq V$ is said to be a dominating set of G if every $v \in V - D$ there exist $u \in D$ such that u dominates v.

An Intuitionistic fuzzy dominating set D of an IFG, G is called minimal dominating set of G if every node $u \in D$, $D - \{u\}$ is not a dominating set in G. An Intuitionistic fuzzy domination number $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

2. INVERSE DOMINATION

In this segment, we investigate some bounds and properties of the inverse domination number in Intuitionisticfuzzy graphs

Definition 3.1. Let $D \subset V$ be a $\gamma(G)$ set of the IFG $G(V,E)$. A dominating set D' contained in $V - D$ is called an ID set of $G(V,E)$ with respect to D. The minimum cardinality among the minimal ID set is called an ID number of $G(V,E)$ and it is denoted by $\gamma'(G)$.

Theorem 3.1: The $G(V,E)$ is anIFG without isolated vertices, then $\gamma(G) \le \gamma'(G)$.

Proof: Let $G(V,E)$ be a IFG without isolated vertices and $D \& D'$ are the $\gamma(G)$, $\gamma'(G)$ sets of $G(V,E)$ respectively. Clearly every $\gamma'(G)$ set D' is a $\gamma(G)$ set D of $G(A,B)$, but D' is not a γ dominating set of $G(V,E)$.Therefore we get

$|D| \le |D'| \Rightarrow \gamma(G) \le \gamma'(G) - - - - (3.1.1)$

Hence proved.

Theorem 3.2: The IFG $G(V,E)$ be a complete IFG, then $\gamma_{if}'(G) = |u|$, here u is the vertex having the second minimum cardinality in $G(V,E)$.

Proof: Let $G(V,E)$ be a complete IFG and v, u are vertex having the least two cardinality among the vertices in $G(V,E)$. In $G(V,E)$ there is a strong edge between every pair of vertices. Since $G(V,E)$ is a complete graph. Clearly $\gamma(G) = |v|$. Since the vertex $v \in V$ is adjacent to all vertices in $V - \{v\}$. The hesitancy sub graph induced by the vertices $V - \{v\}$ is also a complete IFG. Therefore there is a vertex $u \in V - \{v\}$ is adjacent to all other vertices. This implies {u} is a minimal ID set of $G(V,E)$. Hence the ID number $\gamma_{if}'(G) = |u|$.

Example 3.1

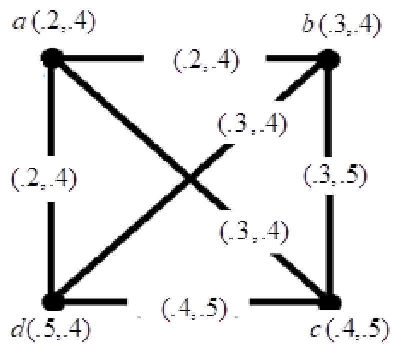

Figure 3.1: Complete IFG $G(V,E)$

In the Complete IFG $G(V,E)$, degree of the vertices are $|a| = 0.4, |b| = 0.45, |c| = 0.45$ and $|d| = 0.55$.An inverse dominating set and inverse domination number of the complete IFG $G(V,E)$ is $D = \{b\} or \{c\} and\ \gamma_{if}'(G) = |b| = 0.45.$

Theorem 3.3: AnIFG $G(A,B)$ be a complete bipartite IFG, then $\gamma_{if}'(G) = |u| + |v|$, here u and v are the vertex having the second minimum cardinality in vertices set V_1 and V_2 in $G(V,E)$.

Proof: Let $G(V,E)$ be a complete bipartite IFG. This implies the vertices of $G(V,E)$ are partition in to V_1 and V_2. Let x and y are the vertex having the minimum cardinality in vertices set V_1 and V_2 in $G(V,E)$. Therefore x dominates vertices in V_1 and y dominates the vertices in V_2. This implies $\{x, y\}$ is a $\gamma(G)$ set of $G(V,E)$. Note that $V - \{x, y\}$ is the complete bipartite IFG. Let u and v are the vertex having the second minimum cardinality in vertices set V_1 and V_2 in $G(V,E)$. Therefore the set $\{u,v\}$ is the dominating set of the graph induced by $< V - \{x, y\} >$, since $V - \{x, y\}$ is the complete bipartite IFG. This implies $\{u,v\}$ is the $\gamma_{if}'(G)$ set of $G(V,E)$. Hence we get $\gamma_{if}'(G) = |u| + |v|$.

Example 3.2

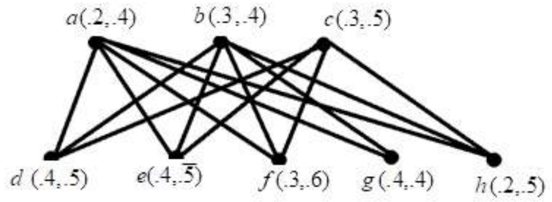

Figure 3.2: Complete Bipartite IFG $G(V,E)$

In the Complete IFG $G(V,E)$, degree of the vertices in V_1 and V_2 are $|a| = .4, |b| = .45, |c| = .4,\ |d| = .45,\ |e| = .45, |f| = .35, |g| = .5$ and $|h| = .35$. The total dominating set $T = \{a, f\} or \{c, f\} or \{a, h\} or \{c, h\}$ and the total domination number of the complete IFG $G(V,E)$ is $\gamma_{if}{}'(G) = |c| + |h| = 0.75$.

Theorem 3.4: Let $G(V,E)$ be a IFG and $\gamma(G), \gamma'(G)$ are domination number and ID number of $G(V,E)$. Then we get $\gamma'_{if}(G) \le \frac{O(G)}{2}$.

Proof: Let $G(V,E)$ be a IFG and $\gamma(G), \gamma'_{if}(G)$ are domination number and inverse domination number of $G(V,E)$. We know that the domination number $\gamma(G)$ such that $\gamma(G) \le \frac{O(G)}{2}$. Let D' is a $\gamma'_{if}(G)$ set of $G(V,E)$. Therefore $\gamma'_{if}(G)$ set is a dominating set in $< V - D >$. This implies $\{V - D - D'\}$ is also an ID set of $< V - D >$. Here $\gamma(G) \le \frac{O(G)}{2}$ therefore we get

$$|D'| = \min\{\ |D'|, |V - D|\ \}$$

$$= \min\{\ |D'|, |V - D|\ \} \quad --------- (3.4.1)$$

$$= Min\{\ |D'|, |V| - |D|\ \}$$

$$\gamma'_{if}(G) \le \frac{O(G)}{2}.$$

Theorem 3.5: Let $G(V,E)$ be a IFG and $\gamma(G), \gamma'(G)$ are domination number and ID number of $G(V,E)$. Then we get $\gamma'_{if}(G) + \gamma(G) \le O(G) - \delta_N(G)$.

Proof: Let $G(V,E)$ be a IFG and $\gamma(G), \gamma'_{if}(G)$ are domination number and ID number of $G(V,E)$. Let u is the vertex having minimum degree in the graph $G(V,E)$, that is $\delta_N(G) = d_N(u)$. Note that the set $\{(V - D) - N(u)\}$ is an ID set not a minimal ID set of $G(A,B)$. Therefore the ID set $D' \subseteq \{(V - D) - N(u)\}$. This implies the ID number is $|D'|$. Hence we get

$$|D'| \le |\{(V - D) - N(u)\}|$$

$$|D'| \le |V| - |D| - |N(u)| \quad ------ (3.5.1)$$

$$\gamma'(G) \le O(G) - \gamma(G) - \delta_N(G)$$

$$\gamma(G) + \gamma'_{if}(G) \le O(G) - \delta_N(G)$$

Hence proved.

Theorem 3.6: Let $G(V,E)$ be a IFG and D is a dominating set of $G(V,E)$. The sub graph $< V - D >$ does not contain any strong edges, Then $\gamma'_{if}(G) + \gamma(G) = O(G)$

Proof: Let $G(V,E)$ be a IFG and $\gamma(G), \gamma'_{if}(G)$ are domination number and ID number of $G(V,E)$. The sub graph $< V - D >$ does not contain any strong edges. Therefore the set $D' = \{(V - D)\}$ is an ID set of the sub graph $< V - D >$. This implies the ID number is $|D'|$. Hence we get

$$|D'| = |\{(V - D)\}|$$

$$|D'| = |V| - |D| \quad --------- (3.6.1)$$

$$\gamma'(G) = O(G) - \gamma(G)$$

$$\gamma(G) + \gamma'(G) = O(G)$$

Theorem 3.7: The subsets $D_1 \subseteq V_1$ and $D_2 \subseteq V_2$ are the dominating sets of the IFG's $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Then the ID number

$$\gamma'(G_1 + G_2) = \max\{|D_1|, |D_2|\}.$$

Proof: Let subsets $D_1 \subseteq V_1$ and $D_2 \subseteq V_2$ are the dominating sets of the IFG $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. In $G_1 + G_2$ there is a strong edge between every pair of vertices in V_1 and V_2. This implies $\gamma(G_1 + G_2) = \min\{|D_1|, |D_2|\}$. Suppose D_1 is γ set of $G_1 + G_2$. Now we prove D_2 is an ID set of $G_1 + G_2$. That is to prove D_2 is a dominating set of $< V - D_1 >$. If $u \in V - D_1$ there is a vertex $v \in D_2$. Suppose $u \in V_2$ the set D_2 dominates u since D_2 is a minimal dominating set of $G_2(V_2, E_2)$. Suppose $u \in V_1$, by the definion of $G_1 + G_2$ we get

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$$(\mu_{12}+\mu_{22})(uv)=\mu_{11}(u)\wedge\mu_{21}(v)$$
$$(\eta_{12}+\eta_{22})(uv)=\eta_{11}(u)\wedge\eta_{21}(v)----(3.7.1)$$
$$(\beta_{12}+\beta_{22})(uv)=\beta_{11}(u)\wedge\beta_{21}(v)$$

This implies there is a strong edge between $u\in V_1$ and $v\in D_2$. Clearly D_2 dominates every vertices in $<V-D_1>$. Here D_2 is a minimal dominating set of $<V-D_1>$. Hence D_2 is a minimal ID set of $<V-D_1>$.

Similarly, we prove if D_2 is γ set of G_1+G_2. D_1 is an ID set of G_1+G_2.Hence the ID number of G_1+G_2 is $\gamma'(G_1+G_2)=\max\{|D_1|,|D_2|\}$.

Example 3.4

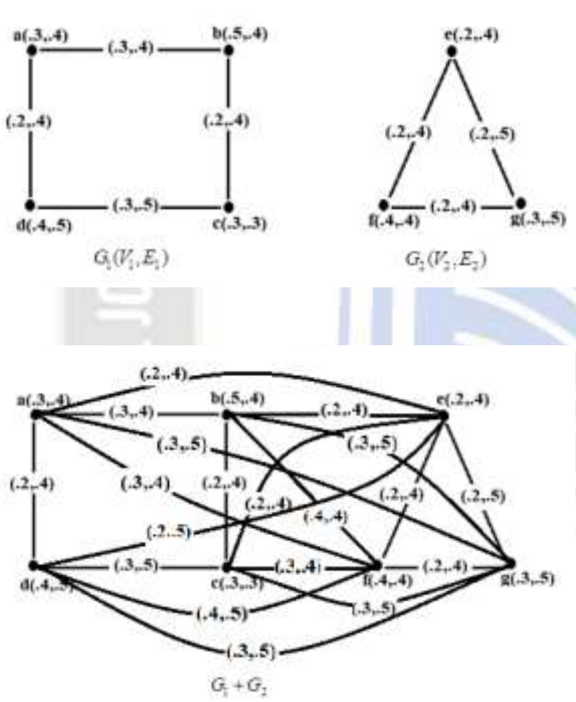

Figure 3.4

In Figure 3.4, the dominating sets of the IFG's $G_1(A_1,B_1)$ *and* $G_2(A_2,B_2)$ are $D_1=\{a,c\}$ *and* $D_2=\{e\}$ .The domination number of $G_1(A_1,B_1)$ *and* $G_2(A_2,B_2)$ are $\gamma(G_1)=0.66$, $\gamma_T(G_2)=0.4$. An ID set of G_1+G_2 is $D=D_1$ and ID number $\gamma'_{If}(G_1+G_2)=0.66$.

Theorem 3.8: The subsets $D_1\subseteq V_1$ and $D_2\subseteq V_2$ are the dominating sets of the IFG's $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Then

(i) If $(D_1\times V_2)$ is a domination set of $(G_1\times G_2)$. Then the ID number $\gamma'_{IF}(G_1\times G_2)=|D_1\times(V_2-D_2)|$.

(ii) If $(V_1\times D_2)$ is a domination set of $(G_1\times G_2)$. Then the ID number $\gamma'_{IF}(G_1\times G_2)=|(V_1-D_1)\times D_2|$

Proof: The subsets $D_1\subseteq V_1$ and $D_2\subseteq V_2$ are the minimal dominating sets of the IFG $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Therefore every vertex $v_1\in V_1$ and $v_2\in V_2$ are adjacent to a vertex in D_1 *and* D_2 respectively. By the known result $(D_1\times V_2)$ *or* $(V_1\times D_2)$ is the minimal dominating set of $(G_1\times G_2)$.

(i). Suppose $(D_1\times V_2)$ is γ set of $G_1\times G_2$. Now we find the dominating set of the subgraph $<(V_1-D_1)\times V_2>$. Assume $(V_1-D_1)\times D_2$ is not an minimal dominating set of $G_1\times G_2$. there exist a vertex $uv\in(V_1-D_1)\times(V_2-D_2)$ is not dominated by the vertex in the set $(V_1-D_1)\times D_2$. There is no strong edge between vertex $uv\in(V_1-D_1)\times(V_2-D_2)$ and every vertex $xy\in(V_1-D_1)\times D_2$. Therefore we get there is no strong edge between $v,y\in V_2$. This implies D_2 is not a minimal dominating set of $G_2(V_2,E_2)$. This is contradict to our assumption D_2 is a minimal dominating set $G_2(V_2,E_2)$. Therefore we get $(V_1-D_1)\times D_2$ is a dominating set of $<(V_1-D_1)\times V_2>$.

(ii). Suppose $(V_1\times D_2)$ is γ set of $G_1\times G_2$. Now we find the dominating set of the subgraph $<(V_1-D_1)\times V_2>$. Assume $D_1\times(V_2-D_2)$ is not an minimal dominating set of $G_1\times G_2$. there exist a vertex $uv\in(V_1-D_1)\times(V_2-D_2)$ is not dominated by the vertex in the set $D_1\times(V_2-D_2)$. There is no strong edge between vertex $uv\in(V_1-D_1)\times(V_2-D_2)$ and every vertex $xy\in D_1\times(V_2-D_2)$. Therefore we get there is no strong edge between $u,x\in V_2$. This implies D_1 is not a minimal dominating set of $G_2(V_2,E_2)$. This is contradict to our assumption D_2 is a minimal dominating set $G_1(V_1,E_1)$. Therefore we get $D_1\times(V_2-D_2)$ is a dominating set of $<(V_1-D_1)\times V_2>$. Hence ID number of the graph $(G_1\times G_2)$ is $\gamma'(G_1\times G_2)=\min\{|D_1\times(V_2-D_2)|,|(V_1-D_1)\times D_2|\}$.

Conclusion

The concepts of an inverse dominating set (ID-Set) and an inverse domination number (ID-number) of anIFG were examined in this work. Continue researching the complete and complete bipartite IFG's inverse domination number. A few findings and limits of the ID-number are examined. We look into the ID-number of the join between two IFGs and the Cartesian product of two IFGs..

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