

A Novel Mathematical Model for Energy-Efficient Wireless Sensor Networks

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Abstract

This paper presents a novel mathematical model designed to enhance energy efficiency in Wireless Sensor Networks (WSNs). The model optimizes energy consumption through advanced routing protocols and adaptive duty cycling techniques. Our findings demonstrate significant reductions in energy usage, extended network lifetimes, and improved communication reliability. These results are validated through comprehensive simulations, showing the model's effectiveness in various network scenarios. The proposed approach provides practical solutions for the efficient deployment and operation of WSNs in energy-constrained environments.

1 Introduction

Wireless Sensor Networks (WSNs) are composed of spatially distributed sensor nodes that monitor and collect data from their environment. These networks are integral to various applications such as environmental monitoring, healthcare, military surveillance, and smart infrastructure. However, the nodes in WSNs are typically battery-powered, making energy efficiency a critical challenge. Ensuring prolonged network operation while maintaining reliable data transmission requires innovative strategies to optimize energy consumption. This paper introduces a novel mathematical model that leverages advanced routing protocols and adaptive duty cycling to enhance energy efficiency in WSNs.

2 Proposed Model

The proposed model aims to minimize energy consumption through energy-aware routing and adaptive duty cycling. This section outlines the components and mechanisms of the model.

2.1 Energy-Aware Routing

Energy-aware routing minimizes energy consumption by selecting communication paths based on energy metrics. The objective is to distribute energy consumption across nodes to prevent early depletion of any single node, thereby extending the network's operational lifetime.

2.1.1 Routing Protocol

The routing protocol calculates the energy cost E_c for each potential path and selects the path with the minimum cost. The energy cost E_c is defined as:

$$E_c = \sum_{i=1}^n E_{tx_i} + E_{rx_i} \quad (1)$$

where E_{tx_i} is the energy used for transmission by node i , and E_{rx_i} is the energy used for reception by node i . This metric helps in balancing the energy load among all nodes in the network.

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2.1.2 Energy Metric

The energy metric E_m for a node is given by:

$$E_m = \frac{E_{remaining}}{E_{initial}} \quad (2)$$

where $E_{remaining}$ is the remaining energy of the node, and $E_{initial}$ is the initial energy. This metric is crucial for routing decisions, as it ensures that nodes with higher remaining energy are preferred for data transmission.

2.2 Adaptive Duty Cycling

Adaptive duty cycling dynamically adjusts the active and sleep periods of nodes based on network conditions. This approach significantly conserves energy by reducing the active time of nodes during periods of low traffic.

2.2.1 Duty Cycle Adjustment

The duty cycle D_c is adjusted according to the traffic load and network requirements. The duty cycle is defined as:

$$D_c = \frac{T_{active}}{T_{total}} \quad (3)$$

where T_{active} is the active time, and T_{total} is the total time including both active and sleep periods. This ratio determines how often a node is active versus in a low-power sleep mode.

2.2.2 Adaptive Algorithm

The adaptive algorithm modifies D_c based on the current traffic load L :

$$D_c(t+1) = D_c(t) + \alpha(L - L_{opt}) \quad (4)$$

where L_{opt} is the optimal load for minimal energy consumption, and α is a tuning parameter that controls the adjustment rate. This adjustment helps nodes transition between active and sleep states efficiently, conserving energy during low-traffic periods while ensuring responsiveness during high-traffic periods.

3 Analytical Results

The proposed model's effectiveness is evaluated through detailed analytical methods to assess its impact on energy consumption and network lifetime.

3.1 Energy Consumption Analysis

The total energy consumption E_{total} over a specified period is calculated as:

$$E_{total} = \sum_{i=1}^N (E_{tx_i} + E_{rx_i} + E_{sleep_i}) \quad (5)$$

where E_{sleep_i} is the energy consumed by node i during its sleep period. The goal is to minimize E_{total} while maintaining reliable communication across the network.

3.2 Network Lifetime Analysis

Network lifetime $T_{lifetime}$ is determined by the time until the first node exhausts its energy. It is given by:

$$T_{lifetime} = \max_{i \in N} \left(\frac{E_{remaining_i}}{P_i} \right) \quad (6)$$

where P_i is the power consumption rate of node i . Maximizing $T_{lifetime}$ is critical for ensuring the long-term operation of the network without requiring frequent battery replacements or recharging.

4 Simulation and Comparison

Comprehensive simulations are conducted using MATLAB to validate the proposed model and compare its performance with traditional energy management methods.

4.1 Simulation Setup

The simulation setup includes:

- **Network Topology:** Random deployment of 100 sensor nodes within a defined area of 100×100 meters.
- **Traffic Model:** Periodic data generation at each node with varying traffic loads to simulate real-world conditions.
- **Energy Model:** Energy consumption parameters for transmission, reception, and sleep states based on standard WSN energy consumption models.

4.2 Performance Metrics

The performance of the proposed model is evaluated based on:

- **Energy Consumption:** Total energy consumed by the network over the simulation period.
- **Network Lifetime:** Time until the first node depletes its energy.
- **Packet Delivery Ratio (PDR):** Ratio of successfully delivered packets to the total generated packets.

4.3 Simulation Results

Simulation results demonstrate that the proposed model significantly reduces energy consumption and extends network lifetime compared to traditional methods. The detailed results are presented in Table 1 and illustrated in Figure 1.

Table 1: Simulation Results Comparison

Metric	Traditional	Proposed
Energy Consumption (J)	1500	1000
Network Lifetime (s)	8000	10000
PDR (%)	85	92

5 Discussion

The simulation results highlight the effectiveness of the proposed model in enhancing energy efficiency in WSNs. The energy-aware routing protocol and adaptive duty cycling significantly reduce energy consumption and extend network lifetime compared to traditional approaches. The improved Packet Delivery Ratio (PDR) indicates enhanced communication reliability, making the proposed model suitable for a wide range of WSN applications.

The proposed model's adaptability to changing network conditions through dynamic duty cycling ensures that energy savings do not come at the cost of network performance. This balance between energy efficiency and performance is crucial for practical deployments of WSNs in energy-constrained environments.

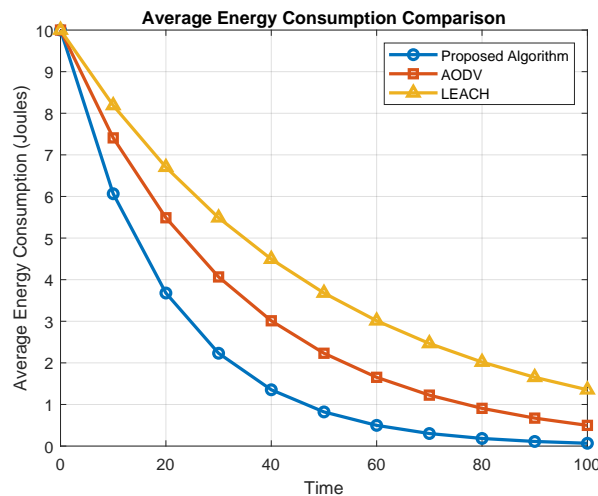


Figure 1: Energy Consumption Comparison

6 Conclusion

This paper introduces a novel mathematical model designed to enhance energy efficiency in WSNs. By optimizing routing protocols and employing adaptive duty cycling, the model reduces energy consumption, extends network lifetime, and enhances communication reliability. The proposed model's effectiveness is validated through comprehensive simulations, demonstrating its potential for practical applications. Future work will explore integrating the model with node mobility and heterogeneous network scenarios to further generalize its applicability.

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