

# $\tilde{O}$ - $\mathcal{I}$ -Locally Closed Sets with Respect to an Ideal Topological Spaces

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**Abstract:** In this paper, we introduce three forms of locally closed sets called  $\tilde{O}$ - $\mathcal{I}$ -locally closed sets,  $\tilde{O}$ - $\mathcal{I}$ - $lc^*$  sets and  $\tilde{O}$ - $\mathcal{I}$ - $lc^{**}$  sets and various properties of  $\tilde{O}$ - $\mathcal{I}$ -locally closed sets,  $\tilde{O}$ - $\mathcal{I}$ - $lc^*$  sets and  $\tilde{O}$ - $\mathcal{I}$ - $lc^{**}$  sets and relation between the above three set and another sets.

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## 1. INTRODUCTION

M. Ganster and I. L. Reilly studied Locally closed sets and LC-continuous functions in the year 1989. Following this attempts, modern mathematics generalized this concept and are being found many generalization of locally closed sets. R. Vaidyanathaswamy studied the localization theory in set topology in 1945. D. Jankovic and T. R. Hamlett studied new topologies from old via ideals in 1990. In this paper, we introduce three forms of locally closed sets called  $\tilde{O}$ - $\mathcal{I}$ -locally closed sets,  $\tilde{O}$ - $\mathcal{I}$ - $lc^*$  sets and  $\tilde{O}$ - $\mathcal{I}$ - $lc^{**}$  sets. Properties of these new concepts are studied as well as their relations to the other classes of locally closed sets are investigated.

## 2. PRELIMINARIES

### Definition 2.1

A subset  $S$  of  $X$  is called

- (i) locally closed [6] (briefly,  $lc$ ) if  $S = U \cap F$ , where  $U$  is open and  $F$  is closed in  $X$ .
- (ii)  $\hat{g}$ -closed set [19] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open. The complement of  $\hat{g}$ -closed set is  $\hat{g}$ -open.
- (iii)  $*g$ -closed set [12] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open. The complement of  $*g$ -closed set is  $*g$ -open.
- (iv)  $\#gs$ -closed set [21] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $*g$ -open. The complement of  $\#gs$ -closed set is  $\#gs$ -open.
- (v)  $\tilde{g}s$ -closed set [15] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\#gs$ -open. The complement of  $\tilde{g}s$ -closed set is  $\tilde{g}s$ -open.
- (vi)  $gs$ -closed set if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open. The complement of  $gs$ -closed set is  $gs$ -open.

The collection of all  $\tilde{g}s$ -open sets is denoted by  $\tilde{g}SO(X)$ .

### Definition 2.2

A subset  $S$  of a space  $X$  is called:

- (i) generalized locally closed (briefly,  $glc$ ) [19] if  $S = V \cap F$ , where  $V$  is  $g$ -open and  $F$  is  $g$ -cld.
- (ii) semi-generalized locally closed [11] (briefly,  $sglc$ ) if  $S = V \cap F$ , where  $V$  is  $sg$ -open and  $F$  is  $sg$ -cld.
- (iii) regular-generalized locally closed [1] (briefly,  $rglc$ ) if  $S = V \cap F$ , where  $V$  is  $rg$ -open and  $F$  is  $rg$ -cld.
- (iv) generalized locally semi-closed (briefly,  $glsc$ ) if  $S = V \cap F$ , where  $V$  is  $g$ -open and  $F$  is semi-cld.
- (v) locally semi-closed (briefly,  $lsc$ ) if  $S = V \cap F$ , where  $V$  is open and  $F$  is semi-cld.

(vi)  $\alpha$ -locally closed (briefly,  $\alpha$ -lc) if  $S = V \cap F$ , where  $V$  is  $\alpha$ -open and  $F$  is  $\alpha$ -cld.

(vii)  $\omega$ -locally closed (briefly,  $\omega$ -lc) if  $S = V \cap F$ , where  $V$  is  $\omega$ -open and  $F$  is  $\omega$ -cld.

The class of all generalized locally closed (resp. generalized locally semi-closed, locally semi-closed,  $\omega$ -locally closed) sets in  $X$  is denoted by  $GLC(X)$  (resp.  $GLSC(X)$ ,  $LSC(X)$ ,  $\omega$ -LC( $X$ )).

An ideal on a topological space  $(X, \tau)$  is a non-empty collection of subsets of  $X$  which satisfies the following properties:

(i)  $A \in I$  and  $B \subset A$  implies  $B \in I$

(ii)  $A \in I$  and  $B \in I$  implies  $A \cup B \in I$ .

An ideal topological space (or An ideal space) is a topological space  $(X, \tau)$  with an ideal  $I$  on  $X$  and is denoted by  $(X, \tau, I)$ . For a subset  $A \subseteq X$ ,  $A^*(I, \tau) = \{x \in X : A \cap U \notin I \text{ for every } U \in \tau(X, x)\}$  is called the local function of  $A$  with respect to  $I$  and  $\tau$ . We simply write  $A^*$  incase there is no chance for confusion. A Kuratowski closure operator  $cl^*(.)$  for a topology  $\tau^*(I, \tau)$  called the \*-topology, finer than  $\tau$  is defined by  $cl^*(A) = A \cup A^*$ .

### Definition 2.3

1) A subset  $S$  of  $X$  is called  $\tilde{O}$ - $J$ -closed (briefly,  $\tilde{O}$ - $J$ -cld) if  $S^* \subseteq P$  whenever  $S \subseteq P$  and  $P$  is  $g_s$ -open. The complement of  $\tilde{O}$ - $J$ -cld is called  $\tilde{O}$ - $J$ -open.

2) A subset  $S$  of  $X$  is called  $\tilde{O}$ - $J$ -locally closed (briefly,  $\tilde{O}$ - $J$ -lc) if  $S = H \cap G$ , where  $H$  is  $\tilde{O}$ - $J$ -open and  $G$  is  $\tilde{O}$ - $J$ -cld.

### 3. $\tilde{O}$ - $J$ -LOCALLY CLOSED SETS

We introduce the following definition.

#### Definition 3.1

1) A subset  $S$  of  $X$  is called  $\tilde{O}$ - $J$ -closed (briefly,  $\tilde{O}$ - $J$ -cld) if  $S^* \subseteq P$  whenever  $S \subseteq P$  and  $P$  is  $\tilde{g}_s$ -open. The complement of  $\tilde{O}$ - $J$ -cld is called  $\tilde{O}$ - $J$ -open.

2) A subset  $S$  of  $X$  is called  $\tilde{O}$ - $J$ -locally closed (briefly,  $\tilde{O}$ - $J$ -lc) if  $S = H \cap G$ , where  $H$  is  $\tilde{O}$ - $J$ -open and  $G$  is  $\tilde{O}$ - $J$ -cld.

The class of all  $\tilde{O}$ - $J$ -locally closed sets in  $X$  is denoted by  $\tilde{O}$ - $J$ -LC( $X$ ).

#### Proposition 3.2

Each  $\tilde{O}$ - $J$ -cld (resp.  $\tilde{O}$ - $J$ -open) is  $\tilde{O}$ - $J$ -lc set but not reverse.

#### Proof

This follows from Definition 3.1.

#### Example 3.3

Let  $X = \{1, 2, 3\}$  and  $\tau = \{\emptyset, \{1\}, \{1, 3\}, X\}$  with  $J = \{\emptyset\}$ . Then the set  $\{2\}$  is  $\tilde{O}$ - $J$ -lc set but it is not  $\tilde{O}$ - $J$ -cld and the set  $\{3\}$  is  $\tilde{O}$ - $J$ -lc set but it is not  $\tilde{O}$ - $J$ -open in  $X$ .

#### Proposition 3.4

Each lc set is  $\tilde{O}$ - $J$ -lc set but not reverse.

#### Proof

This follows from Proposition 3.2.

#### Example 3.5

Let  $X = \{1, 2, 3\}$  and  $\tau = \{\emptyset, \{2, 3\}, X\}$  with  $J = \{\emptyset\}$ . Then the set  $\{2\}$  is  $\tilde{O}$ - $J$ -lc set but it is not lc set in  $X$ .

#### Proposition 3.6

Each  $\tilde{O}$ - $J$ -lc set is  $\tilde{O}$ - $J$ -lc set but not reverse.

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**Proof**

This follows from the fact that every  $\tilde{g}$ s-open set is gs-open set

**Example 3.7**

Let  $X = \{1, 2, 3, 4\}$  and  $\tau = \{\phi, X, \{1\}, \{4\}, \{1, 4\}\}$  with  $\mathcal{I} = \{\phi\}$ . Then the set  $\{2, 3\}$  is  $\tilde{O}$ - $\mathcal{I}$ -lc set but it is not  $\tilde{O}$ - $\mathcal{I}$ -lc set.

**Proposition 3.8**

Each  $\tilde{O}$ - $\mathcal{I}$ -lc set is a (i)  $\omega$ -lc set, (ii) glc set and (iii) sglc set. However the separate reverse is not true.

**Proof**

It is obviously.

**Example 3.9**

Let  $X = \{1, 2, 3\}$  and  $\tau = \{\phi, \{1\}, X\}$  with  $\mathcal{I} = \{\phi\}$ . Then the set  $\{1, 2\}$  is glc set and sglc set but it is not  $\tilde{O}$ - $\mathcal{I}$ -lc set in  $X$ .

**Example 3.10**

Let  $X = \{1, 2, 3\}$  and  $\tau = \{\phi, \{2\}, \{1, 3\}, X\}$  with  $\mathcal{I} = \{\phi\}$ . Then the set  $\{1\}$  is  $\omega$ -lc set but it is not  $\tilde{O}$ - $\mathcal{I}$ -lc set in  $X$ .

**Remark 3.11**

The concepts of  $\alpha$ -lc sets and  $\tilde{O}$ - $\mathcal{I}$ -lc sets are independent of each other.

**Example 3.12**

The set  $\{2, 3\}$  in Example 3.3 is  $\alpha$ -lc set but it is not a  $\tilde{O}$ - $\mathcal{I}$ -lc set in  $X$  and the set  $\{1, 2\}$  in Example 3.5 is  $\tilde{O}$ - $\mathcal{I}$ -lc set but it is not an  $\alpha$ -lc set in  $X$ .

**Remark 3.13**

The concepts of lsc sets and  $\tilde{O}$ - $\mathcal{I}$ -lc sets are independent of each other.

**Example 3.14**

The set  $\{1\}$  in Example 3.3 is lsc set but it is not a  $\tilde{O}$ - $\mathcal{I}$ -lc set in  $X$  and the set  $\{1, 2\}$  in Example 3.5 is  $\tilde{O}$ - $\mathcal{I}$ -lc set but it is not a lsc set in  $X$ .

**Remark 3.15**

The concepts of  $\tilde{O}$ - $\mathcal{I}$ -lc sets and glsc sets are independent of each other.

**Example 3.16**

The set  $\{2, 3\}$  in Example 3.3 is glsc set but it is not a  $\tilde{O}$ - $\mathcal{I}$ -lc set in  $X$  and the set  $\{1, 2\}$  in Example 3.5 is  $\tilde{O}$ - $\mathcal{I}$ -lc set but it is not a glsc set in  $X$ .

**Remark 3.17**

The concepts of  $\tilde{O}$ - $\mathcal{I}$ -lc sets and  $sglc^*$  sets are independent of each other.

**Example 3.18**

The set  $\{2, 3\}$  in Example 3.3 is  $sglc^*$  set but it is not a  $\tilde{O}$ - $\mathcal{I}$ -lc set in  $X$  and the set  $\{1, 2\}$  in Example 3.5 is  $\tilde{O}$ - $\mathcal{I}$ -lc set but it is not a  $sglc^*$  set in  $X$ .

**Definition 3.19** A space  $X$  is said to be an  $\tilde{O}$ - $\mathcal{I}$ -space if every  $\tilde{O}$ - $\mathcal{I}$ -open set is open.

**Theorem 3.20**

For a  $\tilde{O}$ - $\mathcal{I}$ -space  $X$ , the following properties hold:

- (i)  $\tilde{O}\text{-}I\text{-}LC(X) = LC(X)$ .
- (ii)  $\tilde{O}\text{-}I\text{-}LC(X) \subseteq GLC(X)$ .
- (iii)  $\tilde{O}\text{-}I\text{-}LC(X) \subseteq GLSC(X)$ .
- (iv)  $\tilde{O}\text{-}I\text{-}LC(X) \subseteq \omega\text{-}LC(X)$ .

**Proof**

- (i) Since every  $\tilde{O}\text{-}I$ -open set is open and every  $\tilde{O}\text{-}I$ -cld is  $*$ -closed,  $\tilde{O}\text{-}I\text{-}LC(X) \subseteq LC(X)$  and hence  $\tilde{O}\text{-}I\text{-}LC(X) = LC(X)$ .
- (ii), (iii) and (iv) follows from (i), since for any space  $X$ ,  $LC(X) \subseteq GLC(X)$ ,  $LC(X) \subseteq GLSC(X)$  and  $LC(X) \subseteq \omega\text{-}LC(X)$ .

**Definition 3.21**

A subset  $S$  of a space  $X$  is called:

- (i)  $\tilde{O}\text{-}I\text{-}lc^*$  set if  $S = H \cap G$ , where  $H$  is  $\tilde{O}\text{-}I$ -open in  $X$  and  $G$  is  $*$ -closed in  $X$ .
- (ii)  $\tilde{O}\text{-}I\text{-}lc^{**}$  set if  $S = H \cap G$ , where  $H$  is open in  $X$  and  $G$  is  $\tilde{O}\text{-}I$ -cld in  $X$ .

The class of all  $\tilde{O}\text{-}I\text{-}lc^*$  (resp.  $\tilde{O}\text{-}I\text{-}lc^{**}$ ) sets in ideal topological space  $X$  is denoted by  $\tilde{O}\text{-}I\text{-}LC^*(X)$  (resp.  $\tilde{O}\text{-}I\text{-}LC^{**}(X)$ ).

**Proposition 3.22**

Each  $lc$ -set is  $\tilde{O}\text{-}I\text{-}lc^*$  set but not reverse.

**Proof**

It follows from Definition 3.21 (i) and Definition of locally closed set.

**Example 3.23**

The set  $\{2\}$  in Example 3.5 is  $\tilde{O}\text{-}I\text{-}lc^*$  set but it is not a  $lc$  set in  $X$ .

**Proposition 3.24**

Each  $lc$ -set is  $\tilde{O}\text{-}I\text{-}lc^{**}$  set but not reverse.

**Proof**

It follows from Definition 3.21 (ii) and Definition of locally closed set.

**Example 3.25**

The set  $\{1, 3\}$  in Example 3.5 is  $\tilde{O}\text{-}I\text{-}lc^{**}$  set but it is not a  $lc$  set in  $X$ .

**Proposition 3.26**

Each  $\tilde{O}\text{-}I\text{-}lc^*$  set is  $\tilde{O}\text{-}I\text{-}lc$  set but not reverse.

**Proof**

It follows from Definitions 3.1 and 3.21 (i).

**Example 3.27**

The set  $\{1, 2\}$  in Example 3.5 is  $\tilde{O}\text{-}I\text{-}lc$  set but it is not a  $\tilde{O}\text{-}I\text{-}lc^*$  set in  $X$ .

**Proposition 3.28**

Each  $\tilde{O}\text{-}I\text{-}lc^{**}$  set is  $\tilde{O}\text{-}I\text{-}lc$  set but not reverse.

## Proof

It follows from Definitions 3.1 and 3.21 (ii).

### Remark 3.29

The concepts of  $\tilde{O}\text{-}J\text{-}lc^*$  sets and lsc sets are independent of each other.

### Example 3.30

The set  $\{3\}$  in Example 3.5 is  $\tilde{O}\text{-}J\text{-}lc^*$  set but it is not a lsc set in  $X$  and the set  $\{1\}$  in Example 3.3 is lsc set but it is not a  $\tilde{O}\text{-}J\text{-}lc^*$  set in  $X$ .

### Remark 3.31

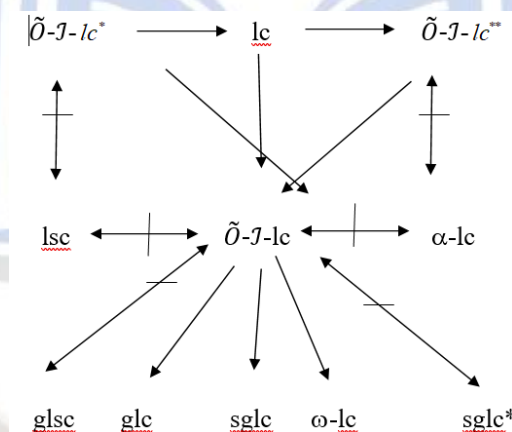
The concepts of  $\tilde{O}\text{-}J\text{-}lc^{**}$  sets and  $\alpha\text{-}lc$  sets are independent of each other.

### Example 3.32

The set  $\{1, 2\}$  in Example 3.5 is  $\tilde{O}\text{-}J\text{-}lc^{**}$  set but it is not a  $\alpha\text{-}lc$  set in  $X$  and the set  $\{1, 2\}$  in Example 3.3 is  $\alpha\text{-}lc$  set but it is not a  $\tilde{O}\text{-}J\text{-}lc^{**}$  set in  $X$ .

### Remark 3.33

From the above discussions we have the following implications where  $A \rightarrow B$  (resp.  $A \nrightarrow B$ ) represents  $A$  implies  $B$  but not conversely (resp.  $A$  and  $B$  are independent of each other).



### Theorem 3.34

Assume that  $\tilde{O}\text{-}J\text{-}C(X)$  is closed under finite intersection. For a subset  $S$  of  $X$ , the following statements are equivalent:

- (i)  $S \in \tilde{O}\text{-}J\text{-}LC(X)$ .
- (ii)  $S = H \cap \tilde{O}\text{-}J\text{-}cl(K)$  for some  $\tilde{O}\text{-}J\text{-}open$  set  $H$ .
- (iii)  $\tilde{O}\text{-}J\text{-}cl(S) - S$  is  $\tilde{O}\text{-}J\text{-}cld$ .
- (iv)  $S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c$  is  $\tilde{O}\text{-}J\text{-}open$ .
- (v)  $S \subseteq \tilde{O}\text{-}J\text{-}int(S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c)$ .

## Proof

(i)  $\Rightarrow$  (ii). Let  $K \in \tilde{O}\text{-}J\text{-}LC(X)$ . Then  $S = H \cap G$  where  $H$  is  $\tilde{O}\text{-}J\text{-}open$  and  $G$  is  $\tilde{O}\text{-}J\text{-}cld$ . Since  $S \subseteq G$ ,  $\tilde{O}\text{-}J\text{-}cl(S) \subseteq G$  and so  $H \cap \tilde{O}\text{-}J\text{-}cl(S) \subseteq S$ . Also  $S \subseteq H$  and  $S \subseteq \tilde{O}\text{-}J\text{-}cl(S)$  implies  $S \subseteq H \cap \tilde{O}\text{-}J\text{-}cl(S)$  and therefore  $S = H \cap \tilde{O}\text{-}J\text{-}cl(S)$ .

(ii)  $\Rightarrow$  (iii).  $S = H \cap \tilde{O}\text{-}J\text{-}cl(S)$  implies  $\tilde{O}\text{-}J\text{-}cl(S) - S = \tilde{O}\text{-}J\text{-}cl(S) \cap H^c$  which is  $\tilde{O}\text{-}J\text{-}cld$  since  $H^c$  is  $\tilde{O}\text{-}J\text{-}cld$  and  $\tilde{O}\text{-}J\text{-}cl(S)$  is  $\tilde{O}\text{-}J\text{-}cld$ .

(iii)  $\Rightarrow$  (iv).  $S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c = (\tilde{O}\text{-}J\text{-}cl(S) - S)^c$  and by assumption,  $(\tilde{O}\text{-}J\text{-}cl(S) - S)^c$  is  $\tilde{O}\text{-}J\text{-}open$  and so is  $S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c$ .

(iv)  $\Rightarrow$  (v). By assumption,  $S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c = \tilde{O}\text{-}J\text{-}int(S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c)$  and hence  $S \subseteq \tilde{O}\text{-}J\text{-}int(S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c)$ .

(v)  $\Rightarrow$  (i). By assumption and since  $S \subseteq \tilde{O}\text{-}J\text{-}cl(S)$ ,  $K = \tilde{O}\text{-}J\text{-}int(S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c) \cap \tilde{O}\text{-}J\text{-}cl(S)$ . Therefore,  $S \in \tilde{O}\text{-}J\text{-}LC(X)$ .

### Theorem 3.35

For a subset  $S$  of  $X$ , the following statements are equivalent:

- (i)  $S \in \tilde{O}\text{-}J\text{-}LC^*(X)$ .
- (ii)  $S = H \cap K^*$  for some  $\tilde{O}\text{-}J$ -open set  $H$ .
- (iii)  $S^* - S$  is  $\tilde{O}\text{-}J$ -cld.
- (iv)  $S \cup (S^*)^c$  is  $\tilde{O}\text{-}J$ -open.

### Proof

(i)  $\Rightarrow$  (ii). Let  $S \in \tilde{O}\text{-}J\text{-}LC^*(X)$ . There exist an  $\tilde{O}\text{-}J$ -open set  $S$  and a  $\star$ -closed set  $G$  such that  $S = H \cap G$ . Since  $S \subseteq H$  and  $S \subseteq S^*$ ,  $S \subseteq H \cap S^*$ . Also, since  $S^* \subseteq G$ ,  $H \cap S^* \subseteq H \cap G = S$ . Therefore  $S = H \cap S^*$ .

(ii)  $\Rightarrow$  (i). Since  $H$  is  $\tilde{O}\text{-}J$ -open and  $S^*$  is a  $\star$ -closed set,  $S = H \cap S^* \in \tilde{O}\text{-}J\text{-}LC^*(X)$ .

(ii)  $\Rightarrow$  (iii). Since  $S^* - S = S^* \cap H^c$ ,  $S^* - S$  is  $\tilde{O}\text{-}J$ -cld.

(iii)  $\Rightarrow$  (ii). Let  $H = (S^* - S)^c$ . Then by assumption  $H$  is  $\tilde{O}\text{-}J$ -open in  $X$  and  $S = H \cap S^*$ .

(iii)  $\Rightarrow$  (iv). Let  $G = S^* - S$ . Then  $G^c = S \cup (S^*)^c$  and  $S \cup (S^*)^c$  is  $\tilde{O}\text{-}J$ -open.

(iv)  $\Rightarrow$  (iii). Let  $H = S \cup (S^*)^c$ . Then  $H^c$  is  $\tilde{O}\text{-}J$ -cld and  $H^c = S^* - S$  and so  $S^* - S$  is  $\tilde{O}\text{-}J$ -cld.

### Theorem 3.36

Let  $S$  be a subset of  $X$ . Then  $S \in \tilde{O}\text{-}J\text{-}LC^{**}(X)$  if and only if  $S = H \cap \tilde{O}\text{-}J\text{-}cl(S)$  for some open set  $H$ .

### Proof

Let  $S \in \tilde{O}\text{-}J\text{-}LC^{**}(X)$ . Then  $S = H \cap G$  where  $H$  is open and  $G$  is  $\tilde{O}\text{-}J$ -cld. Since  $S \subseteq G$ ,  $\tilde{O}\text{-}J\text{-}cl(S) \subseteq G$ . We obtain  $S = S \cap \tilde{O}\text{-}J\text{-}cl(S) = H \cap G \cap \tilde{O}\text{-}J\text{-}cl(S) = H \cap \tilde{O}\text{-}J\text{-}cl(S)$ .

Converse part is trivial.

### Corollary 3.37

Let  $S$  be a subset of  $X$ . If  $S \in \tilde{O}\text{-}J\text{-}LC^{**}(X)$ , then  $\tilde{O}\text{-}J\text{-}cl(S) - S$  is  $\tilde{O}\text{-}J$ -cld and  $S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c$  is  $\tilde{O}\text{-}J$ -open.

### Proof

Let  $S \in \tilde{O}\text{-}J\text{-}LC^{**}(X)$ . Then by Theorem 3.40,  $S = H \cap \tilde{O}\text{-}J\text{-}cl(S)$  for some open set  $H$  and  $\tilde{O}\text{-}J\text{-}cl(S) - S = \tilde{O}\text{-}J\text{-}cl(S) \cap H^c$  is  $\tilde{O}\text{-}J$ -cld in  $X$ . If  $G = \tilde{O}\text{-}J\text{-}cl(S) - S$ , then  $G^c = S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c$  and  $G^c$  is  $\tilde{O}\text{-}J$ -open and so is  $S \cup (\tilde{O}\text{-}J\text{-}cl(S))^c$ .

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