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Evenvertex Oblong Mean Labeling of Union of Graphs

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Abstract: The n^{th} oblong number is denoted by O_n and is defined by $O_n = n(n+1)$. Suppose G = (V, E) be loop-free, not having multiple edges, finite, and non-directed graph with |V(G)| = p and |E(G)| = q. An evenvertex oblong mean labeling is an injective function $\gamma_{evom} : V(G) \rightarrow \{0, 2, 4, \ldots, 2O_q\}$ where O_q is the q^{th} oblong number that induces a bijective edge labeling $\gamma^*_{O_n}$: E(G) $\rightarrow$ $\{O_1, O_2, \ldots, O_q\}$ defined by $\gamma^*_{O_n}(uv) = \frac{\gamma_{evom}(u)+\gamma_{evom}(v)}{2}$ for all e = uv $\in$ E(G). Any graph that accommodates evenvertex oblong mean labeling is known as evenvertex oblong mean graph. In the present paper, evenvertex oblong mean labeling of union of some special graph with any evenvertex oblong mean graph is investigated.

Keywords: *oblong number, evenvertex oblong mean labeling, evenvertex oblong mean graphs.*

1. Introduction

In the context of a graph G=(V, E) with p vertices & q edges, we refer to a loop-free, not having multiple edges, finite, and non-directed graph. The expressions prescribed here are employed in accordance with Harary's conventions [4]. For number-theoretic terminology, we adhere to the definitions outlined in [1].A graph labeling involves assigning integers to vertices(edges/both), dependent on particular criteria. When the input space encompasses the collection of edges (both/ vertices), the labeling is termed an edge (total/vertex) labeling.The survey in [2] comprehensively explores various hierarchical approaches to graph labeling. The notion of mean labeling was innovated and explored by Somasundaram and Ponraj [7], while Manickam and Marudai unveiled the formulation odd mean labeling [5]. Gayathri and Gobi, in [3], explored even mean labeling. Oblong Sum labeling, was introduced by K. Murugan et al. [6]Building on the aforementioned articles, M. P. Syed Ali Nisaya and K. Somasundari introduced evenvertex oblong mean labeling in [8]. The exploration of evenvertex oblong mean labeling for further graphs is detailed in [9]. In this paper, the author delves into the discrete union of some special graphs with an evenvetex oblong mean graph.

2. Initial Descriptions

Definition 2.1: A path $P_n$is attained by joining u_s to the consecutive vertices $u_{s+1}$for $1 \leq s \leq n-1$.

Definition 2.2 : A Star $K_{1,n}$ is an one point union of n hanging vertices.

Definition 2.3 : A Bistar $B_{m,n}$ is attained by attaching m and n hanging vertices on the either side of a path P_2.

Definition 2.4: The Disjoint union of two graphs namely, $G_1$and G_2 with vertex sets and edges sets respectively as $V_1, E_1$and V_2, E_2 denoted by $G_1 \cup G_2$ with vertex sets $V_1 \cup V_2$ and edge sets $E_1 \cup E_2$ is formed by combining the vertex and edge set of the original graphs, ensuring that no vertices are shared among them. On other words, every vertex and edge in the resulting graph belongs exclusive to one of the original graphs.

Definition 2.5: The product of a number and its successor is called an oblong number; algebraically, it takes the form $n(n+1)$. The oblong numbers are 2, 6, 12, 20, 30, 42, ….

Definition 2.6: An evenvertex oblong mean labeling is an injective function $\gamma_{evom} : V(G) \to \{0, 2, 4, \dots, 2O_q\}$ where O_q is the q^{th} oblong number that induces a bijective edge labeling $\gamma^*_{O_n}: E(G) \to \{O_1, O_2, \dots, O_q\}$ defined by $\gamma^*_{O_n}(uv) = \frac{\gamma_{evom}(u)+\gamma_{evom}(v)}{2}$ for every single e = uv $\in$ E(G). The graph that accomadates evenvertex oblong mean labeling is known as evenvertex oblong mean graph.

Illustration. 2.7: Evenvertex oblong mean graph is given in the figure below .

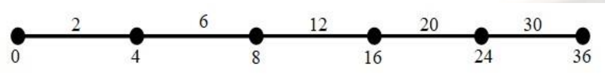

Fig. 2.8

3. Principal Outcomes:

Proposition 3.1: 2 and 6 are not used as vertex labels in any even vertex oblong mean graph. Since 0, 4 and 8 has to be used as vertex label to get the first oblong number 2 and second oblong number 6.

Theorem: 3.2: If G be any evenvertex oblong mean graph. Then, $G \cup K_{1,n}$ is an evenvertex oblong mean graph.

Proof: Hypothesize G be any evenvertex oblong mean graph with p vertices and q edges with the injective vertex labeling as $\gamma_{evom}: \{0, 2, 4, \dots, 2O_q\}$ which induces the edge labels as $O_1, O_2, \dots, O_q$.

Let $G' = G \cup K_{1,n}$.

Suppose $V(K_{1,n}) = \{v, v_s: 1 \le s \le n\}$.

Then, G'has $p + n + 1$ vertices and $q + n$ edges.

Define $\gamma'_{evom}: V(G') \to \{0, 2, 4, \dots, 2O_q, 2O_q + 2, \dots, 2O_{q+n}\}$ by

$\gamma'_{evom}(w) = \gamma_{evom}(w)$ for each $w \in G$

$\gamma'_{evom}(v) = 2$

for $1 \le s \le n$, $\gamma'_{evom}(v_s) = 2[(q + s)(q + s + 1) - 1]$

Then, the induced edge labelling

$\gamma'^*_{O_n}: E(G') \to \{O_1, O_2, \dots, O_q, O_{q+1}, \dots, O_{q+n}\}$ is defined by

$\gamma'^*_{O_n}(e) = \gamma'_{O_n}(e) = O_h, 1 \le h \le q$ for each $e \in G$

And $\gamma'^*_{O_n}(vv_s) = O_{q+s}, 1 \le s \le n$

Hence, we get the distinct edge labels as $O_1, O_2, \dots, O_{q+n}$.

Illustration 3.3: The evenvertex of $F(P_5) \cup K_{1,5}$ is given below.

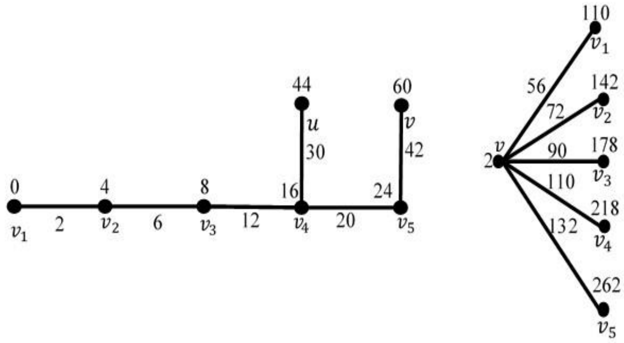

Fig. 3.4

Proposition 3.5: In any evenvertex oblong mean graph, with p vertices and q edges the $O_q$is not used as any vertex label.

Theorem 3.6: If G be any evenvertex oblong mean graph. Then $G \cup P_n$is an evenvertex oblong mean graph.

Proof: Hypothesize G be any evenvertex oblong mean "graph with p vertices and q edges with injective vertex labelling as $\gamma_{evom}: \{0, 2, 4, \dots, 2O_q\}$ which induces the edge labels as $O_1, O_2, \dots, O_q$.

Let $G' = G \cup P_n$.

Let $V(P_n) = \{v_s: 1 \le s \le n\}$ and $E(P_n) = \{(v_s v_{s+1}): 1 \le s \le n - 1\}$

Then, G' has $p + n$ vertices and $q + n - 1$ edges.

Define" $\gamma'_{evom}: V(G') \to \{0, 2, 4, \dots, 2O_q, 2O_q + 2, \dots, 2O_{q+n-1}\}$ by

$\gamma'_{evom}(w) = \gamma_{evom}(w)$ for each $w \in G$

$\gamma'_{evom}(v_s) = \begin{cases} q^2 + (2s - 1)q + s^2 - 1 & if\ s\ is\ odd \\ q^2 + (2s + 1)q + s^2 & if\ s\ is\ even \end{cases}$ where $q = |E(G)|$

Then, the induced edge labelling $\gamma'^*_{O_n}: E(G') \to \{O_1, O_2, \dots, O_q, O_{q+1}, \dots, O_{q+n}\}$ is defined by

$\gamma'^*_{O_n}(e) = \gamma'_{O_n}(e) = O_h, 1 \le h \le q$ for each $e \in G$

And $\gamma'^*_{O_n}(v_s v_{s+1}) = O_{q+s}, 1 \le s \le n - 1$

Hence, we get the distinct edge labels as $O_1, O_2, \dots, O_{q+n}$.

Illustration 3.7: The evenvertex oblong mean labeling of $GB(P_5) \cup P_7$

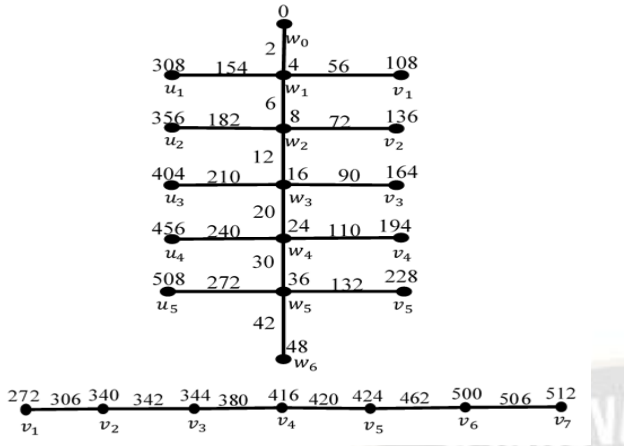

Fig. 3.8

Theorem 3.9: If G be any evenvertex oblong mean graph. Then, $G \cup B_{m,n}$is an evenvertex oblong mean graph.

Proof: Hypothesize G be any evenvertex oblong mean graph with p vertices and q edges with injective vertex labelling as $\gamma_{evom}:\{0,2,4,\ldots,2O_q\}$ which induces the edge labels as $O_1, O_2, \ldots, O_q$.

Let $G' = G \cup B_{m,n}$.
Suppose $V(B_{m,n}) = \{u, v, u_s: 1 \le s \le m,\ v_h: 1 \le h \le n\}$
and $E(B_{m,n}) = \{(u,v)(u,u_s: 1 \le s \le m)(v, v_h: 1 \le h \le n)\}$
Then, G' has $p + m + n + 2$ vertices and $q + m + n + 1$ edges.
Define $\gamma'_{evom}: V(G') \to \{0,2,4,\ldots,2O_q, 2O_q + 2, \ldots, 2O_{q+m+n+1}\}$ by
$\gamma'_{evom}(w) = \gamma_{evom}(w)$ for each $w \in G$
$\gamma'_{evom}(u) = q^2 + q$ where $q = |E(G)|$
$\gamma'_{evom}(u_s) = 2(q+s+1)(q+s+2) - \gamma'_{evom}(u)$ where $1 \le s \le m$
$\gamma'_{evom}(v) = q^2 + 5q + 4$ where $q = |E(G)|$
$\gamma'_{evom}(v_h) = 2(q+m+h+1)(q+m+h+2) - \gamma'_{evom}(v)$ where $1 \le h \le n$
Then, the induced edge labelling $\gamma'^{*}_{O_n}: E(G') \to \{O_1, O_2, \ldots, O_q, O_{q+1}, \ldots, O_{q+n}\}$ is defined by
$\gamma'^{*}_{O_n}(e) = \gamma'_{O_n}(e) = O_a, 1 \le a \le q$ for each $e \in G$
$\gamma'^{*}_{O_n}(u,v) = O_{q+1}$,
$\gamma'^{*}_{O_n}(u,u_s) = O_{q+s+1}, 1 \le s \le m$
and $\gamma'^{*}_{O_n}(v,v_h) = O_{q+m+h+1}, 1 \le h \le n$
Hence, we get the distinct edge labels as $O_1, O_2, \ldots, O_{q+m+n+1}$.

Illustration 3.10: The evenvertex oblong mean labeling of $Y(P_7) \cup B_{3,7}$ is given below

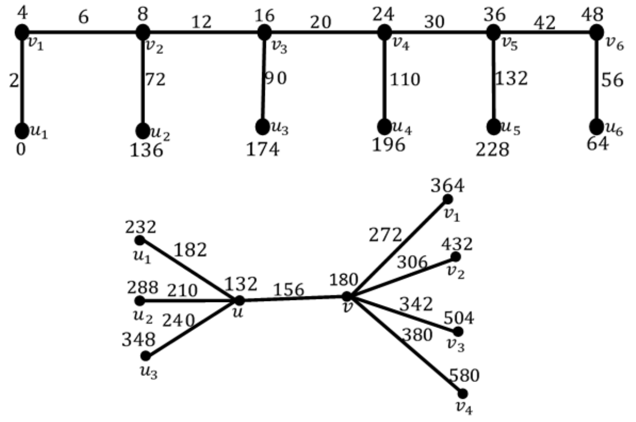

Fig. 3.11

4. Conclusion:

In wrapping up our study, we have meticulously examined the evenvertex oblong mean labeling of several graph structures. Our investigation not only confirms the labeling patterns but also introduces insightful observations tailored for smaller n values. The simplicity of our findings carries profound implications, providing a solid foundation for future exploration.

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