International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 8s
***Article Received:** 25 June 2023 **Revised:** 12 July 2023 **Accepted:** 30 August 2023*

Independence Reinforce Number as an Optimization Tool

Dr. D. D. Pandya[1*], Dr. S.G.Sonchhatra[2], Dr. T. M. Chhaya[3] , P D Maheta[4], H M Rathod[5], S D Bhanderi[6]

[1*]General Department, Government Engineering college, Rajkot, Gujarat, India. ddpandya@gecrajkot.ac.in
[2]General Department, Rajkot, Gujarat, India.Email: sgsonchhatra@iaar.co
[3]General Department,Lukhdhirji Engineering college(Diploma),Morbi, Gujarat, India. Email tarunchhaya47@gmail.com
[4]Computer Engineering Department, Government Engineering college, Rajkot, Gujarat, India. pdmaheta@gecrajkot.ac.in
[5]Computer Engineering Department, Government Engineering college, Rajkot, Gujarat, India. hmrathod@gecrajkot.ac.in
[6]Computer Engineering Department, Government Engineering college, Rajkot, Gujarat, India. sdbhanderi@gecrajkot.ac.in

***Corresponding Author:** Dr. D. D. Pandya
*General Department, Government Engineering college, Rajkot, Gujarat, India.ddpandya@gecrajkot.ac.in

Abstract
Independent domination is having a chief contribution in theory of graphs. We can find its direct and indirect application in theory of matching, graphs colouring and trees. Computer applications developed by flow chart of this variant play an important role. Independence bondage number and Independence reinforcement number can be directly applied for the optimization. In this paper results regarding these two parameters have been discussed with illustrations for better understanding. These results can be applied directly in the optimization for the applications like wireless communication, electrical network and social network theory. Programming to the mathematical modeling can lead easiest optimization for stake holders.

Index Terms—Independence set or i set ,Iplus number , Iminus number Max I Set , critical graphs.

I. INTRODUCTION

Due to the application, domination has been discussed by many authors [4-8]. Fink introduced bondage number for domination in gaphs[3,9]. many research papers have been published regarding these two parameters. Authors have shown their interest in independent domination also [10-12] D. D. Pandya and D. K. Thakar [1] introduced the same concept for Independence number and they lebelled them as Independence bondage number or Iplus number and Independence Reinforcement number or Iminus number respectively. Here we are going to discuss social network graph and wireless network communication graph. Results in these papers includes edge addition, edge removal and relation between vertices and edges.

II. DEFINITIONS

Definition 2.1 : Iplus number
The minimum number of edges which must be removed from a graph $\Re$ to increases the independence number or i number of $\Re$ is known as Iplus number of graph $\Re$. It is expressed as $I\,B(\Re)$

Definition 2.2 : Iminus number
The smallest number of edges whose addition in graph $\Re$ decrease i number of $\Re$ is called Iminus number of graph $\Re$ expressed as $I\,R(\Re)$

Definition 2.3 : Max I set
Max I set is the i set whose cardinality is maximum.

III. IPLUS NUMBER

Lemma 3.1 : $I\,B(\Re)$ will be always than or equal to $l+m$ for the graph $\Re$ if ψ is the new graph obtained after removing l edges from the $\Re$ and $I\,B(\psi)=m$

Theorem 3.2 : If v^1 and v^2 are adjacent vertices of a graph $\Re$ then $I\,B(\Re)\ \le\ M-1$ where M is the number of incident edges with vertices v^1 and v^2

Proof: Consider the graph $\Re$ with adjacent vertices v^1 and v^2 and ψ is the graph obtained from $\Re$ after removing edges incident to v^1 and v^2 except the common edge between two.

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 8s
***Article Received:** 25 June 2023 **Revised:** 12 July 2023 **Accepted:** 30 August 2023*

Here if W is max I set of ψ then we can say that any one of the two vertices v^1 and v^2 will be in W.

Now if $\overline{\psi}$ is the graph obtained by removing edge v^1v^2 from ψ then $\beta_0(\overline{\psi}) > \beta_0(\psi)$ means $IB(\psi) = 1$. So using the lemma 3.1, $IB(\Re) \leq M-1$ where M is the number of incident edges with vertices v^1 and v^2

Theorem 3.3 : If v^1 and v^2 are vertices of graph $\Re$ such that distance between them is 2 and v^3 is in the common neighbor hood of v^1 and v^2 then $IB(\Re) \leq M-2$ where M is the number of incident edges with vertices v^1, v^2 *and* v^3.

Proof: v^1 and v^2 are vertices of graph $\Re$ such that distance between them is 2 and v^3 is in the common neighbor hood of v^1 and v^2. If we define the graph ψ after removing edges incident to v^1, v^2 *and* v^3 from $\Re$ except the common edge v^1v^3 *and* v^2v^3 .

Here if W is max I set of ψ then we can say that both vertices v^1 and v^2 belong to W and v^3 does not belong to W. Now if $\overline{\psi}$ is the graph obtained by removing edge v^1v^3 & v^2v^3 from ψ then $\beta_0(\overline{\psi}) > \beta_0(\psi)$ means $IB(\psi) = 2$.

So considering the statement of lemma 3.1, $IB(\Re) \leq M-2$ where M is the number of incident edges with vertices v^1, v^2 *and* v^3.

Theorem 3.4 : $|IB(\Re)|$ will be always less than or equal to the minimum cardinality of the set Φ. Where Φ is the set of degree of vertex whose removal does not increase or decrese independence number.

Proof: Consider the vertex ρ whose removal does not increase or decrease Independence number means mathematically $\beta_0(\Re-\rho) = \beta_0(\Re)$. Let there exist a max I set ψ of the graph $\Re$ which does not include ρ . Let Φ be the new graph after removing all edges incident to ρ then $\psi \cup \rho$ is I set in graph Φ. So $|IB(\Re)|$ will be always less than or equal to the number of edges incident with ρ. Hence proved.

Theorem 3.5 : Suppose $\Re$ is the graph then $|IB(\Re)| \leq \min(\Omega, \Phi)$ where $\Omega = \beta_0(\Re)$ and Φ is the set of degree of vertex whose removal does not increase or decrese independence number.

Proof: Consider the vertex ρ whose removal does not increase or decrease I number means mathematically $\beta_0(\Re-\rho) = \beta_0(\Re)$. Let there exist a max I set ψ of the graph $\Re$ which does not include ρ. Now if we remove $\min(\alpha, \beta)$ edges then $\psi \cup \rho$ is an I set in new graph where α equals to $\beta_0(\Re)$ edges incident to ρ & β equals to number of edges incident to ρ Which proves the theorem.

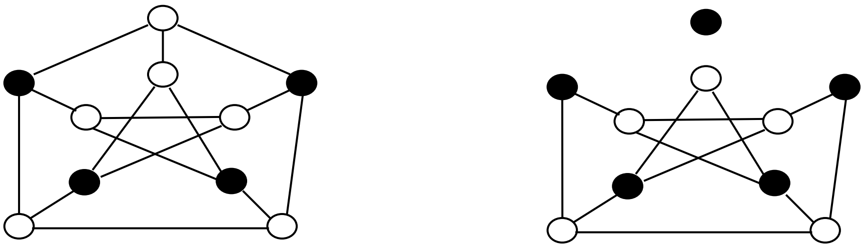
Illustration 3.6 : Here we show, $IB(\Re) \leq M-1$ where M is the number of incident edges with vertices v^1 and v^2 with $M = 1+2 = 3$

Figure 3.6.a

Illustration 3.7 : Here we show, $IB(\Re) \leq M-2$ where M is the number of incident edges with vertices v^1, v^2 *and* v^3 with $M = 1+1+1 = 3$

Figure 3.7.a

Illustration 3.8 : Here we show, $IB(\Re) \leq 3$, for every vertex of the graph $\Re$



$I(\Re)=4$ $\qquad I(\psi)=5$

Figure 3.8.a

Illustration 3.9 : Here we consider the graph in which $I\,B(\Re)=1$

$I(\Re)=1$ $\qquad I(\psi)=2$

Figure 3.9.a

IV. IMINUS NUMBER

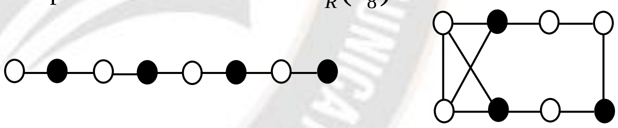
Theorem 4.1 : The cardinality of $I\,R(\Re)$ will be less than or equal to Δ if the graph $\Re$ is having exactly Δ max I set.

Proof: Suppose $\Re$ is the graph having θ max I set named as $\Phi^1, \Phi^2, \Phi^3,,,,,, \Phi^\theta$. Now if we add θ edges $e^1, e^2, e^3,,,,, e^\theta$ between two vertices of $\Phi^1, \Phi^2, \Phi^3,,,,,, \Phi^\theta$ respectively to form a new graph ψ having max I set as δ then cardinality of δ will be less than or equal to $\beta(\Re)$.

Let us assume cardinality is equal to $\beta(\Re)$ then δ is an I set in $\Re$ also. Here cardinality of δ is equal to $\beta(\Re)$ and δ is an max I set in $\Re$. So δ will be one of the $\Phi^1, \Phi^2, \Phi^3,,,,,, \Phi^\theta$ say Φ^1.

The subgraph induced by vertices of δ in ψ will be same as the subgraph induced by vertices of Φ^1 in $\Re$. But δ is an I set in ψ having no edges between two vertices. But according to starting of assumption, there is an edge e^1 in Φ^1. Which is the contradiction. So only one option is there and so cardinality of δ will be less than to $\beta(\Re)$ which proves the statement.

Illustration 4.2 : consider an example of P_8. For every Independent set we can see $I_R(P_8)=3$

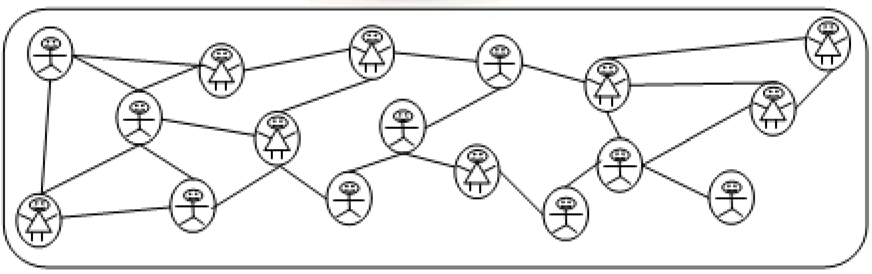


$I(\Re)=4$ $\qquad I\,R(\psi)=3$

Figure 4.2.a

V. APPLICATION

Application 5.1 Social Network Theory



Kelleher presented research on dominating sets in social network graphs.[13]. In social network theory we are interested in relationships among member of group. Relationships can be defined in terms of dichotomous

International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 11 Issue: 8s
***Article Received:** 25 June 2023 **Revised:** 12 July 2023 **Accepted:** 30 August 2023*

property. Here this property between any two holds or not.

One can define a graph for this social network in which all these actors will play roll of vertices and their relation will be edge between them. If two actors are in direct relationship means in graph those two vertices will be adjacent to each other. If any two actors are not having direct relationship means in the graphical representation we will not find these vertices adjacent. However connected graph is also a concept where vertices are not connected directly but indirectly connection is there same as relationship between actors may be indirect. Logic of I plus number or I minus number for the optimization of these graph may be used. If one can develop programme for this social network by converting graphs into augmented matrix then optimization will be easy and quick task.

Application 5.2 : optimization of field through complete grid graph

Optimization is the key to utilize minimum resources to gain maximum benefits. Lighting area, watering farm with fountain method, broad casting coverage with minimum radio station, work assignment problems all these field may be taken care of with the optimization using complete grid graph allocating area according to field. Following table represents Iplus number and Iminus number of complete grid graphs which is useful for all above fields for effectiveness with cost cutting. One may take help of programming as it is symmetric and generalized.

$Graph\,G$	Iplus number ($I\ \ B(G)$)	Iminus umber ($I\,R(G)$)
K_n – complete graph	1	Not possible
S_n – star graph	$n-1$	1
C_n	1, for odd no of vertices 2, for even no of vertices	2, for odd no of vertices 6, for even no of vertices 5, no of vertices is 5
P_n – path	2, for odd no of vertices 1, for even no of vertices	1, for odd no of vertices 3, for even no of vertices more than 3
Grid graphs $(P_m \times P_n)$		
m=2	2	2
m=3	2, for even no of n 3, for odd no of n	2
m=4	2	2
m=5	2	2

REFERENCES

1. "Domination related parameters and critical graphs", Ph.D Thesis, Saurashtra University, Dhaval D. Pandya and Dr. D. K. Thakkar ,2015.
2. Teresa W. Haynes, Stephen T. Hedetniemi, Peter J. Slater. "Fundamental of Dominatin in Graphs". Marcel Dekker Inc. New york.
3. J. K. Fink and M. S. Jacobson, "n-domination in graphs" Graph Thoery With Application To Algorithms And Computer Science. Pages: 283-300, (Kalamazoo M I 1984). Wiley.
4. D. K. Thakkar and J. C. Bosamiya,"Graph critical with respect to variants of domination" Ph.D thesis, Saurashtra university ,2011.
5. D. K. Thakkar and G.J.Vala " A study of some topics in Graph Theory ", Ph. D thesis, Saurashtra university, 2010.
6. D. K. Thakkar and J. C. Bosamiya,"Graph critical with respect to variants of domination" Ph.D thesis, Saurashtra university ,2011.
7. D. K. Thakkar and J. V. Changela, "Mathematical Modelling"Ph.D thesis, Saurashtra university ,2011. ,
8. Vala G. J , " A study of some topics in Graph Theory ", Ph. D thesis, Saurashtra James
9. university, 2010.
10. [9] J. F. Fink, M.S. Jacobson, L.F.Kinch and J.Roberts, "The bondage number of graph", Discrete Math. 86(1990)47-57
11. S.Ao. "Independent domination critical graphs", Master Thesis, University of Victoria, Victoria BC, Canada, (1984)
12. Tim Nieberg, "Independent and Dominating Sets in Wireless Communication Graphs" Ph. D thesis, Netherland, 2006.
13. L. L. Kelleher. "Domination in Graphs and its application to Social Network Theory . PhD thesis, Northeastern University. 1985
14. D. D. Pandya, "Bondage numbers for independence in graphs" , Journal of Emerging Technologies and Innovative Research, Vol 9, Issue 6, 2022.