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Advancements in Common Fixed Point Theorems for ϵ-Chainable Fuzzy Metric Spaces with Absorbing Maps

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Abstract : This paper explores advancements in common fixed point theorems within the context of ϵ-chainable fuzzy metric spaces, specifically utilizing absorbing maps. We extend the theoretical framework by introducing new conditions and methodologies that leverage the properties of absorbing maps to establish common fixed points. Our research builds on and enhances the foundational work in fuzzy metric space theory, providing a comprehensive analysis of ϵ-chainable spaces. The results demonstrate the effectiveness of these new approaches in identifying common fixed points, thereby offering significant contributions to the field. The developed theorems not only reinforce existing knowledge but also pave the way for future studies and applications in mathematical and computational disciplines where fuzzy metrics and chainability are essential.

Key words: *Fuzzy Metric Spaces, Fuzzy-2 Metric Spaces.*

1 INTRODUCTION

In this paper, we establish several common fixed point theorems for both set-valued and single-valued mappings in fuzzy metric spaces and fuzzy 2-metric spaces. We introduce the concept of fuzzy metric spaces in various forms, focusing on different types and their properties. Many of our results pertain to either commuting mappings or mappings that exhibit weak commutativity as introduced by Seesa. In 1986, Jungck introduced the notion of compatible maps, which has since been extensively used to prove existence theorems in common fixed point theory. Additionally, Pant and Pant explored common fixed points of pairs of non-compatible maps and the property E.A in fuzzy metric spaces. Our results generalize numerous significant fixed point theorems and broaden the scope for studying common fixed points under contractive-type conditions. Despite these advancements, fixed point theorems in fuzzy metric spaces remain an area with substantial potential for further exploration.

2 PRELIMINARIES

Definition 2.1 *[15] Let X be a non- empty set.Then a function A with domain X and value in* $[0,1]$ *is said to be a fuzzy set in X.*

Definition 2.2 *A $t-$ norm or more precisely triangular norm $*$ is a binary operation defined on* $[0,1]$ *such that for all $a, b, c, d \in$* $[0,1]$ *following conditions are satisfied:*

1. $a * 1 = 1$
2. $a * b = b * a$
3. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$
4. $a * (b * c) = (a * b) * c$

Definition 2.3 [3] *The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary non empty set $*$ is continuo $t-$ norm and M is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$ and $s, t > 0$.*

1. $M(x, y, 0) = 0$.
2. $M(x, y, t) = 1 \ \forall \ t > 0 \Leftarrow x = y$.
3. $M(x, y, t) = M(y, x, t)$
4. $M(x, y, t) * M(y, z, s) \leq M(x, z, s + t) = M(y, x, t)$ where $t \in [0,1]$
5. $M(x, y, .): [0, \infty] \rightarrow [0,1]$ is left continuous.

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Definition 2.4 [14] *A sequence $\{x_n\}$ in a fuzzy metric space $(X, M,*)$ is called cauchy if for every $\epsilon > 0$ and $t > 0$ there exists an integer $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) = 1 - \epsilon$, $\forall n, m \geq n_0$.A fuzzy metric space $(X, M,*)$ is said to be completer if every cauchy sequence in X converges to point in X. A sequence $\{x_n\}$ in X is convergent to $x \in X$ if $lim_{n\to\infty} M(x_n, x_m, t) > 1 - \epsilon$ for each $t > 0$,there exist $n_0 \in \mathbb{N}$*

Definition 2.6 [6] *A pair (A, B) of self maps of fuzzy metric space $(X, M,*)$ is said to be reciprocal continuous if $lim_{n\to\infty} ABx_n = Ax$ and $lim_{n\to\infty} BAx_n = Bx$ whenever there exists a sequence $x_n \in X$ such that $lim_{n\to\infty} Ax_n = Bx_n = x$ for some $x \in X$.*

If A and B are continuous then they are obviously reciprocally continuous but not converse need not to be true.

Definition 2.7 [14]*Let $(X, M,*)$ be a fuzzy metric space and $\epsilon > 0$. A finite sequence $x = x_0, x_1, x_2 \dots, x_n = y$ is called ϵ chain from $x \quad to \quad y$ if $(x_i, x_{i-1} M, t) > 1 - \epsilon$ for all $t > 0$ and $i = 1,2,3, \dots n$.*

A fuzzy metric space $(X, M,*)$ is called ϵ- chain from $x \quad to \quad y$.

Definition 2.8 [13]*Let P and Q be two self maps on a fuzzy metric space $(X, M,*)$ then P is called $Q -$ absorbing if there exists a positive integer $R > 0$ such that $M(Qx, QPx, t) \geq M(Qx, QPx, \frac{t}{R})$ for al $x \in X$. Similarly Q is called $P -$ absorbing if there exists a positive integer $R > 0$, such that $M(Px, PQx, t) \geq M(Px, Qx, \frac{t}{R})$ for all $x \in X$.*

Proposition 2.9 [14]*Let P and Q be two self maps on a fuzzy metric space $(X, M,*)$. Assume that (P, Q) is reciprocal continuous then (P, Q) is semi compatible if and only if (P, Q) is compatible.*

Lemma 2.10 *Let $(X, M,*)$ be fuzzy metric space then for all $x, y \in X$, $(M, X, .)$ is non decreasing.*

Lemma 2.11 *Let $(X, M,*)$ be fuzzy metric space if there exists $R \in (0,1)$ such that $M(x, y, kt) \geq M(x, y, t)$ for all $x, y \in X$, and $t > 0$ then $x = y$.*

Lemma 2.12 [8]*A sequence $\{x_n\}$ in a fuzzy metric space $(X, M,*)$ converges to a point $x \in X$ if and only if $M(x_n, x, t) = 1 \; \forall \; t > 0$.*

Example 2.13[6] *Let $X = [0,1]$ be a metric space we define $P, Q: X \to X$ by*

$$Px = \begin{cases} 1, & if \quad x \neq 1 \\ 0, & if \quad x = 1 \end{cases} and \; Qx = \begin{cases} 1 for \quad x \in X \\ \end{cases}$$

Then the maps P and $Q -$ absorbing for any $R > 1$ but the pair of maps (P, Q) do not commute at their coincidence point $x = 0$.

Lemma 2.14[6] *If for all $x, y \in X$ $t > 0$ and $0 < k < 1$ $M(x, y, kt) \geq M(x, y, t)$ then $x = y$*

Proof. Suppose that there exists $0 < k < 1$ such that $M(x, y, kt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$. Then $M(x, y, t) \geq M(x, y, \frac{t}{k})$ and so $M(x, y, t) \geq M(x, y, \frac{t}{k^n})$ for positive integer n taking limit as $M_{lim\to\infty}(x, y, t) \geq 1$ and hence $x = y$.

Following theorem is proved by Syed Shahnawaz Ali and Jainendra Jain[14]

Theorem 2.17 *Let A, B, S, T, P and Q be self mappings of a complete $\epsilon -$ chainable fuzzy metric space $(X, M,*)$ with continuous $t -$ norm satisfying the conditions:*

(a) $P(X) \subseteq ST(X)$ and $Q(X) \subseteq AB(X)$
(b) Q is ST absorbing
(c) $AB = BA$, $ST = TS$, $PB = BP$ and $QT = TQ$
(d) $\exists \; k \in (0,1)$ such that

$$M(Px, Qy, kt) \geq min\{M(ABx, STy, t), M(Px, ABy, t), \frac{M(ABx, Qx, t) + M(Px, STy, t)}{2}, M(STy, Qy, t)\}$$

for every $x, y \in X$ and $t > 0$. If (P, AB) is reciprocally continuous semi compatible maps. Then A, B, S, T, P and Q have unique common fixed point in X.

3 Main result

Theorem 3.1 *Let A, B, S, T, L and M be self mappings of a complete $\epsilon -$ chainable fuzzy metric space $(X, M,*)$ with continuous $t -$ norm defined by $a * b = min\{a, b\}$ with satisfying:*

1. $L(X) \subseteq ST(X)$ and $M(X) \subseteq AB(X)$
2. M is ST absorbing
3. $ST = TS$, $LB = BL$, $MT = TM$
4. $\exists \quad k \in (0,1)$ such that for some $\phi \in \Phi$ every $x, y \in X$ and $t > 0$ where $a, b \geq 0$ with a and b can not be simultaneously 0.

$$\phi\{M(Lx, My, kt), \frac{M(ABx, Lx, t) + M(Lx, STy, t)}{2}, M(STy, ABx, t),$$

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$$M(STy, My, kt), \frac{aM(ABy, My, t) + bM(ABy, STy, t)}{aM(STy, My, t) + b}\} \geq 0 \quad \ldots (iv)$$

If $\{L, AB\}$ is reciprocally continuous semi compatible maps, then A, B, S, T, L and M have a unique fixed point X.

Proof. Let $x_0 \in X$ be any arbitrary be any point from (i) there exists $x_1, x_2 \in X$ such that $Lx_0 = STx_1 = y_0$ and $Mx_1 = ABx_2 = y_1$

Inductively we can construct sequence $\{x_n\}$ and $\{y_n\}$ in X.
such that $Lx_{2n} = STx_{2n+1} = y_{2n}$ and $Mx_{2n+1} = ABx_{2n+2} = y_{2n+1}$ for $n = 0,1,2 \ldots$
Taking $x = x_{2n}, y = x_{2n+1}$ for $t > 0$ in (iv)

$$\phi\{M(Lx_{2n}, Mx_{2n+1}, kt), \frac{M(ABx_{2n}, Lx_{2n}, t) + M(Lx_{2n}, STx_{2n+1}, t)}{2},$$

$$, M(STx_{2n+1}, ABx_{2n}, t), M(STx_{2n+1}, Mx_{2n+1}, kt),$$

$$\frac{aM(ABx_{2n+1}, Mx_{2n+1}, t) + bM(STx_{2n+1}, ABx_{2n+1}, t)}{aM(STx_{2n+1}, Mx_{2n+1}, t) + b}\} \geq 0$$

$$\phi\{M(y_{2n}, y_{2n+1}, kt), \frac{M(y_{2n-1}, y_{2n}, t) + M(y_{2n}, y_{2n}, t)}{2},$$

$$M(y_{2n}, y_{2n-1}, t), M(y_{2n}, y_{2n+1}, kt), \frac{aM(y_{2n}, y_{2n+1}, t) + bM(y_{2n}, y_{2n}, t)}{aM(y_{2n}, y_{2n+1}, t) + b}\} \geq 0$$

$$\phi\{M(y_{2n}, y_{2n+1}, kt), \frac{M(y_{2n-1}, y_{2n}, t) + 1}{2}, M(y_{2n}, y_{2n-1}, t), M(y_{2n}, y_{2n+1}, kt), 1\} \geq 0$$

ϕ is non- increasing in second argument

$$\phi\{M(y_{2n}, y_{2n+1}, kt), M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M(y_{2n}, y_{2n+1}, kt)\} \geq 0$$

$$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t)$$

Similarly we put $x = x_{2n+2}, y = x_{2n+1}$ in (iv) we have

$$\phi\{M(Lx_{2n+2}, Mx_{2n+1}, kt), \frac{M(ABx_{2n+2}, Lx_{2n+2}, t) + M(Lx_{2n+2}, STx_{2n+1}, t)}{2}),$$

$$M(STx_{2n+1}, ABx_{2n+2}, t), M(STx_{2n+1}, Mx_{2n+1}, kt),$$
$$\frac{aM(ABx_{2n+1}, Mx_{2n+1}, t) + bM(ABx_{2n+1}, STx_{2n+1}, t)}{aM(STx_{2n+1}, Mx_{2n+1}, t) + b}\} \geq 0$$

$$\phi\{M(y_{2n+2}, y_{2n+1}, kt), \frac{M(y_{2n+1}, y_{2n+2}, t) + M(y_{2n+2}, y_{2n}, t)}{2}),$$

$$M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, kt), \frac{aM(y_{2n}, y_{2n+1}, t) + bM(y_{2n}, y_{2n}, t)}{aM(y_{2n}, y_{2n+1}, t) + b}\} \geq 0$$

$$\phi\{M(y_{2n+2}, y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, kt) \geq 0$$

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$$M(y_{2n+2}, y_{2n+1}, kt) \geq M(y_{2n+1}, y_{2n}, t)$$

therefore for all n even or odd, we have

$$M(y_n, y_{n+1}, kt) \geq M(y_n, y_{n-1}, t)$$

$$M(y_n, y_{n+1}, t) \geq M(y_n, y_{n-1}, \tfrac{t}{k}) \geq M(y_n, y_{n-1}, \tfrac{t}{k^2}) \dots \geq M(y_n, y_{n-1}, \tfrac{t}{k^n})$$
$$n \to \infty$$

So, $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ and for any $t > 0$. For each $\epsilon > 0$ and each $t > 0$ we can $L = ABz$. Choose $n_0 \in N$such that $M(y_n, y_{n+1}, t) > 1 - \epsilon$ for $m, n \in \mathbb{N}$, we suppose $m \geq n$. Then we have that

$$M(y_n, y_m, t) \geq M(y_n, y_{n+1}, \frac{t}{m-n}) * M(y_{n+1}, y_{n+2}, \frac{t}{m-n})$$

$$* \dots * M(y_n, y_{n-1}, \tfrac{t}{m-n})$$

$$M(y_n, y_m, t) \geq (1-\epsilon) * (1-\epsilon) * \dots * (1-\epsilon)$$

$$\geq (1-\epsilon)$$

And hence $\{y_n\}$ is a cauchy sequence in X. Since X is complete therefore $\{y_n\} \to z$ in X and its subsequence $\{ABx_{2n}\}$,$\{Mx_{2n+1}\}$, $\{STx_{2n+1}\}$, $\{Lx_{2n}\}$ also converges to z. Since X is ϵ chain from x_n to x_{n+1} that is there exists a finite sequence $x_n = y_1, y_2, \dots y_l = x_{2n+1}$ such that $M(y_i, y_{i-1}, t) > 1 - \epsilon$ for all $t > 0$ and $i = 1,2, \dots$.

Thus we have

$$M(x_n, x_{n+1}, t) \geq M(y_1, y_2, \tfrac{t}{l}) * M(y_2, y_3, \tfrac{t}{l}) * \dots * M(y_{l-1}, y_l, \tfrac{t}{l})$$

$$(1-\epsilon) * (1-\epsilon) * \dots (1-\epsilon)$$

$$\geq (1-\epsilon)$$

for $m > n$

$$M(x_n, x_m, t) \geq M(x_n, x_{n+1}, \frac{t}{m-n}) * M(x_{n+1}, x_{n+2}, \frac{t}{m-n}) * \dots * M(x_{m-1}, x_m, \frac{t}{m-n})$$

$$(1-\epsilon) * (1-\epsilon) * \dots (1-\epsilon)$$

$$\geq (1-\epsilon)$$

And so $\{x_n\}$ is a Cauchy sequence in X and hence there exists $x \in X$ such that $x_n \to z$. By the reciprocally continuity and semi compatibility of maps (L, AB) we have

$$lim_{n\to\infty} L(AB)x_{2n} = Lz, \quad lim_{n\to\infty} AB(L)x_{2n} = ABz$$

$$and \quad lim_{n\to\infty} L(AB)x_{2n} = ABz,$$

which implies

$$lim_{n\to\infty} L(AB)x_{2n} = ABz$$

Step I:- By putting $x = z, y = x_{2n+1}$ in (iv) we get

$$\phi\{M(Lz, Mx_{2n+1}, kt), \frac{M(ABz, Lz, t) + M(Lz, STx_{2n+1}, t)}{2}, M(STx_{2n+1}, ABz, t),$$

$$M(STx_{2n+1}, Mx_{2n+1}, kt), \frac{aM(ABx_{2n+1}, Mx_{2n+1}, t) + bM(ABx_{2n+1}, STx_{2n+1}, t)}{aM(STx_{2n+1}, Mx_{2n+1}, t) + b}\} \geq 0$$

Letting $n \to \infty$

$$\phi\{M(Lz, z, kt), \frac{M(Lz, Lz, t) + M(Lz, z, t)}{2}, M(z, Lz, t), M(z, z, kt),$$

$$\frac{aM(z, z, t) + bM(z, z, t)}{aM(z, z, t) + b}\} \geq 0$$

$$\phi\{M(Lz, z, kt), \frac{1 + M(Lz, z, t)}{2}, M(z, Lz, t), 1, 1\} \geq 0$$

ϕ is non-decreasing in second, fourth and fifth arguments

$$M(Lz, z, kt) \geq M(Lz, z, t)$$

Thus we get

$$Lz = z = ABZ$$

Step II:-

By putting $x = Bz, y = x_{2n+1}$ in (iv)

we get,

$$\phi\{M(LBz, Mx_{2n+1}, kt), \frac{M(AB(Bz), L(Bz), t) + M(L(Bz), STx_{2n+1}, t)}{2}),$$

$$M(STx_{2n+1}, AB(Bz), t), M(STx_{2n+1}, Mx_{2n+1}, kt),$$

$$\frac{aM(ABx_{2n+1}, Mx_{2n+1}, t) + bM(ABx_{2n+1}, STx_{2n+1}, t)}{aM(STx_{2n+1}, Mx_{2n+1}, t) + b}\} \geq 0$$

Since,

$$AB = BA, LB = BL$$

therefore $AB(Bz) = B(ABz) = Bz$ and $L(Bz) = B(Lz) = Bz$ letting $n \to \infty$ we get

$$\phi\{M(Bz, z, kt), \frac{M(Bz, Bz, t) + M(Bz, z, t)}{2}), M(Bz, z, t), M(z, z, kt),$$

$$\frac{aM(z, z, t) + bM(z, z, t)}{aM(z, z, t) + b}\} \geq 0$$

$$\{M(Bz, z, kt), \frac{1 + M(Bz, z, t)}{2}), M(z, Bz, t), 1, 1\} \geq 0$$

ϕ is non-decreasing in second, fourth and fifth arguments

$$\phi\{M(Bz, z, kt), \frac{M(Bz, Bz, t) + M(Bz, z, t)}{2}), M(z, Bz, t), M(Bz, z, t),$$

$$\frac{aM(Bz, z, t) + bM(Bz, z, t)}{aM(Bz, z, t) + b}\} \geq 0$$

$$\phi\{M(Bz, z, kt), M(z, Bz, t), M(z, Bz, t), M(Bz, z, t)\} \geq 0$$

i.e.

$$M(Bz, z, kt) \geq M(Bz, z, t)$$

By lemma 2.14

$$Lz = Az = Bz = z$$

Since $L(X) \subseteq ST(X)$ there exist $u \in X$ such that $z = Lz = STu$.

Step III:-

By putting $x = x_{2n}, y = u$ in (iv)

we get

$$\phi\{M(Lx_{2n}, Mu, kt), \frac{M(ABx_{2n}, Lx_{2n}, t) + M(Lx_{2n}, STu, t)}{2}), M(STu, ABx_{2n}, t),$$

$$M(STu, Mu, kt), \frac{aM(ABu, Mu, t) + bM(ABu, STu, t)}{aM(STu, Mu, t) + b}\} \geq 0$$

Letting $n \to \infty$

$$\phi\{M(z, Mu, kt), \frac{M(z,z,t) + M(z,z,t)}{2}), M(z,z,t), M(z, Mu, kt),$$

$$\frac{aM(z, Mu, t) + bM(z,z,t)}{aM(z, Mu, t) + b}\} \geq 0$$

$$\phi\{M(z, Mu, kt), \tfrac{1+1}{2}), 1, M(z, Mu, kt), 1\} \geq 0$$

ϕ is non-decreasing in second, fourth and fifth arguments

$$\phi\{M(z, Mu, kt), M(z, Mu, t), M(z, Mu, t), M(z, Mu, t), M(z, Mu, t)\} \geq 0$$

i.e.

$$M(Bz, z, kt) \geq M(Bz, z, t)$$

$$z = Mu = STu$$

Since M is $ST -$ absorbing then we have

$$M(STu, STMu, t) \geq M(STu, Mu, \tfrac{t}{R}) = 1$$

Step IV:-

By putting $x = x_{2n}, y = z$ in (iv) we get

$$\phi\{M(Lx_{2n}, Mz, kt), \frac{M(ABx_{2n}, Lx_{2n}, t) + M(Lx_{2n}, STz, t)}{2}), M(STz, ABx_{2n}, t),$$

$$M(STz, Mz, kt), \frac{aM(ABz, Mz, t) + bM(ABz, STz, t)}{aM(STz, Mz, t) + b}\} \geq 0$$

Letting $n \to \infty$

$$\phi\{M(z, Mz, kt), \frac{M(z,z,t) + M(z,z,t)}{2}), M(z,z,t),$$

$$M(z, Mz, kt), \frac{aM(z, Mz, t) + bM(z,z,t)}{aM(z, Mz, t) + b}\} \geq 0$$

$$\phi\{M(z, Mz, kt), \tfrac{1+1}{2}), 1, M(z, Mz, kt)1\} \geq 0$$

ϕ is non-decreasing in second, fourth and fifth arguments

$$\phi\{M(z, Mz, kt), M(z, Mz, t), M(z, Mz, t), M(z, Mz, t), M(z, Mz, t)\} \geq 0$$

Hence by lemma 2.14

$$z = Mz = STz$$

Step V:-

By putting $x = x_{2n}, y = Tz$ in (iv)
we get,

$$\phi\{M(Lx_{2n}, MTz, kt), \frac{M(ABx_{2n}, Lx_{2n}, t) + M(Lx_{2n}, STTz, t)}{2}), M(STTz, ABx_{2n}, t),$$

$$M(STTz, MTz, kt), \frac{aM(ABTz, MTz, t) + bM(ABTz, STTz, t)}{aM(STTz, MTz, t) + b}\} \geq 0$$

Since $ST = TS$ and $MT = TM$ therefore $M(Tz) = T(Mz) = Tz\ ST(Tz) = T(STz) = Tz$
and letting $n \to \infty$ we get

$$\phi\{M(z, Tz, kt), \frac{M(z, z, t) + M(z, Tz, t)}{2}), M(Tz, z, t),$$

$$M(Tz, Tz, kt), \frac{aM(z, Tz, t) + bM(z, Tz, t)}{aM(Tz, Tz, t) + b}\} \geq 0$$

$$\phi\{M(z, Tz, kt), \frac{1 + M(z, Tz, t)}{2}), M(Tz, z, t), 1, \frac{(a + b)M(z, Tz, t)}{(a + b)}\} \geq 0$$

$$\phi\{M(z, Tz, kt), \frac{1 + M(z, Tz, t)}{2}), M(Tz, z, t), 1, M(z, Tz, t)\} \geq 0$$

ϕ is non-decreasing in second, fourth and fifth arguments

$$\phi\{M(z, Tz, kt), M(z, Tz, t), M(z, Tz, t), M(z, Tz, t), M(z, Tz, t)\} \geq 0$$

i.e.

$$\phi\{M(z, Tz, kt) \geq M(z, Tz, t)\}$$

Hence by lemma 2.14

$$z = Tz = Mz = STz$$

Therefore,

$$z = Az = Bz = Sz = Tz = Lz = Mz$$

That is z is a fixed point of A, B, X, T, L and M.

UNIQUENESS:-

Let w be another fixed point of A, B, X, T, L and M; therefore putting $x = z$ and $y = w$ in (iv) we get

$$\phi\{M(Lz, Mw, kt), \frac{M(ABz, Lz, t) + M(Lz, STw, t)}{2}), M(STw, ABz, t),$$

$$M(STw, Mw, kt), \frac{aM(ABw, Mw, t) + bM(ABw, STw, t)}{aM(STw, Mw, t) + b}\} \geq 0$$

$$\phi\{M(z, w, kt), \frac{M(z, z, t) + M(z, w, t)}{2}), M(w, z, t), M(w, w, kt),$$

$$\frac{aM(w, w, t) + bM(w, w, t)}{aM(w, w, t) + b}\} \geq 0$$

$$\phi\{M(z, w, kt), \frac{1 + M(z, w, t)}{2}), M(w, z, t)1, 1\} \geq 0$$

ϕ is non-decreasing in second, fourth and fifth arguments

$$\phi\{M(z,w,kt), \frac{M(z,w,t)+M(z,w,t)}{2}), M(z,w,t), M(z,w,t)\} \geq 0$$

i.e.

$$M(z,w,kt) \geq M(z,w,t)$$

Hence by lemma 2.14 $z = w$, hence z is a unique fixed point of X. This completes the proof.

Corollary 3.2 *Let A, B, S and T be self mapping of a complete $\epsilon -$ chainable fuzzy metric space $(X, M, *)$ with continuous $t -$ norm defined by $a * b = min\{a,b\}$ satisfying $(i) - (iv)$ of theorem and there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t > 0$, where $a, b \geq 0$.*

1. $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$
2. B is absorbing T
3. $\exists \quad k \in (0,1)$ such that for some $\phi \in \Phi$ every $x, y \in X$ and $t > 0$.

$$\phi\{M(Ax,By,kt), \frac{M(Ax,Sx,t)+M(Ax,Ty,t)}{2}), M(Ty,Sx,t),$$

$$M(Ty,By,kt), \frac{aM(Ay,By,t)+bM(Ay,Ty,t)}{aM(By,Ty,t)+b}\} \geq 0$$

If A is absorbing S and both are reciprocal then A, B, S, T have a unique fixed point in X.

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