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# Literature Review on Vague Set Theory in Different Domains

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*Abstract*—Problem of decision making is a crucial task in every business. This decision making job is found very difficult when it is depends on the imprecise and vague environment, which is frequent in recent years. Vague sets are an extension of Fuzzy sets. In the fuzzy sets, each object is assigned a single value in the interval [0,1] reflecting its grade of membership. This single value does not allow a separation of evidence for membership and evidence against membership. Gau et al. proposed the notion of vague sets, where each object is characterized by two different membership functions: a true membership function and a false membership function. This kind of reasoning is also called interval membership, as opposed to point membership in the context of fuzzy sets. In this paper, reviews the related works on the decision making by using vague sets in different fields.

*Keywords-Vague set, Fuzzy set, Application*

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## I. INTRODUCTION

Vague or fuzzy concepts are fundamental to natural language, playing a central role in communications between individuals within a shared linguistic context. In fact Russell [1] even goes so far as to claim that all natural language concepts are vague. Yet often vague concepts are either viewed as problematic because of their susceptibility to Sorities paradoxes or at least as somehow 'second rate' when compared with the more precise concepts of the physical sciences, mathematics and formal logic. This view, however, does not properly take account of the fact that vague concepts seem to be an effective means of communicating information and meaning. Sometimes more effective, in fact, than precise alternatives. Somehow, knowing that 'the robber was tall' is more useful to the police patrolling the streets, searching for suspects, than the more precise knowledge that 'the robber was exactly 1.8 metres in height'. But what is the nature of the information conveyed by fuzzy statements such as 'the robber was tall' and what makes it so useful? It is an attempt to answer this and other related questions that will be the central theme of this volume. Throughout, we shall unashamedly adopt an Artificial Intelligence perspective on vague concepts and not even attempt to resolve longstanding philosophical problems such as Sorities paradoxes. Instead, we will focus on developing an understanding of how an intelligent agent can use vague concepts to convey information and meaning as part of a general strategy for practical reasoning and decision making. Such an agent could be an artificial intelligence program or a human, but the implicit assumption is that their use of vague concepts is governed by some underlying internally consistent strategy or algorithm. For simplicity this agent will be referred to using the pronoun You. This convention is borrowed from Smets work on the Transferable Belief Model although the focus of this work is quite different. We shall immediately attempt to reduce the enormity of our task by restricting the type of vague concept to be considered. For the purposes of this volume we shall restrict our attention to concepts as identified by words such as adjectives or nouns that can be used to describe an object or instance.

## II. VAGUE SET THEORY

In this section, we present now some preliminaries on the theory of vague sets (VS).

**Definition 1:** A vague set A in the universe of discourse U is characterized by two membership functions given by:

1- A truth membership function

$$t_A : U \rightarrow [0,1]$$

and

2- A False membership function

$$f_A : U \rightarrow [0,1]$$

where $t_A(u)$ is a lower bound of the grade of membership of u derived from the 'evidence for u', and $f_A(u)$ is a lower bound of the negation of u derived from the 'evidence against u and

$$t_A(u) + f_A(u) \le 1$$

Thus the grade of membership of u in the vague set A is bounded by a sub interval $[t_A(u), 1 - f_A(u)]$ of [0, 1]. This indicates that if the actual grade of membership is μ(u), then

$$t_A(u) \le t_A(u)$$

The vague set is written as

$$A = \{(u, [t_A(u), f_A(u)]) \mid u \in U\}$$

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where the interval $[t_A(u), 1 - f_A(u)]$ is called the 'vague value' of u in A and is denoted by $V_A(u)$.

It is worth to mention here that interval-valued fuzzy sets (i-v fuzzy sets) [2] are not vague sets. In i-v fuzzy sets, an interval valued membership value is assigned to each element of the universe considering the 'evidence for u' only, without considering 'evidence against u'. In vague sets both are independently proposed by the decision maker. This makes a major difference in the judgment about the grade of membership.

**Definition 2:** A vague set A of a set U is called

1- the zero vague set of U if $t_A(u) = 0$ and $f_A(u) = 1$ for all $u \in U$

2- the unit vague set of U if $t_A(u) = 1$ and $f_A(u) = 0$ for all $u \in U$

3- the $\alpha$ -vague set of U if tA(u) = $\alpha$ and fA(u) = 1− $\alpha$ for all u 2 U, where $\alpha \in (0, 1)$.

Let D[0, 1] denotes the family of all closed sub-intervals of [0, 1]. Now we define refined minimum (briefly, rmin) and order " $\leq$ " on elements $D_1 = [a_1, b_1]$ and $D_2 = [a_2, b_2]$ of D[0, 1] as:

$$\text{rmin } (D_1, D_2) = [\min\{a_1, a_2\}, \min\{a_1, a_2\}]$$
$$D_1 \leq D_2 \leftrightarrow a_1 \leq a_2 \wedge b_1 \leq b_2$$

Similarly we can define $\geq$, = and rmax. Then concept of rmin and rmax could be extended to define rinf and rsup of infinite number of elements of D[0, 1].

It is that L = {D[0, 1], rinf, rsup, $\leq$} is a lattice with universal bounds [0, 0] and [1, 1].

### III. APPLICATION OF VAGUE SET THEORY IN DIFFERENT DOMAINS

The following table 1 represents the related works done on the different domains by using Vague Set Theory.

TABLE 1: RELATED WORKS USING VAGUE SET THEORY ON THE DIFFERENT DOMAINS

| AUTHORS | PAPER TITLE | METHODS | APPLICATION |
|---|---|---|---|
| Wei, Bo, et al [3] | Fuzzy GML Modeling Based on Vague Soft Sets | Vague Soft Set (Vague Soft Set GML (Geography Markup Language) Document Type Definition (DTD), Vague Soft set GML ) | Spatial Information and Geomatics |
| Matawale, Chhabi Ram, Saurav Datta, and S. S. Mahapatra [4] | Supplier/partner selection in agile supply chain: Application of vague set as a decision making tool | Linguistic Variables, degree of similarity and probability of vague sets | Supplier Selection Decision-Making Process |
| LUO Hui, WANG | A Vague Decision | Vague Decision rule by Vague | Aeronautics |
| Youren, CUI Jiang [5] | Method for Analog Circuit Fault Diagnosis Based on Description Sphere | Distance weight | |
| Anjana Pandey K.R.Pardasani [6] | A Model for Vague Association Rule Mining in Temporal Databases | Vague Association Rule | Temporal Databases |
| Guoxin Zhang, Yunjun Guan [7] | An Evaluation Model with Effect of Rewards and Penalties Based on Vague Sets | Similarity Principle | Organizations |
| Shibao Lu, Jianhua Wang, Haijun Bao [8] | Study on Urban Water Security Evaluation Based on the Vague Set Similarity Model | Vague Set Similarity Principle | Energy, Water Resources |
| Karan Singh, Samajh Singh Thakur, Mangi Lal [9] | Vague Rough Set Techniques for Uncertainty Processing in Relational Database Model | Membership and Non-Membership Values for SQL-Like Query Language | Database Model |
| Zhigang Tang, Dongyuan Liu [10] | Fuzzy Multiple Objectives Decision Fusion Based on Vague Sets | target decision matrix, Probability, Objective Decision making | Decision Making problems |
| Amit Kumar, Shiv Prasad Yadava, and Surendra Kumar [11] | Fuzzy Reliability of a Marine Power Plant Using Interval Valued Vague Sets | interval valued trapezoidal vague | Marine Power Plant |
| Terrence Shebuel Arvind, Vivek Badhe [12] | Incorporating Vague Set Theory for Decision Making Process in Association Rule Mining | Subversive Rules, Puissant Rules | Software and Web Science |
| Ashit Kumar Dutta, and Ranjit Biswas [13] | A Study of Vague Search to Answer Imprecise Query | $\beta$-value of an interval, vague relation, $\alpha$-vague-equality search, vague-proximity search, vague-search, vague-predicate, vague-query | Database Management System |
| B. Chetia and P. K. Das [14] | Application of Vague Soft Sets in students' evaluation | Vague soft sets Concepts | Student's Evaluation |
| Shiheng Guan, Anmin Wang | Dynamic weighted | Vague Dynamic | Decision Making Problems |

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| [15] | multi-criteria fuzzy decision-making based on vague sets | Weighting Function | |
| Musheer Ahmad [16] | On Theory Of Intuitionistic Fuzzy Sets (Or Vague Sets) | Intuitionistic Fuzzy Sets (IFS), Indeterministic Part, Vague Set | Decision Making Problems |
| Anjana Pandey, K.R.Pardasani [17] | A Model for Mining Course Information using Vague Association Rule | Vague Association Rule | Course Information |
| Dr. Hakimuddin Khan [18] | Fuzzy Sets Verses Vague Sets | Intuitionistic Fuzzy Set, | Decision Making Problems |
| S. Srikrishna [19] | Acceptance Single Sampling Plan using Vague Parameters | Triangular Vague sets, ambiguity proportion parameter | Single Sampling Problem |
| Zhao Ai-wu, Guan Hong-jun [20] | The Assessment Method of Supply Chain Risks based on Vague Sets | Vague sets | Supply Chain Risk Assessment Problem |
| Ping Wang & Kuo-Ming Chao & Chi-Chun Lo [21] | Satisfaction-based Web service discovery and selection scheme utilizing vague sets theory | Dempster–Shafer (D-S) evidence theory | Web Service Ranking Problem, QoS-aware Service Discovery and Selection (QSDS) schemes |
| Pushpinder Singh, Manjit Verma, Amit Kumar [22] | A Novel Method for Ranking of Vague Sets for Handling the risk analysis of Compressor System | Ranking of trapezoidal vague sets, inter-valued fuzzy sets | Risk Analysis Problem |
| Shyi-Ming Chen [23] | Similarity Measures Between Vague Sets and Between Element**s** | Similarity Measure | Handling Behavior Analysis Problems |
| Jingli Lu, Xiaowei Yan, Dingrong Yuan, Zhangyan Xu [24] | A New Similarity Measure for Vague Sets | Similarity Measures between two vague sets | Data Processing and Analysis |
| Shunqi Zeng, Zhenzhi Lin, Fushuan Wen, Gerard Ledwich [25] | A new approach for power system black-start decision-making with vague set theory | vague-valued fuzzy measure | power system black-start decision-making |
| Zhang Kun, Chen Mei-yi and Li Zhuang [26] | Similarity Measure of Vague Set Based on Uncertainty | similarity measure between Vague values based on Uncertainty | Data Mining |
| Arun Kumar Singh and Akhilesh Tiwari [27] | Exploring the Utility of Vague Concept for | Vague Association Rule | Decision Making problems |

| | Uncertainty and Hesitation Management | | |
|---|---|---|---|
| Sanaz Pourdarab, Ahmad Nadali and Hamid Eslami Nosratabadi[28] | Determining the Knowledge Management Strategy Using Vague Set Group Decision | Vague set decision making, Possibility Theory | Knowledge Management Strategy |
| Hui-Yu Wang, Shyi-Ming Chen [29] | Artificial Intelligence Approach to Evaluate Students' Answer scripts Based on the Similarity Measure between Vague Sets | Similarity Measure | Assessment Problem |
| Shikha Verma, Jitender Kumar [30] | Fuzzy Reliability of a Gas Power Plant Using Interval valued Trapezoidal Vague Set | interval valued vague sets, interval valued trapezoidal vague set | Gas Power Plant |
| Rajesh Dangwal, M.K. Sharma and Anita[31] | A Comparative Study of Health Status Index Using Vague Sets and Interval Valued Vague Sets | Vague Sets, interval valued vague sets | Human health Problem |
| D. Pandey,M. K. Sharma [32] | Vague Set Theoretic Approach to Fault Tree Analysis | Vague numbers | Fault Tree Analysis |
| J. Mishra, S. Ghosh [33] | Uncertain Query Processing using Vague Set or Fuzzy Set: Which One Is Better? | similarity measures, SQL statement | processing uncertain queries problems |
| Arun Kumar Singh, Akhilesh Tiwari [34] | Vague Set Based Association Rule Mining for Profitable Patterns | Vague Association Rule, | Profit pattern mining |
| Prateek Shrivastava, Akhilesh Tiwari [35] | On the Use of Vague Set Theory and Genetic Algorithm for Hesitation Information Mining | Association rule mining, Genetic Algorithm | Information Mining |
| Hongxu Wang, Fujin Zhang, Yunsheng Xu [36] | Evaluating New Varieties of Wheat with the Application of Vague Optimization | similarity measure, Vague optimization | Evaluating new varieties of wheat |

339

| | Methods | | |
|---|---|---|---|
| Bai Hua [37] | The Combined Use of Evidence Theory and Vague Sets to Interpret Multimodal Inspection Data | ague sets/ D-S evidence theory, true-membership function and false-membership function | Multimodal inspection problem |
| L.L. Shi and J. Ye [38] | Study on Fault Diagnosis of Turbine Using an Improved Cosine Similarity Measure for Vague Sets | Cosine Similarity Measure | Fault Diagnosis |
| Faxin Zhao and Z.M. Ma [39] | Similarity Measures of Vague Sets Based on the Set-theoretic Approach | Similarity measure, Set-theoretic approach | Information Processing |
| Khaleed Alhazaymeh and Nasruddin Hassan [40] | Interval-Valued Vague Soft Sets and Its Application | equality, subset, intersection, union, AND operation, OR operation, complement, and null | decision making and medical diagnosis problems |

## IV. CONCLUSION

The theory of vague sets has been the base for the foundation of mathematics and so is considered as one of the most significant branches in mathematics. The fact that any mathematical concept can be interpreted with the help of set theory has not only increased its versatility but has established this theory to be the universal language of mathematics. In this literature review paper, the vague/ interval valued vague, Vague Association Rule, Similarity Measure were used in the various applications for Decision Making process.

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**340**

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