*International Journal on Recent and Innovation Trends in Computing and Communication*
*ISSN: 2321-8169 Volume: 11 Issue: 8*
***Article Received:** 25 May 2023 **Revised:** 12 June 2023 **Accepted:** 30 July 2023*

# Resource Allocation Algorithm for OFDMA System based on Bidirectional Multi-Relay

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**Abstract—** A novel approach to optimize power allocation and subcarrier pairing in a high signal-to-noise ratio (SNR) scenario within a two-way multi-relay orthogonal frequency division multiple access (OFDMA) system has been introduced. Unlike conventional methods where relays operate on individual subcarriers, our scheme allows all relays to transmit signals across each subcarrier pair, thereby leveraging significant space diversity. Operating under a constraint of total system power, our proposed scheme initially assigns power to each relay using Cauchy inequality under the assumption of fixed total relay power. Subsequently, employing a dichotomous approach, we determine the power allocation between the source node and the relay node by maximizing the equivalent channel gain across various subcarrier pairs. Finally, we employ convex programming to allocate power to different subcarrier pairs, while utilizing the Hungarian algorithm to pair subcarriers effectively, thereby maximizing system capacity. Given the inherent complexity of power allocation algorithms in two-way multi-relay networks, conventional methods lack optimal solutions with low complexity. However, our algorithm significantly mitigates the complexity associated with power allocation, particularly within a system comprising 40 subcarriers. Simulation results underscore the superiority of our proposed scheme over conventional relay selection approaches, particularly those wherein relays operate on individual subcarriers.

**Key words :** OFDMA, Two-Way Multi-Relay, Power Allocation, Subcarrier Spacing.

## I. INTRODUCTION

In the cooperative communication model, a group of relays forwards the signal received from the source node to the destination node. It not only reduces the interruption probability of the signal, but also enables the receiving end to obtain spatial diversity gain. However, the relay model in actual application generally uses half-duplex technology. In the same time period, the relay either receives signals or sends signals, which reduces the total spectrum efficiency of the system by half. In order to avoid this loss of spectrum efficiency, the relay technology of two-way transmission has been widely studied [1-5].

Literature [5, 6, 7] studied the resource allocation algorithm in the two-way multi-relay scenario, and optimized the power of the source node and each relay node. Literature [8, 9] proposed a relay selection algorithm, under the condition of limited total power, first select the optimal relay from multiple relays, and then obtain the maximum system output signal-to-noise ratio through the power allocation algorithm. Although the single-relay selection simplifies the optimization model of the system[10, 11], it also removes the signal diversity gain that can be obtained at the receiving end, which is not the optimal algorithm. Applying two-way relay technology to multi-carrier OFDMA systems is even more challenging. Literature [12, 13] studied the power allocation and subcarrier pairing algorithm based on relay selection in the multi-relay multi-carrier scenario, and proposed a method to maximize the system transmission rate when the total power is limited or the independent power of each node is limited. The algorithm does not make full use of the diversity gain of the relay. Literature [14, 15] proposed a resource allocation algorithm in a two-way multi-relay multi-user OFDMA system. In addition to the three system resources to be optimized in [16], this algorithm also considers the selection of sub-carrier pairs by different users, but it also wastes the diversity effect of multiple relays. In order to avoid interference between relays, most of the research on the resource allocation algorithm of multi-relay OFDMA systems is based on the relay selection algorithm. Different relays will use different sub-carrier pairs, and the same sub-carrier pair will only pass through a certain relay forwards.

Different from the resource allocation algorithm based on relay selection, this paper proposes a joint algorithm of power allocation and sub-carrier pairing that allocates a sub-carrier pair to multiple relay nodes. When multiple relays

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forward the same subcarrier pair in the algorithm, all relays will occupy all subcarrier pairs, and the same subcarrier pair will be forwarded through all relays. This approach increases the complexity of the optimization problem of the multi-relay OFDMA system. But it improves the diversity gain of the system brought by the relay. Under the condition that the total power of the system is limited, this algorithm first uses Cauchy's inequality to optimize the distribution of the optimal power of each relay when the total power of all relays is a fixed value, simplifying the original optimal system model; then applying dichotomy to calculate the power allocation between the source node and the relay node by maximizing the equivalent channel gain under the sub-carrier pair, and finally allocate the power of different sub-carrier pairs through convex planning, and combine the sub-carrier pairing to obtain the maximum capacity of the system. The simulation results show that the proposed method when compared with the optimal single-relay selection algorithm and the relay selection-based algorithm, the average channel capacity is significantly improved.

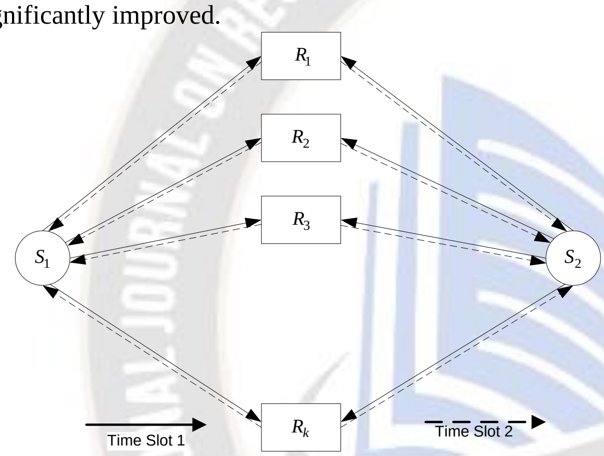


**Figure 1. Bidirectional multi-relay OFDMA system model**

## II. SYSTEM MODEL

### A. RELAY MODEL

The two-hop two-way multi-relay OFDMA system model studied in this paper is shown in Figure 1. Two user nodes $S_1$ and $S_2$ communicate through K two-way relay nodes. The transmission bandwidth is divided into N sub-carriers, and each sub-carrier is divided into N sub-carriers, sub-system bandwidth and independent Rayleigh fading. All relay nodes adopt half-duplex AF relay mode, and can obtain instantaneous channel information of all sub-carriers. The communication process of the system is divided into two time slots. In the first time slot, all relay nodes receive the signals broadcasted by user nodes $S_1$ and $S_2$ , and combine the received signals; in the second time slot , All relay nodes amplify the received signal, and then forward it to 2 user nodes through the sub-carrier matching the first time slot.

For the $k^{th}$ relay $R_k$, it may be assumed that the $i^{th}$ subcarrier is used to receive the signal in the first time slot, and the $j^{th}$ subcarrier is used to forward the signal in the second time slot, and this subcarrier pair is recorded as $SP(i,j)$, then the signal received by the relay $R_k$ in the first time slot is

$$y_{i,j}^{k} = \sqrt{P_{i,j}^{s_1}} h_{i,j}^{k} s_1 \sqrt{P_{i,j}^{s_2}} g_{i,j}^{k} s_2 + n_{i,j}^{k} \tag{1}$$

where, $s_1$ and $s_2$ respectively represent the signals sent by 2 users, and the power is normalized to 1; $P_{i,j}^{s_1}$ and $P_{i,j}^{s_2}$ represent the transmission power of 2 users respectively; $h_{i,j}^{k}$ and, $g_{i,j}^{k}$ represent 2 respectively The channel gains between each user and the relay $R_k$, assuming that they all obey the complex Gaussian distribution with zero mean, the variances are respectively $\sigma_{i,j}^{s_1,k}$ and $\sigma_{i,j}^{s_2,k}$;$n_{i,j}^{k}$ is the additive at the relay $R_k$ Gaussian white noise, its variance is $\sigma_{i,j}^{k}$.

### B. PROBLEM DESCRIPTION

In the second time slot, the relay amplifies the received signal and forwards it to 2 users. Assuming that the channel state does not change in the first and second time slots, then the subcarrier pair $SP(i,j)$ in other words, the signal received by user $S_1$ is

$$y_{i,j}^{1} = \sum_{k=1}^{K} y_{i,j}^{k} \beta_{i,j}^{k} h_{i,j}^{k} + n_{i,j}^{s_1} \tag{2}$$

where, $n_{i,j}^{s_1}$ represents the additive white Gaussian noise at the node $S_1$ , and its variance is $\sigma_{i,j}^{s_1}, \beta_{i,j}^{k}$ represent the power amplification factor, and $P_{i,j}^{k}$ represents the transmit power when the signal is relayed and forwarded, then the expression of $\beta_{i,j}^{k}$ is

$$\beta_{i,j}^{k} = \sqrt{\frac{P_{i,j}^{k}}{P_{i,j}^{s_1}\left|h_{i,j}^{k}\right|^2 + P_{i,j}^{s_2}\left|g_{i,j}^{k}\right|^2 + \sigma_{i,j}^{k}}} \tag{3}$$

Substituting (1) and (3) into (2), we can get

$$y_{i,j}^{1} = \sum_{k=1}^{K} \frac{\sqrt{P_{i,j}^{k}} h_{i,j}^{k}}{\sqrt{P_{i,j}^{s_1}\left|h_{i,j}^{k}\right|^2 + P_{i,j}^{s_2}\left|g_{i,j}^{k}\right|^2 + \sigma_{i,j}^{k}}} \cdot \left(\sqrt{P_{i,j}^{s_1}} h_{i,j}^{k} s_1 + \sqrt{P_{i,j}^{s_2}} g_{i,j}^{k} s_2 + n_{i,j}^{k}\right) + n_{i,j}^{s_1} \tag{4}$$

where, the first item in brackets is the signal sent by user $S_1$ in the first time slot. This interference can be completely eliminated by coding technique. After this sub-interference is eliminated, user $S_1$ is in the sub-carrier pair $SP(i,j)$ , the received signal-to-noise ratio under is

$$SNR_{i,j}^{1} = \frac{P_{i,j}^{s_2}\left|\sum_{k=1}^{K} \frac{\sqrt{P_{i,j}^{k}} h_{i,j}^{k} g_{i,j}^{k}}{P_{i,j}^{s_1}\left|h_{i,j}^{k}\right|^2 + P_{i,j}^{s_2}\left|g_{i,j}^{k}\right|^2 + \sigma_{i,j}^{k}}\right|^2}{\sigma_{i,j}^{s_1} + \sum_{k=1}^{K}\left|\frac{\sqrt{P_{i,j}^{k}} h_{i,j}^{k}}{P_{i,j}^{s_1}\left|h_{i,j}^{k}\right|^2 + P_{i,j}^{s_2}\left|g_{i,j}^{k}\right|^2 + \sigma_{i,j}^{k}}\right| \sigma_{i,j}^{k}} \tag{5}$$

Applying the same method, the signal received by user $S_2$ can be obtained as

$$y_{i,j}^{2} = \sum_{k=1}^{K} \frac{\sqrt{P_{i,j}^{k}} g_{i,j}^{k}}{P_{i,j}^{s_1}\left|h_{i,j}^{k}\right|^2 + P_{i,j}^{s_2}\left|g_{i,j}^{k}\right|^2 + \sigma_{i,j}^{k}} \left(\sqrt{P_{i,j}^{s_1}} h_{i,j}^{k} s_1 + \sqrt{P_{i,j}^{s_2}} g_{i,j}^{k} s_2 + n_{i,j}^{k}\right) + n_{i,j}^{s_2} \tag{6}$$

where, $n_{i,j}^{s_2}$ represents the additive white Gaussian noise at the user receiving node $S_2$. The received signal-to-noise ratio of user $S_2$ is

$$SNR_{i,j}^2 = \frac{P_{i,j}^{s_1}\left|\sum_{k=1}^{K}\frac{\sqrt{P_{i,j}^k}h_{i,j}^k g_{i,j}^k}{P_{i,j}^{s_1}\left|h_{i,j}^k\right|^2+P_{i,j}^{s_2}\left|g_{i,j}^k\right|^2+\sigma_{i,j}^k}\right|^2}{\sigma_{i,j}^{s_2}+\sum_{k=1}^{K}\left|\frac{\sqrt{P_{i,j}^k}g_{i,j}^k}{P_{i,j}^{s_1}\left|h_{i,j}^k\right|^2+P_{i,j}^{s_2}\left|g_{i,j}^k\right|^2+\sigma_{i,j}^k}\right|\sigma_{i,j}^k} \quad (7)$$

where, $\sigma_{i,j}^{s_2}$ represents noise variance.

According to Shannon's formula, the system capacity under sub-carrier pair SP(i, j) is [17]

$$R_{i,j} = \frac{1}{2}\mathrm{lb}\left(1+SNR_{i,j}^1\right)+\frac{1}{2}\mathrm{lb}\left(1+SNR_{i,j}^2\right) \quad (8)$$

Since the capacity expression of the two-way multi-relay OFDMA system in (8) is too complicated, in order to reduce the computational complexity, suppose $\sigma_{i,j}^k = \sigma_{i,j}^{s_1} = \sigma_{i,j}^{s_2} = N_0$, so that the transmission power of the two user nodes is the same, i.e., $P_{i,j}^{s_1} = P_{i,j}^{s_2} = P_{i,j}^s$ and defined

$$l_{i,j}^k = \max\left(h_{i,j}^k, g_{i,j}^k\right) \quad \forall k = 1,2,\dots,K \quad (9)$$

Can be obtained

$$SNR_{i,j}^1 = SNR_{i,j}^2 = SNR_{i,j} = \frac{P_{i,j}^S\left|\sum_{k=1}^{K}\frac{\sqrt{P_{i,j}^k}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^{s_1}\left|h_{i,j}^k\right|^2+P_{i,j}^{s_2}\left|g_{i,j}^k\right|^2+N_0}}\right|^2}{N_0+\sum_{k=1}^{K}\left|\frac{\sqrt{P_{i,j}^k}\left|l_{i,j}^k\right|^2}{\sqrt{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}}\right|^2 N_0} \quad (10)$$

Substituting equation (10) into equation (8), we get

$$R_{i,j} = \mathrm{lb}(1+SNR_{i,j}) \quad (11)$$

Therefore, the capacity optimization model of the two-hop two-way multi-relay OFDMA system studied in this paper can be expressed as

$$\max R = \sum_{i=1}^{N}\sum_{j=1}^{N} s_{i,j}R_{i,j}$$

$$\text{s.t.} \quad C_1: \sum_{i=1}^{N}\sum_{j=1}^{N} S_{i,j}\left(P_{i,j}^s+\sum_{k=1}^{N}P_{i,j}^k\right) \le P_t$$

$$C_2: s_{i,j} \in \{0,1\} \ \forall i,j = 1,2,\dots,N$$

$$C_3: \sum_{i=1}^{N} s_{i,j} = 1, \ \forall j, \ \sum_{j=1}^{N} s_{i,j} = 1, \ \forall i \quad (12)$$

where, the constraint condition $C_1$ indicates that the maximum total output power of the system is $P_t$, a $N \times N$-dimensional decision matrix, $s = \{s_{i,j}\}$ indicates the pairing of the i-th sub-carrier and the j-th sub-carrier, if the i-th sub-carrier and the j-th sub-carrier are paired If the carriers match, then, $s_{i,j} = 1$; if the i-th sub-carrier does not match the j-th sub-carrier, then, $s_{i,j} = 0$. Constraint $C_2$ ensures that $s_{i,j} = 0$ are only 0 or 1. Constraint $C_3$ ensures that each sub-carrier can only be paired with another sub-carrier, and there will be no repeated pairing.

## III. JOINT POWER AND SUB-CARRIER ALLOCATION ALGORITHM

The $s_{i,j}$ in equation (12) can only be 0 or 1. Therefore, the optimization problem is a mixed-integer nonlinear optimization problem [18]. If it is solved directly, it will have higher computational complexity. Therefore, this article first optimizes the internal $SNR_{i,j}$ of $R_{i,j}$, and then solves the external power allocation and sub-carrier pairing problems.

### A. RELAY POWER ALLOCATION ALGORITHM

It can be seen from equation (11) that for a fixed subcarrier pair $SP(i,j)$, to obtain the maximum $R_{i,j}$ is equivalent to obtaining the maximum $SNR_{i,j}$. It is assumed here that the value of $P_{i,j}^s$ and, $\sum_{k=1}^{K}P_{i,j}^k$ is a fixed value. Then, in order to obtain the maximum $SNR_{i,j}$, the power of each relay must be optimally allocated, so the optimization of $SNR_{i,j}$ is maximized, the problem can be reduced to

$$\max SNR_{eq} = \frac{\left|\sum_{k=1}^{K}\frac{\sqrt{P_{i,j}^k}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}}\right|^2}{1+\sum_{k=1}^{K}\left|\frac{\sqrt{P_{i,j}^k}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}}\right|^2}$$

$$s.t. \ P_{i,j}^r = \sum_{k=1}^{K}P_{i,j}^k \quad (13)$$

In order to make the power expression of each relay related to $P_{i,j}^r$, an auxiliary complex variable $a_{i,j}^k$, is introduced. The use of this variable reduces the difficulty of analyzing the optimization problem, so that the power of each relay is expressed as

$$P_{i,j}^k = \frac{\left|a_{i,j}^k\right|^2}{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2}P_{i,j}^r \quad (14)$$

Substituting equation (14) into equation (13), $SNR_{eq}$ can be re-expressed as

$$SNR_{eq} = \frac{\left|\sum_{k=1}^{K}\frac{\left|a_{i,j}^k\right|}{\sqrt{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2}}\frac{\sqrt{P_{i,j}^r}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}}\right|^2}{1+\sum_{k=1}^{K}\frac{\left|a_{i,j}^k\right|^2}{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2}\frac{P_{i,j}^r\left|l_{i,j}^k\right|^2}{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}} \quad (15)$$

Because of $\frac{\sum_{k=1}^{K}\left|a_{i,j}^k\right|^2}{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2} = \frac{\sum_{k=1}^{K}\left|a_{i,j}^k\right|}{\sum_{k=1}^{K}\left|a_{i,j}^m\right|^2}$, formula (15) can be rewritten as

$$SNR_{eq} = \frac{\left|\sum_{k=1}^{K}\frac{\left|a_{i,j}^k\right|}{\sqrt{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2}}\frac{\sqrt{P_{i,j}^r}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}}\right|}{\frac{\sum_{k=1}^{K}\left|a_{i,j}^k\right|^2}{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2}+\frac{\sum_{k=1}^{K}\left|a_{i,j}^k\right|^2}{\sum_{m=1}^{K}\left|a_{i,j}^m\right|^2}\frac{P_{i,j}^r\left|l_{i,j}^k\right|^2}{P_{i,j}^s\left|h_{i,j}^k\right|^2+P_{i,j}^s\left|g_{i,j}^k\right|^2+N_0}}$$

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$$= \frac{\frac{\left|\frac{\sum_{k=1}^{K}|a_{i,j}^k|}{\sqrt{\sum_{m=1}^{K}|a_{i,j}^m|^2}}\frac{\sqrt{P_{i,j}^r}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}}\right|^2}{\frac{\sum_{k=1}^{K}|a_{i,j}^k|^2 P_{i,j}^r|l_{i,j}^k|^2+P_{i,j}^{s_1}|h_{i,j}^k|^2+P_{i,j}^{s_2}|g_{i,j}^k|^2+N_0}{\sum_{m=1}^{K}|a_{i,j}^m|^2 \quad P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}}}{} = \frac{\left|\sum_{k=1}^{K}|a_{i,j}^k|\frac{\sqrt{P_{i,j}^r}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}}\right|^2}{\sum_{k=1}^{K}|a_{i,j}^k|^2\frac{P_{i,j}^r|l_{i,j}^k|^2+P_{i,j}^{s_1}|h_{i,j}^k|^2+P_{i,j}^{s_2}|g_{i,j}^k|^2+N_0}{P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}} \tag{16}$$

The following variables are defined to simplify the expression of $SNR_{eq}$.

$$c_{i,j}^k = \frac{\sqrt{P_{i,j}^r}h_{i,j}^k g_{i,j}^k}{\sqrt{P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}} \tag{17}$$

$$d_{i,j}^k = \frac{P_{i,j}^r|l_{i,j}^k|^2+P_{i,j}^{s_1}|h_{i,j}^k|^2+P_{i,j}^{s_2}|g_{i,j}^k|^2+N_0}{P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0} \tag{18}$$

$$|b_{i,j}^k|^2 = |a_{i,j}^k|^2 d_{i,j}^k \tag{19}$$

Substituting formula (17) ~ formula (19) into formula (16), we can get

$$SNR_{eq} = \frac{\left|\sum_{k=1}^{K}|a_{i,j}^k|c_{i,j}^k\right|^2}{\sum_{k=1}^{K}|a_{i,j}^k|^2 d_{i,j}^k} = \frac{\left|\sum_{k=1}^{K}|b_{i,j}^k|\frac{c_{i,j}^k}{\sqrt{d_{i,j}^k}}\right|^2}{\sum_{k=1}^{K}|b_{i,j}^k|^2} \tag{20}$$

According to Cauchy's inequality, the maximum value of the numerator (20) is

$$\left|\sum_{k=1}^{K}|b_{i,j}^k|\frac{c_{i,j}^k}{\sqrt{d_{i,j}^k}}\right|^2 \le \sum_{k=1}^{K}|b_{i,j}^k|^2\sum_{k=1}^{K}\left|\frac{c_{i,j}^k}{\sqrt{d_{i,j}^k}}\right|^2 = \sum_{k=1}^{K}|b_{i,j}^k|\sum_{k=1}^{K}\frac{(c_{i,j}^k)^2}{(d_{i,j}^k)^2} \tag{21}$$

If and only if, $|b_{i,j}^k|$ and, $\frac{c_{i,j}^k}{\sqrt{d_{i,j}^k}}$ are linearly related, the equal sign holds. Here, if the two are equal, we can get

$$|b_{i,j}^k| = \frac{c_{i,j}^k}{\sqrt{d_{i,j}^k}} \tag{22}$$

According to (21), the maximum value of $SNR_{eq}$ can be obtained as $\sum_{k=1}^{K}\frac{(c_{i,j}^k)^2}{(d_{i,j}^k)^2}$, at this time the equal sign is established, so it can be derived from (22)

$$(b_{i,j}^k)^2 d_{i,j}^k = (c_{i,j}^k)^2$$
$$\Rightarrow |a_{i,j}^k|^2(d_{i,j}^k)^2 = (c_{i,j}^k)^2$$
$$\Rightarrow |a_{i,j}^k|d_{i,j}^k = c_{i,j}^k$$
$$\Rightarrow |a_{i,j}^k| = \frac{c_{i,j}^k}{d_{i,j}^k} = \frac{\sqrt{P_{i,j}^r}h_{i,j}^k g_{i,j}^k\sqrt{P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}}{P_{i,j}^r|l_{i,j}^k|^2+P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0} \tag{23}$$

So the power that each relay should be allocated can be calculated by $|a_{i,j}^k|^2$.

$$P_{i,j}^k = \frac{|a_{i,j}^k|^2}{\sum_{k=1}^{K}|a_{i,j}^m|^2}P_{i,j}^r \tag{24}$$

## B. POWER ALLOCATION ALGORITHM FOR FIXED SUB-CARRIER PAIRS

After the calculation in Section 3, the optimal algorithm for each power within $\sum_{k=1}^{K}P_{i,j}^k$ under the sub-carrier pair $SP(i,j)$ can be obtained when, $P_{i,j}^s$ and $\sum_{k=1}^{K}P_{i,j}^k$ are fixed values, and then the power occupied by $P_{i,j}^s$ and $\sum_{k=1}^{K}P_{i,j}^k$ under the sub-carrier pair will be allocated.

According to (21), we can get

$$SNR_{eq} = \sum_{k=1}^{K}\frac{(c_{i,j}^k)^2}{(d_{i,j}^k)^2} = \sum_{k=1}^{K}\frac{P_{i,j}^r|h_{i,j}^r|^2|g_{i,j}^k|^2}{P_{i,j}^r|l_{i,j}^k|^2+P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0} \tag{25}$$

Substituting equation (25) into equation (10), we get

$$SNR_{i,j} = \sum_{k=1}^{K}\frac{P_{i,j}^s P_{i,j}^r|h_{i,j}^r|^2|g_{i,j}^k|^2}{P_{i,j}^r|l_{i,j}^k|^2+P_{i,j}^s|h_{i,j}^k|^2+P_{i,j}^s|g_{i,j}^k|^2+N_0}\frac{1}{N_0} = \sum_{k=1}^{K}\frac{\frac{P_{i,j}^s P_{i,j}^r|h_{i,j}^r|^2|g_{i,j}^k|^2}{N_0^2}}{\frac{P_{i,j}^r|l_{i,j}^k|^2}{N_0}+\frac{P_{i,j}^s|h_{i,j}^k|^2}{N_0}+\frac{P_{i,j}^s|g_{i,j}^k|^2}{N_0}+1} \tag{26}$$

In the case of a large signal-to-noise ratio, the constant 1 [19] in the denominator of equation (26) can be ignored, and is defined as

$$P_{i,j} = 2P_{i,j}^s + P_{i,j}^r \tag{27}$$

$$P_{i,j}^s = \alpha_{i,j}P_{i,j};\ P_j^r = (1-2\alpha_{i,j})P_{i,j};\ \alpha_{i,j}\in(0,0.5) \tag{28}$$

Equation (26) can be expressed as

$$SNR_{i,j} \approx \sum_{k=1}^{K}\frac{\frac{P_{i,j}^s P_{i,j}^r|h_{i,j}^r|^2|g_{i,j}^k|^2}{N_0^2}}{\frac{P_{i,j}^r|l_{i,j}^k|^2}{N_0}+\frac{P_{i,j}^s|h_{i,j}^k|^2}{N_0}+\frac{P_{i,j}^s|g_{i,j}^k|^2}{N_0}} = P_{i,j}\sum_{k=1}^{K}\frac{\frac{\alpha_{i,j}(1-2\alpha_{i,j})|h_{i,j}^r|^2|g_{i,j}^k|^2}{N_0^2}}{\frac{(1-2\alpha_{i,j})|l_{i,j}^k|^2}{N_0}+\frac{\alpha_{i,j}|h_{i,j}^k|^2}{N_0}+\frac{\alpha_{i,j}|g_{i,j}^k|^2}{N_0}} \tag{29}$$

Define the equivalent channel gain $\omega_{i,j}$ of the sub-carrier pair $SP(i,j)$ as

$$\omega_{i,j} = \sum_{k=1}^{K}\frac{\frac{\alpha_{i,j}(1-2\alpha_{i,j})|h_{i,j}^r|^2|g_{i,j}^k|^2}{N_0^2}}{\frac{(1-2\alpha_{i,j})|l_{i,j}^k|^2}{N_0}+\frac{\alpha_{i,j}|h_{i,j}^k|^2}{N_0}+\frac{\alpha_{i,j}|g_{i,j}^k|^2}{N_0}} \tag{30}$$

According to formula (30), $\omega_{i,j}$ are monotonic functions of variables$\alpha_{i,j}$, and the optimal solution can be calculated by dichotomy. After the optimal value of $\omega_{i,j}$ is obtained, the power allocated by the source node and the relay node can be calculated according to the power $P_{i,j}$of the allocated specific subcarrier to $SP(i,j)$ , In this way, the optimal power allocation under the current $SP(i,j)$ is obtained.

## C. JOINT POWER AND SUBCARRIER ALLOCATION ALGORITHM

Substituting equation (29) and equation (30) into equation (11), the channel capacity under sub-carrier pair $SP(i,j)$ can be obtained as

$$R_{i,j} = lb\left(1 + SNR_{i,j}\right) = lb\left(1 + \omega_{i,j} P_{i,j}\right) \quad (31)$$

Therefore, the problem of maximizing the capacity of the two-way relay OFDMA system proposed in equation (12) can be rewritten as

$$\max R = \sum_{i=1}^{N} \sum_{j=1}^{N} s_{i,j} lb\left(1 + \omega_{i,j} P_{i,j}\right)$$
$$s.t. C_1: \sum_{i=1}^{N} \sum_{j=1}^{N} s_{i,j} P_{i,j} \le P_t$$
$$C_2: s_{i,j} \in \{0,1\} \; \forall\, i,j = 1,2,\dots,N$$
$$C_3: \textstyle\sum_{i=1}^{N} s_{i,j} = 1, \forall\, j, \sum_{j=1}^{N} s_{i,j} = 1, \forall i \quad (32)$$

The optimization goal in equation (32) is a hybrid shaping model, which is too difficult to solve directly. Therefore, this section first transforms it into linear programming and then solves it. The constraint $C_2$ in formula (32) requires that $s_{i,j}$ can only be integers 0 or 1. In order to linearize the objective function, define a new decision matrix, $\tilde{s} = \{\tilde{s}_{i,j}\}$, where $\tilde{s}_{i,j}$ is obtained [ 0,1], and use, $\frac{\tilde{P}_{i,j}}{\tilde{s}_{i,j}}$ to replace $P_{i,j}$, then the original optimization objective function can be transformed into

$$\max R = \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{s}_{i,j} lb\left(1 + \omega_{i,j} \frac{\tilde{P}_{i,j}}{\tilde{s}_{i,j}}\right)$$
$$s.t. C_1: \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{P}_{i,j} \le P_t$$
$$C_2: \tilde{s}_{i,j} \in [0,1] \; \forall i,j = 1,2,\dots,N$$
$$C_3: \textstyle\sum_{i=1}^{N} \tilde{s}_{i,j} = 1. \forall j, \sum_{j=1}^{N} \tilde{s}_{i,j} = 1, \forall i \quad (33)$$

It can be seen from equation (33) that the objective optimization function is a concave function about $\tilde{P}_{i,j}$, and the constraint conditions also meet the convex programming conditions. Therefore, the global optimum can be found in the Lagrangian dual domain of the function variable.

The Lagrangian function of equation (33) is as follows

$$L(\tilde{s}, \tilde{P}, \lambda) = \textstyle\sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{s}_{i,j}\, lb\left(1 + \omega_{i,j} \frac{\tilde{P}_{i,j}}{\tilde{s}_{i,j}}\right) - \lambda\left(\sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{P}_{i,j} - P_t\right) \quad (34)$$

where, $\tilde{P}$ represents the matrix about. $\{\tilde{P}_{i,j}\}$, and λ is the Lagrangian coefficient of the constraint condition $C_1$ in formula (33). The dual programming of equation (34) is

$$g(\lambda) = \max L(\tilde{s}, \tilde{P}, \lambda)$$
$$s.t.\; C_1: \tilde{s}_{i,j} \in [0,1], \forall i,j = 1,2,\dots,N$$
$$C_2: \textstyle\sum_{i=1}^{N} \tilde{s}_{i,j} = 1, \; \forall j, \sum_{j=1}^{N} \tilde{s}_{i,j} = 1, \forall i \quad (35)$$

According to the KKT condition, the power allocated to $SP(i,j)$ in the sub-carrier pair can be obtained as

$$\tilde{P}_{i,j} = \left[\frac{1}{\lambda \ln 2} - \frac{1}{\omega_{i,j}}\right]^{+} \tilde{s}_{i,j} \quad (36)$$

$$P_{i,j} = \frac{\tilde{P}_{i,j}}{\tilde{s}_{i,j}} = \left[\frac{1}{\lambda \ln 2} - \frac{1}{\omega_{i,j}}\right]^{+} \quad (37)$$

where, $[x]^{+} = \max(0, x)$.

Substituting formula (37) into formula (34), and taking $\lambda$ as a constant, then the optimization goal of sub-carrier pairing is

$$\max \sum_{i=1}^{N} \sum_{j=1}^{N} s_{i,j} \left( lb\left(1 + \omega_{i,j} P_{i,j}\right) - \lambda \sum_{i=1}^{N} \sum_{j=1}^{N} P_{i,j} \right)$$
$$s.t. C_1:\; s_{i,j} \in \{0,1\} \; \forall i,j = 1,2,\dots,N$$
$$C_2 : \textstyle\sum_{i=1}^{N} s_{i,j} = 1, \forall j, \sum_{j}^{N} s_{i,j} = 1, \forall i \quad (38)$$

The optimal decision matrix, $s = \{s_{i,j}\}$ can be calculated by applying the Hungarian algorithm.

The minimum value of the dual programming equation (35) is to obtain the optimal value of the Lagrangian function equation (35). Here the sub-gradient algorithm is used to calculate

$$\min g(\lambda), s.t. \lambda \ge 0 \quad (39)$$

Obviously, according to the literature [9], the optimization problem in equation (38) always has an integer binary optimal solution. Therefore, this optimization problem is transformed into a typical two-dimensional knapsack problem, and the optimal solution can be obtained by the Hungarian algorithm, and its complexity is $O(N^3)$. The specific implementation steps of the joint power and subcarrier allocation algorithm proposed in this paper are as follows.

**Step 1** Obtain the instantaneous channel information, and the destination node obtains the instantaneous channel information of each channel through the training sequence.

**Step 2** Calculate the equivalent channel gain $\omega_{i,j}$ of the sub-carrier pair $SP(i,j)$ according to equation (30), and use the dichotomy to obtain $\alpha_{i,j}^{*}$ in $(0,1]$, so that the equivalent channel gain $\omega_{i,j}$ takes the maximum value.

**Step 3** Select the initial value of the appropriate Lagrangian factor $\lambda_0$ and $\lambda_n$, where n is the number of iterations of the sub-gradient algorithm.

**Step 4** Substitute the current $\lambda_n$ into equation (37) to calculate the power allocation value $P_{i,j}$ in the case of different sub-carrier pairs $SP(i,j)$.

**Step 5** Define the decision matrix, $s = \{s_{i,j}\}$ means that the i-th sub-carrier is paired with the j-th sub-carrier pair, and $s_{i,j} = 0$ means that the i-th sub-carrier is not paired with the j-th sub-carrier pair. According to the calculated $P_{i,j}$ and $\omega_{i,j}$, the Hungarian algorithm is used for sub-carrier matching, and the decision matrix s is calculated.

**Step 6** Update λ with the calculated decision matrix $s$ and power distribution $P_{i,j}$, and use the subgradient method to take $\lambda_{n+1} = \left[\lambda_n - st_n\left(P_t - \sum_{i=1}^{N} \sum_{j=1}^{N} s_{i,j} P_{i,j}\right)\right]$ to update the iteration factor, where $st_n$ is a variable related to $n$, used to control the step size of each iteration.

**Step 7** Compare the values of $\lambda_{n+1}$ and $\lambda_n$. If the value of $|\lambda_{n+1} - \lambda_n|$ is less than a fixed constant, it means that the values calculated in this iteration are the optimal solutions. Go to step 8; otherwise, use $\lambda_{n+1}$ instead of $\lambda_n$, go to step 4 and repeat the iterative process.

*International Journal on Recent and Innovation Trends in Computing and Communication*
***ISSN: 2321-8169 Volume: 11 Issue: 8***
***Article Received:** 25 May 2023 **Revised:** 12 June 2023 **Accepted:** 30 July 2023*

**Step 8** According to the decision matrix $s = \{s_{i,j}\}$ obtained in this iteration, use the calculated values of $P_{i,j}$ and $\alpha_{i,j}$ to calculate the corresponding $P_{i,j}^s$ and $P_{i,j}^k$.

**Step 9** Broadcast the resource allocation information to all nodes.

The resource allocation algorithm of the two-way relay OFDMA system studied in this paper adopts the way that the relays share subcarriers, so that all relays share the subcarriers to increase the diversity gain of the receiving node, and apply Cauchy's inequality to simplify the relationship between the source node and the relay node. Power allocation reduces the complexity of the algorithm. Similarly, only a small amount of decision-making signaling information needs to be transmitted. Most of the calculations are done locally at the node, which greatly reduces additional consumption. In addition, this paper uses Cauchy's inequality combined with dichotomy to simplify the traditional optimization algorithm, avoiding a large number of equivalent channel gain calculations, and reducing the complexity of the algorithm while providing approximately the same performance as the traditional optimization algorithm.

## IV. SIMULATION RESULTS AND ANALYSIS

Joint power and subcarrier allocation algorithm for multi-relay and multi-user OFDMA system is analyzed using matlab simulation and compared with existing algorithms. The algorithms compared in the analysis are the optimal relay selection algorithm proposed in [20] and the relay sub-carrier algorithm and equal power allocation algorithm used in [21]. In the simulation environment, it is assumed that all sub-carrier channels obey Rayleigh fading, and the channel gain between each node obeys a zero-mean complex Gaussian distribution. However, the channel variance between 2 source nodes and relay nodes is 1, the channel between 2 source nodes is not used, and the variance of all noise is 1.

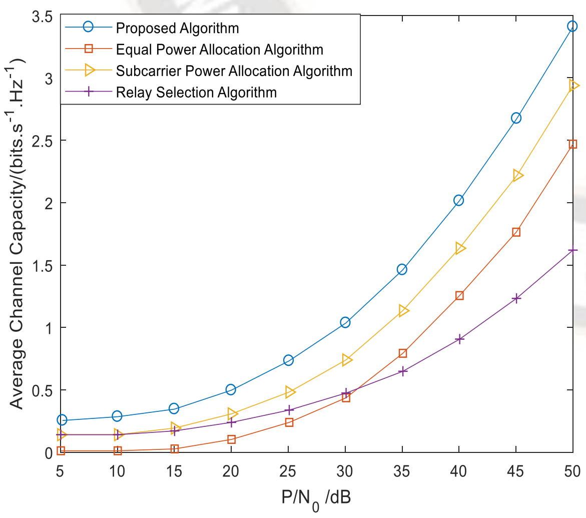


**Figure 2. Comparison of average channel capacity of different resource allocation algorithms**

Fig. 2 shows the comparison of the average channel capacity of different resource allocation algorithms, where the number of relays is 4 and the number of subcarrier pairs is 30. The parameter $P$ on x-axis in Fig. 2 represents the total transmit power available for the system, and $P/N_0$ represents the transmit power normalized to the noise at the receiver. The ordinate represents the optimization goal of this article, the average channel capacity of the system. From Fig. 2 it is observed that four different subcarrier pairing algorithms are compared. The proposed joint optimization algorithm is compared with equal power allocation algorithm, in this algorithm all relay nodes and transmitting nodes have the same transmit power on all subcarrier pairs; subcarrier power allocation proposed in [21], in this algorithm, each pair of sub-carriers can only be used by one relay, and all relays cannot use the same sub-carrier; relay selection algorithm proposed in [20], in this algorithm the instantaneous channel state information selects an optimal one from all relays, and then allocates system resources. Fig. 2 illustrates that the system resource joint allocation algorithm proposed in this paper has good system performance, and this performance gain is brought about by the relay shared sub-carrier mechanism. If relays are allowed to share subcarriers, multiple relay nodes can use the same subcarrier to forward the signal transmitted by the source node. Compared with the literature [20, 21], a subcarrier pair can only be allocated to one relay, which is equivalent to the difference between multi-relay and single-relay in a single carrier network. Allowing multiple relays to forward the same subcarrier pair at the same time providing additional cooperative diversity gains for the system. Although it increases the complexity of the algorithm, it can further improve the spectrum efficiency of the system. In contrast, one subcarrier pair in [10, 11] is allocated to one relay. Although this approach simplifies the problem, it sacrifices the performance of the system.

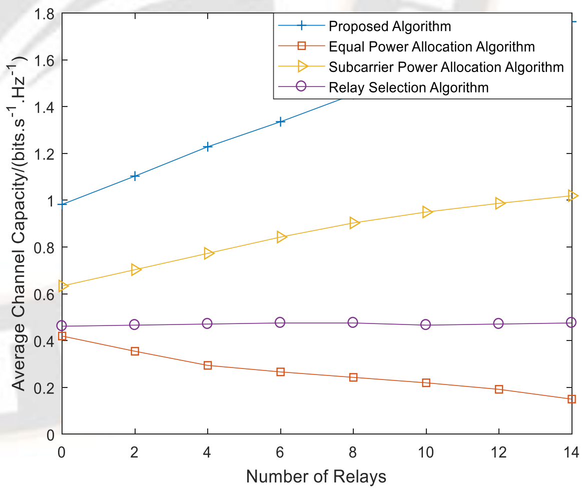


**Figure 3. Comparison Of Average Channel Capacity Under Different Number Of Relays**

In Fig. 3 average channel capacity of several resource allocation algorithms is compared with different numbers of relays, where $\frac{P}{N_0} = 30dB$, and the number of sub-carriers is 40. It is observed from Fig. 3 that compared to the other three resource allocation algorithms, the algorithm proposed in this paper obviously improves the average channel capacity of the

*International Journal on Recent and Innovation Trends in Computing and Communication*
***ISSN: 2321-8169 Volume: 11 Issue: 8***
***Article Received:** 25 May 2023 **Revised:** 12 June 2023 **Accepted:** 30 July 2023*

system. Similarly, with the increase in number of relays, the proposed algorithm can flexibly allocate system resources and increase the system capacity linearly. The subcarrier power allocation algorithm increase the system capacity as the number of relays increases, but because relays use sub-carriers separately and do not make full use of the diversity of multiple relays, the curve rise rate is relatively slow. The relay selection algorithm selects the optimal relay before allocating system resources, so the performance of the algorithm is less affected by the number of relays. For power algorithms such as equal power, the increase in the number of relays leads to a decrease in the power allocated by each node, and system resources cannot be allocated reasonably according to channel conditions. Therefore, the performance of this algorithm gradually decreases with the increase of relays.

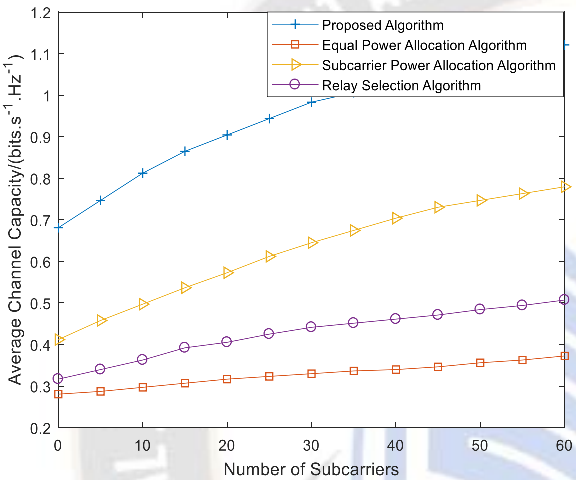


**Figure 4. Comparison of average channel capacity under different number of subcarriers**

Fig. 4 compares the average channel capacity of different resource allocation algorithms with different subcarrier numbers, where, $\frac{P}{N_0} = 30dB$, and the number of relays is 8. It is observed from Fig. 4 that compared to the other three resource allocation algorithms, the system average channel capacity performance of the proposed algorithm is better. This performance gain comes from the mechanism of relay sharing sub-carriers. The system diversity gain increase linearly, with the increase in subcarrier pairs, and it is clearly observed from proposed and subcarrier power allocation algorithm curves.

## V. CONCLUSION

In the two-way multi-relay OFDMA network, a joint algorithm for power allocation and sub-carrier pairing under high signal-to-noise ratio is proposed in this paper. Different from the traditional algorithm based on relay selection, the algorithm in this paper allows all relays to forward the same sub-carrier pair, thereby increasing the additional diversity gain of the system. When the total power of the system is limited, the algorithm first uses Cauchy's inequality to optimize the distribution of the optimal power of each relay when the total power of all relays is fixed, simplifying the original optimal system model; then applying the dichotomy to pass the maximum power allocation between the source node and the relay node by quantifying the equivalent channel gains under different sub-carrier pairs; finally, allocate the power of different sub-carrier pairs through convex planning, and apply the Hungarian algorithm to pair the sub-carriers to obtain the maximum system capacity. Due to the complexity of the power allocation algorithm in the two-way multi-relay network, there is currently no optimal power allocation method with lower computational complexity. The algorithm proposed in this paper greatly reduces the complexity of power allocation, and the correctness of the algorithm is verified by simulation. Compared with the simulation results, it is shown that when the same sub-carrier pair is relayed and forwarded, the combined power and sub-carrier joint allocation brings great gains to the energy efficiency of the system.

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