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A Study on Effective Mathematics Education Through Various Modelling Strategies

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Abstract— In India, as well as many other countries, mathematical modelling and its application in mathematics education are gaining popularity. The growing body of literature on the subject uncovers a variety of techniques to mathematical modelling as well as similar theories, and also perspectives on the use of mathematical modelling in mathematical learning, in definitions of meanings of designs and modelling, conceptual backdrops of modelling, and the essence of questions used in teaching modelling. This journal is about mathematical modelling, or, to put it another way, implementations and modelling. Beginning with Henry Pollak's research lecture, this has been a significant theme in math education over several decades. The term "applications and modelling" refers to both the products and the processes involved in the interaction of the physical world and mathematics. The purpose of this paper is to summarise some critical parts, particularly those related to implementation teaching and modelling.

Keywords- Mathematical Modelling, Learning, Education, Effective.

I. INTRODUCTION

Mathematical modelling has grown in popularity as a teaching strategy in math education at different levels, right up from elementary up till the higher education, over the last two decades. Mathematical modelling has been used in educational settings to help students improve their ability to solve real-world problems. In recent years, many studies on modelling at various levels of school achievement have been conducted (e.g., Delice & Kertil, 2014; Kertil, 2008), and mathematical modeling has become more prominent in education curriculum.
"All tap the potential in learners' minds and the external transcription systems of these systems," say Lesh and Doerr (2003) (e.g., ideas, thoughts, their representations, rules, and regulations as well as materials). To comprehend and analyze various natural systems, a framework is used. Lehrer and Schauble (2007) identified a model with an attempt to make a comparison between an undamped variable and a previously observed or recognisable system. As a direct consequence, humans use approaches to understand and make sense of absolute situations. They define this process as model-based thinking, emphasising the importance of developing a robust model. Model-based able to think levels are also described as hierarchical.

According to Lesh and Doerr (2003a), a framework consists of both- internal and external notation systems (e.g., innovation, their representations, regulations, and materials). Nature's complex systems are studied and interpreted using models. A model, according to Lehrer and Schauble (2003), is an attempt to create an analogy between an unfamiliar system and one that is already known or familiar. As a result, people use models to interpret and make sense of absolute situations. Model-based thinking, as defined by Lehrer and Schauble (2007), is a developmental process. The tiers of model-based thinking are also described as hierarchical.

II. MATHEMATICAL MODELLING

According to Haines and Crouch (2007), mathematical modelling is a continuous cycle in which fundamental issues are transcribed into mathematical logic, solved using a system of symbols i.e. visual language, and the solutions evaluated in a real-world system. Verschaffel, Greer, and De Corte (2002) define mathematical modelling as a procedure for expressing real-world situations and relationships using mathematics. Both perspectives emphasise the importance of using mathematics to examine the essential characteristics of real life scenarios beyond their physiological properties.
According to Lesh and Doerr (2003a), mathematical modelling is the process in which current cognitive systems

and models are used to create and develop new models in new contexts. According to this definition, modelling is the process of creating a physical, symbolic, or abstract model of a situation. According to Gravemeijer and Stephan (2002), mathematical modelling is indeed not constrained to expressing specific cases of real world in mathematical language utilising predetermined models. It involves reinterpreting and rewriting mathematical concepts and representations in order to link them to specific events in the circumstance. To successfully articulate a real situation in mathematical language, pupils should have higher-level math skills beyond supercomputing and arithmetical abilities, such as spatial reasoning, analysis, and prediction.

2.1 Mathematical Modelling Approaches

Different methods towards using modelling in mathematics teaching have been proposed from various theoretical perspectives, and no single point of view among educators has emerged (Kaiser, Borromeo Ferri, Blum & Stillman, 2011). These parallels and differences must be clarified in order to explain different points of view and reach a consensus. Classification systems are the most common way of displaying modelling approaches. According to this scheme, viewpoints are divided into I realistic or applied modelling, (ii) context - specific modelling, (iii) academic modelling, (iv) socio-critical modelling, (v) epistemological or theoretical modelling, as well as (vi) cognitive modelling. In general, modelling in mathematics education is divided into two categories: (i) modelling work for the purpose of teaching maths and (ii) modelling as a way of teaching mathematics.

Table 1: Comparison between traditional problem solving method and Mathematical Modeling

Traditional Method	Modeling Method
It was a process used for reaching a conclusion using data	Different interpretations for multiple cycles
The problem's context is either an idealised or realistic real-life situation.	Real-world setting
Students are required to use equations, methodologies, techniques, and mathematical ideas that have been taught to them.	During the modelling process, students go through the stages or ties of development, review, and revision as key mathematical concepts and structures.
Individual efforts were emphasised.	The importance of group projects was highlighted (social interaction, exchange of mathematical ideas, etc.)
Extrapolated from reality	The nature of the project is interdisciplinary.
Students must be able to comprehend mathematical symbols and structures.	Students attempt to make mathematical descriptions of meaningful real world situations through modelling processes.
Particular problem-solving strategies and methods such as developing a unique approach and transferring onto a figure) that can be applied to other problems are taught.	Students consciously developed open-ended and multiple solution strategies in response to the problem's specifications.
There is only one correct answer.	There is the possibility of multiple solution approaches and solutions (models).

2.2 Objective of the Study

The objective of the study is to:

- Understand the fundamental concept of mathematical modelling
- Examine the various mathematical modelling approaches
- Compare and contrast the traditional and modelling methods of mathematics learning

III. METHODOLOGY

3.1 Research Design

This study used a variety of modelling strategies to evaluate the effectiveness of mathematics education. The study used a developed a semi design and a quantitative approach.
This scientific method was used because the independent variables were difficult to control. The researchers are trying and see if the instructors' treatment had a positive influence or not, as assessed by before and after the tests. This justification is consistent with the notion that the quasi-experimental method is designed to determine how researchers attempt to control factors so that the study variables influence the variance.

3.2 Population and sample

The study included all secondary school students in New Delhi during the academic year 2021/2022. The specimen consisted of nine students in class 12th from a New Delhi high school. The Stratified Random Sampling technique was considered as the ideal method for this type of situation. Random selection sampling is the procedure used for selecting samples by taking into account the strata within each school in order to accurately represent each sub-group.

3.3 Result and Analysis

3.3.1 Model 1

Problem 1: An individual person swims 45 kilometres downstream and 25 kilometres upstream in five hours. What is the current velocity?

Students' traditional methods rely on the formulas for distance and velocity. And then attempt on solving the issue.
We taught them a few techniques or steps for solving the problem using mathematical modelling. Actually, there are three basic concepts in boat and stream problems: time, distance, and speed. Now, before we solve the problem, we'll go over some key formulas that will help them solve it quickly.

Let A km/hr is speed of boat and B km/hr is speed of stream then

- In downstream, Speed of boat = $A + B$
- In upstream, Speed of boat = $A + B$

Let A km/hr is the boat speed in downstream and in upstream is B km/hr then

- $\frac{A+B}{2}$ will be the speed of boat in stagnant water
- $\frac{A-B}{2}$ will be the speed of stream

If B is the stream speed and A is the speed of either the man or a boat in stagnant water and then time taken is:
Downstream = $\frac{Distance}{A+B}$
Upstream = $\frac{Distance}{A-B}$
Total time taken in going downstream and upstream =Upstream time + Downstream time

Step 1
The issue is described as follows: During this process, we check whatever is given to us and what needs to be discovered. So, according to the above problem, downstream and upstream rate and time is give and we have to calculate speed of current?
Mathematical description: Now, we know that the distance in upstream is 45 kilometres in five hours and the downstream distance is 25 kilometres in five hours. The transition in distance over time is known as velocity. As a result, we've Velocity "V"=distance/time
It can also be written as Downstream Velocity=45/5km/hr and Upstream Velocity =25/5km/hr

Velocity of current = $\frac{Downstream\ Velocity - Upstream\ velocity}{2}$ km/hr

Step 2
Now, the solution is:
Downstream rate of Man=45/5 =9km/hr
Upstream rate of Man =25/5=5km/hr
Velocity of current = $\frac{9-5}{2} = \frac{4}{2}$ = 2km/hr
Step 3
Swimming with the stream takes 5 hours, and swimming in the opposite direction takes the same time. We must also quantify river flow velocity, which is calculated using the direct formula (V1-V2)/2, if we need to calculate current velocity.

Table 2: Representing time taken to solve the problem

Student's name	Students' speed in the traditional method	Students' speed in the mathematical modelling
Akash Singh	12 min	9 min
Anamika Mishra	8min	5 min
Sanadeed Fatima	13min	10 min
Vipin Kumar	14 min	11 min
Shashank Kumar	9min.	6min.
Mariyam Zaidi	8min	5min.
Sanjay Sharma	10 min	8 min
Shwet Yadav	9 min	5 min
Bhavna Singh	7 min	5 min

We should emphasise a wide range of applied mathematics experiences, improve mathematical concepts by describing facts, Explain how theories are used and how problems are related to real-world situations. Educators also consider NCTM policies for mathematics teaching.

Also learn about the common core state standards for mathematics, which were recently developed in the United States (2010) Mental habits and productive thinking are the focus of modelling. It helps students make better judgments by connecting classroom maths to everyday life. Most schools, according to the findings, do not involve children in problem solving. modification, or effective presentation. Solving word problems in mathematics from textbooks is also a challenge. The explanation in the syllabus is neither interesting nor detailed. There is no connection to everyday problems, particularly among the upper classes. To help pupils understand the complexity of mathematics, we described issues at various phases of the educational process. Consider some mathematical models and mathematical modelling teaching techniques. Problems involving fractions, shapes, quantification, probability, and statistics emphasise pattern recognition and problem solving. Also consider the role of teachers in assisting students in participating in activities and completing difficult tasks. Students' mathematical attitudes were improved, as was their habit of discovering new mathematical ideas and facts. As a result, the teacher's efficiency should improve. All higher education is founded on secondary school students acquiring an interest in the subject at this age. If we put in less effort here, the subject will get more complex.

3.3.2 Model 2

I present some examples of how to engage students in mathematical activities in this section. Also, use a diagram to show the relationship between the problem and the real world. The following steps were involved in these problems: constructing, mathematically describing, and simplifying.

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We've explained what a matrix is and how it relates to real-life scenarios. Is there a library at your school, we inquired? Yes, they said. Let's say there are 25 roses in one first table, 20 lilies on the second table, and 15 lotuses on the third table. The above records are then arranged as follows: [25 20 15]. I've given them questions about matrix operations, variety of matrices and asked them to describe them.. Traditional procedures such as substitution, elimination, and cross multiplication, were used by students to solve equations.

Problem 1
If,

$$2x + y + 2z = 0$$
$$2x - y + z = 10$$
$$x + 3y - z = 5$$

Then find the value of x, y and z?
Solution:
To find the values of x, y, and z we have to solve the above given equations
Numerical explanation
First, we write the above equation as

$$P = Q^{-1}R$$

Where,

$$Q = \begin{bmatrix} 2 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 \\ 10 \\ 5 \end{bmatrix}$$

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now, formula to find Q^{-1} is

$$Q^{-1} = Adjoint\ of\ Q/|Q|$$

First we have to find cofactor to calculate adjoint

$$C_{ij} = (-1)^{i+j}M_{ij}$$

Where M_{ij} is minor.
Minor $M_{11} = \begin{bmatrix} -1 & 1 \\ 3 & -1 \end{bmatrix} = -2$
Similarly, we can calculate other Minor M_{ij}
Now, we can calculate the cofactor by the above mentioned formula.

Adjoint of $Q = \begin{bmatrix} -2 & 7 & 3 \\ 3 & -4 & 2 \\ 7 & -5 & -4 \end{bmatrix}$

$$Q^{-1} = 1/13 \begin{bmatrix} -2 & 7 & 3 \\ 3 & -4 & 2 \\ 7 & -5 & -4 \end{bmatrix}$$

Now we will solve the equation to determine the values of x, y, and z.

$$P = Q^{-1}R$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1/13 \begin{bmatrix} -2 & 7 & 3 \\ 3 & -4 & 2 \\ 7 & -5 & -4 \end{bmatrix} * \begin{bmatrix} 0 \\ 10 \\ 5 \end{bmatrix}$$

Then we get x= 85/13, y= -30/13, z= -70/13.

Table 3: Representing time taken to solve the problem

Name of students	Students' speed in the traditional method	Students' speed in mathematical modelling
Akash Singh	16 min	13 min
Anamika Mishra	13min	10 min
Sanadeed Fatima	17min	13 min
Vipin Kumar	13 min	9 min
Shashank Kumar	10min.	8 min.
Mariyam Zaidi	10min	7 min.
Sanjay Sharma	12 min	10 min
Shwet Yadav	9 min	6 min
Bhavna Singh	8 min	6 min

IV. CONCLUSION

After reviewing the above research, we believe that mathematics education has a lot of room to grow, and that mathematical modelling is the greatest weapon (or link) for integrating mathematics in real-world problems. However, much work remains to be done to improve math education for both students and teachers. In model 2, we attempted to distinguish between traditional methods and mathematical modelling, as well as show how speed is influenced in both methods using examples. We also talked about how to teach math. What exactly should be taught? We attempted to take cognisance of the role of the teacher because teachers have a significant impact on students' lives and students rely heavily on them for their studies. As a result, teachers should have a thorough understanding of the material and be able to connect mathematics to real-world problems. Modelling in mathematical education: mathematical modelling is also very functional in higher education. We'll go over some examples of where it's useful.

Engineering is a mathematical branch

Higher-level education in the field of mathematics, particularly in the engineering area, is very different and distinguished from school learning. Students of the engineering field apply their knowledge and understanding in a logical, analytical, and graphical manner. Because they study the application of mathematics in engineering. Engineering allows students to interact to technology and helps them develop innovative ideas while also improving their math skills.

How mathematical modelling related with engineering?

1. Students design and develop an approach and strategy to address the issues with the help of mathematical engineering. They also inculcate the ability to properly shape their abstract thinking and imagination. Their ability to be creative is boosted.

2. Modeling provides an effective system for resolving issues. Prediction of solution can be done mathematically as well as with the help of modelling, but proper strategies are required. As a result, mathematical modelling is a valuable engineering tool. Essentially, modelling is a step-by-step procedure that includes the following steps:
   - The problem must be identified.
   - Gather and analyse the pertinent data.
   - Finally, create a model that meets all of the problem's requirements. Also innovate and create tools that verify and confirm to the predictability of the model. And also leave some space to modify the model according to the conditions.

When resources and options are insufficient, modelling really is the only way to fix the problem. Engineers use a method to come up with a solution in which they collect all relevant information about the problem, correlate it with appropriate information for the advancement of a working model, and try to solve using variables such as time, cost and expense, size, as well as performance.

Mathematical modelling is becoming more prevalent in all aspects of life. From elementary school to higher education in mathematics. It plays an important role in various aspects of life. Content was also linked to real-life issues. However, it necessitates a thorough understanding of the content as well as relevant information. Mathematical modelling creates an environment in which students can solve problems mathematically and develop a mathematical mindset. Increased student interest is a good thing because it allows students to connect textbook material to their daily lives and attempt to create and solve models. It can also be used in the field of research. There can be no progress without research, and all development is dependent on it. However, in order to improve research quality, we need some technology and systems that make research work easier and more interesting, and mathematical modelling provides a better system, innovative ideas, and logically process.

A conducive environment for research is now in place. We have a lot of talented people, but some things are lacking, such as economic factors, inspiration and motivation, and technological knowledge. We need a lot of research work in India, but it's difficult due to a lack of curriculum, workshops, and high-quality learning and teaching. In India, mathematical modelling is now taught in schools, but it still requires more work because it connects us to our daily lives and ensures successful problem-solving planning.

People can translate real-life situations into mathematical language with the help of mathematical modelling. Verifies results on actual problem using some variables related to situations. There is no formal mathematical modelling procedure for obtaining a solution. The modelling process is cyclic, according to (Haines & Crouch).

REFERENCES

[1] Delice, A., & Kertil, M. (2014). Investigating the representational fluency of pre-service mathematics teachers in a modeling process. International Journal of Science and Mathematics Education. doi: 10.1007/s10763- 013-9466-0
[2] Kertil, M. (2008). Examination of the problem-solving skills of mathematics teacher candidates in the modeling process (Master's thesis, Marmara University, Department of Educational Sciences, Department of Secondary Science and Mathematics Education, Istanbul). Retrieved from http://tez.yok.gov.tr
[3] Lesh, R., Cramer, K., Doerr, H. M., Post, T., & Zawojewski, J. S. (2003). Model development sequences. In R. Lesh, & H. M. Doerr (Eds.), Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum
[4] Lehrer, R., & Schauble, L. (2007). A developmental approach for supporting the epistemology of modeling. In
[5] W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), Modeling and applications in mathematics education (pp. 153-160). New York, NY: Springer.
[6] Haines, C., & Crouch, R. (2007). Remarks on a modelling cycle and interpretation of behaviours.
[7] Verschaffel, L., Greer, B., & De Corte, E. (2002). Everyday knowledge and mathematical modeling of school word problems. In K. P. Gravemeijer, R. Lehrer,H. J. van Oers, & L. Verschaffel (Eds.), Symbolizing, modeling and tool use in mathematics education (pp. 171-195). Dordrecht, The Netherlands: Kluwer Academic Publishers.
[8] Kaiser, G., Blum, W., Borromeo Ferri, R., & Stillman, G. (2011). Preface. In G. Kaiser, W. Blum, R. BorromeoFerri, & G. Stillman (Eds.), Trends in teaching and learning of mathematical modelling: ICTMA14 (pp. 1-5). Dordrecht, The Nedherlands: Springer.
[9] Dede, A.T., Hıdıroğlu, Ç.N., & Güzel, E.B. (2017). Examining of Model Eliciting Activities Developed by Mathematics Student Teachers. Journal on Mathematics Education, 8(2), 223-242.
[10] Erbaş, A. K., Kertil, M., Çetinkaya, B., Çakıroğlu, E., Alacacı, C., & Baş, S. (2014). Mathematical modeling in Mathematics education: Basic concepts and different approaches. Educational Sciences: Basic Concepts and Different Approaches Practice, 14(4), 1607–1627.
[11] Eric, C M., Dawn, N K., Wanty,W. & Seto, C.(2015). A Case Study on Developing a Teacher's Capacity in Mathematical Modelling. The Mathematics Educator 16(1), 45-74.
[12] Achmetli, K., Schukajlow, S., & Rakoczy, K. (2019). Multiple solutions to solve real-world problems and students' procedural and conceptual knowledge. International Journal of Science and Mathematics Education,100, 43–60.

[13] Alghamdi, A., Jitendra, A. K., & Lein, A. E. (2020). Solving multiplication and division word problems with schematic diagrams: The role of schema-based instruction in supporting mathematical thinking skills of students with mathematics learning disabilities. ZDM Mathematics Education, this issue (in press).