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Fuzzy Lattice Ks-Operator Normal Group

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ABSTRACT
In this paper an algebraic structure fuzzy lattice KS- operator group is defined and derived its properties. We all know that all subgroups of an abelian group are normal. Here the same condition of commutativity is used to define this structure but it is in form of a function as fuzzy set is a function. The basic set on which this structure is defined is a KS operator fuzzy group and also a lattice hence the name.

Keywords- Lattice group, fuzzy lattice group, K operator group, fuzzy lattice k^2 operator group, fuzzy lattice KS operator group.

1. Introduction

A revolutionary thing happened in the history of mathematics by introduction of Fuzzy set theory by Zadeh [11]. Rosenfield[7] then utilized this theory in algebra and introduced fuzzy groups..In 1994 Ajmal and Thomas [1] combined this fuzzy algebra with the algebraic structure lattice and extended the concept to next level. Nanda [6] used partial ordering in fuzzy set and defined fuzzy lattice. Satya Saibaba[8] used ordering of lattice relation and defined fuzzy lattice ordered group. Goguen [2] generalized the concept of fuzzy lattice by replacing image set of fuzzy set [0,1] by a complete lattice. Solairau and Nagrajan [9] defined fuzzy Q- modules. Murdai and Rajendran [5] defined a new form of fuzzy lattice. Gu[3] done the fuzzy algebra work on operator groups. Subramaniam , Nagrajan & Chellapa [10] brought the research to the next level by giving m fuzzy group concept.Lokhande and Makandar [4] defined the concept of one operator fuzzy algebraic structures. In this paper we introduced the two operator fuzzy lattice normal group

2. PRELIMINARIES

Definition : Fuzzy group-
Let $\mu: X \to [0, 1]$ be a fuzzy set & $G \in þ(X)$ = Set of all fuzzy sets on X. A fuzzy set μ on G is called a fuzzy group if i) $\mu(x y) \geq \min\{\mu(x), \mu(y)\}$ ii) $\mu(x^{-1}) \geq \mu(x)$, for all x, y$\in$ G .

Definition : Lattice group
A lattice group is a system (G,.,≤) if i) (G, ·) is a group ii) (G,≤) is a lattice . For a, b, x, y$\in$ G

Definition : Fuzzy lattice group
Let $\mu: X \to [0, 1]$ be a fuzzy set & G is a lattice , $G \in þ(X)$. A function μ on G is said to be a fuzzy lattice group if
i) $\mu(x y) \geq \text{mini}\{\mu(x), \mu(y)\}$
ii) $\mu(x^{-1}) \geq \mu(x)$
iii) $\mu(x \vee y) \geq \text{mini}\{\mu(x), \mu(y)\}$
iv) $\mu(x \wedge y) \geq \text{mini}\{\mu(x), \mu(y)\}$
for all x, y$\in$ G

Definition: K operator group
Let G be a group, K be any set if i) $k x \in G$. for all x$\in$ G, $k \in K$. Then G is called a K operator group.

Definition : Fuzzy K operator group
Let $\mu: X\to [0, 1]$ be a fuzzy set & G be a K group G. A fuzzy set on G, $G \in þ(X)$) is called a fuzzy K operator group if i) $\mu(kxky)) \geq \text{mini}\{\mu(k x), \mu(ky)\}$
ii) $\mu(kx)^{-1} \geq \mu(k x)$ for all x, y$\in$ G, $k \in K$

Definition : Fuzzy lattice K^2-operator group
μ: X to [0, 1], $G \in þ(X)$, K C X. G is a lattice and a K^2-operator group. A function μ on G is said to be a fuzzy lattice K-operator group if
i) $\mu(kxky) \geq \text{mini}\{\mu(kx),\mu(ky)\}$
ii) $\mu((kx)^{-1} \geq \mu(kx)$
iii) $\mu(kx \vee ky) \geq \text{mini}\{\mu(kx),\mu(ky)\}$
iv) $\mu(kx \wedge ky) \geq \text{mini}\{\mu(kx),\mu(ky)\}$ For all x, y $\in$ G

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Definition: Fuzzy lattice KS-operator group

μ: X to [0, 1], G ϵ þ(X), K,S C X. G is a lattice and a KS-operator group. A function μ on G is said to be a fuzzy lattice KS-operator group if

i) $\mu(kxsy) \geq \text{mini}\{\mu(kx),\mu(sy)\}$
ii) $\mu((kx)^{-1} \geq \mu(kx)$, $\mu((sx)^{-1} \geq \mu(sx)$
iii) $\mu(kx \vee sy) \geq \text{mini}\{\mu(kx),\mu(sy)\}$
iv) $\mu(kx \wedge sy) \geq \text{mini}\{\mu(kx),\mu(sy)\}$ For all x, y ϵ G

Definition : Fuzzy lattice K operator normal group

A fuzzy lattice K-operator group is said to be a fuzzy lattice K-operator normal group if $\mu((kx)(ky)) = \mu((ky)(kx)$ for all x, yϵ G ,kϵ K

Definition : Fuzzy lattice KS operator normal subgroup

A fuzzy lattice KS-operator group is said to be a fuzzy lattice KS-operator normal subgroup if $\mu((kx)(ky)) = \mu((ky)(kx)$ & $\mu((sx)(sy)) = \mu((sy)(sx)$
for all x, yϵ G ,kϵ K, s ϵS

3. PROPERTIES OF FUZZY LATTICE KS OPERATOR NORMAL GROUP

Proposition 3.1: Intersection of two fuzzy lattice KS operator normal subgroups is again a fuzzy lattice KS operator normal subgroup.

Proof-Let A and B be two fuzzy lattice KS operator normal groups on G.

$\mu_{A \cap B}((kx)(ky)) = \mu_A((kx)(ky)) \wedge \mu_B((kx)(ky))$
$= \mu_A(ky)(kx) \wedge \mu_B(ky)(kx)$
$= \mu_{A \cap B}((ky)(kx)$
$\mu_{A \cap B}((sx)(sy)) = \mu_A((sx)(sy)) \wedge \mu_B((sx)(sy))$
$= \mu_A(sy)(sx) \wedge \mu_B(sy)(sx)$
$= \mu_{A \cap B}((sy)(sx)$

Proposition 3.2: If $\{A_i\}$ is a family of fuzzy lattice KS operator normal subgroups of G then $\cap A_i$ is a fuzzy lattice KS operator normal group of G

where $\cap A_i = \{ kx , \wedge \mu_{Ai}(kx) / x \in G, k \in K \}$

Proof- $(\cap \mu_{Ai})((kx)(ky)) = \wedge \mu_{Ai}((kx)(ky))$
$= \wedge \mu_{Ai}(ky\, kx)$
$= (\cap \mu_{Ai})(ky\, kx)$
$(\cap \mu_{Ai})((sx)(sy)) = \wedge \mu_{Ai}((sx)(sy))$
$= \wedge \mu_{Ai}(sy\, sx)$
$= (\cap \mu_{Ai})(sy\, sx)$

Proposition 3.3: A fuzzy lattice KS operator group is a normal if and only if A is constant on conjugate classes of G.

Proof- Let A be a fuzzy lattice KS operator normal group of G.

$\mu_A(ky)^{-1}(kx)(ky)) = \mu_A((ky)^{-1}(ky)(kx)$
$= \mu_A(e\,kx)$
$= \mu_A(kx)$
$\mu_A(sy)^{-1}(sx)(sy)) = \mu_A((sy)^{-1}(sy)(sx)$
$= \mu_A(e\,sx)$
$= \mu_A(sx)$

Hence A is constant on conjugate classes of G.

Conversely $\mu_A(kx\,ky) = \mu_A(kx\,ky\,e)$
$= \mu_A(kx\,ky\,kx(kx)^{-1})$
$= \mu_A(kx\,(ky\,kx)\,(kx)^{-1})$
$= \mu_A(ky\,kx)$
$\mu_A(sx\,sy) = \mu_A(sx\,sy\,e)$
$= \mu_A(sx\,sy\,sx\,(sx)^{-1})$
$= \mu_A(sx\,(sy\,sx)\,(sx)^{-1})$
$= \mu_A(sy\,sx)$

Therefore the fuzzy lattice KS operator group is a normal.

Proposition 3.4: For fuzzy lattice KS operator normal group

$\mu_A(ky)^{-1}(kx)(ky)) = \mu_A(ky)(kx)(ky)^{-1}$
$\mu_A(sy)^{-1}(sx)(sy)) = \mu_A(sy)(sx)(sy)^{-1}$

Proof- Let A be a fuzzy lattice KS operator normal subgroup of G.

$\mu_A(ky)^{-1}(kx)(ky)) = \mu_A(ky)^{-1}(ky)(kx))$
$= \mu_A(e)(kx))$

$= \mu_A\ (k\,x))$
$= \mu_A\ (k\,x\,e)$
$= \mu_A\ ((k\,x)(k\,y)(ky)^{-1}$
$= \mu_A\ (k\,y)(k\,x)\ (k\,y)^{-1}$
$\mu_A\ (s\,y)^{-1}\ (s\,x)\ (s\,y) = \mu_A\ (s\,y)^{-1}\ (s\,y)\ (s\,x))$
$= \mu_A\ (\,e)\ (s\,x)$
$= \mu_A\ (s\,x)$
$= \mu_A\ (s\,x\,e)$
$= \mu_A\ ((s\,x)(s\,y)(s\,y)^{-1}$
$= \mu_A\ (s\,y)(s\,x)\ (s\,y)^{-1}$

Proposition 3.5: For a fuzzy lattice K^2 operator group and $\mu_A\ (k\,x\,.\,k\,y\,) = \mu_A\ (k\,x\,y)$, A is normalized if and only if μ_A (ke) = 1 & μ_A (se) = 1

Proof- Let A is normalized.
μ_A (kx) =1 for some x ϵG
But $\mu_A\ (k\,x\,.\,k\,x^{-1}\,) \geq$ mini { μ_A (kx), $\mu_A\ (\,k\,x^{-1}\,))$
$\geq$ mini { μ_A (kx), μ_A (k x))
$= \mu_A$ (kx)
$\mu_A\ (k\,x\,.\,x^{-1}\,) \geq \mu_A$ (kx)
μ_A (ke) $\geq \mu_A$ (kx)
μ_A (kx) $\leq \mu_A$ (ke) for all x ϵG
$1 \leq \mu_A$ (ke)
Therefore μ_A (ke) =1
Similarly μ_A (sx) $\leq \mu_A$ (se) for all x ϵG
$1 \leq \mu_A$ (se)
Therefore μ_A (se) =1
Conversely μ_A (ke) =1& μ_A (se) = 1
Hence A is normalized.

Proposition 3.6: If A and B are fuzzy lattice KS operator normal subgroups of group G_1 and G_2 respectively, then A× B is fuzzy lattice KS operator normal subgroup of $G_1 \times G_2$.

Proof- The direct product of fuzzy lattice KS operator normal group is a fuzzy lattice KS operator normal group.
Let A & B be fuzzy lattice KS operator normal groups of group G_1 and G_2 respectively.
Then A × B is a fuzzy lattice KS operator group.
Let $x = (\,x_1, x_2\,)$, $y = (\,y_1, y_2)\ \epsilon\ G_1 x\, G_2$
$\mu_{A\times B}\ (\,k\,x\,ky) = \mu_{A\times B}\ (k(x_1, x_2)\ k(y_1, y_2))$
$= \mu_{A\times B}\ ((kx_1, kx_2)\ (ky_1, ky_2))$
$= \mu_{A\times B}\ (kx_1\,ky_1,\ kx_2ky_2)$
$=$ mini { $\mu_A\ (kx_1ky_1), \mu_B(\,kx_2ky_2)\}$
$=$ mini { $\mu_A\ (ky_1kx_1), \mu_B(\,ky_2kx_2)\}$
$= \mu_{A\times B}\ (ky_1kx_1,\ ky_2kx_2)$
$= \mu_{A\times B}\ ((ky_1, ky_2)\ (kx_1, kx_2))$
$= \mu_{A\times B}\ (k(y_1, y_2)\ k(x_1, x_2))$
$= \mu_{A\times B}\ (\,ky\,kx)$
$\mu_{A\times B}\ (\,sxsy) = \mu_{A\times B}\ (s(x_1, x_2)\ s(y_1, y_2))$
$= \mu_{A\times B}\ ((sx_1, sx_2)\ (sy_1, sy_2))$
$= \mu_{A\times B}\ (sx_1\,sy_1,\ sx_2sy_2)$
$=$ mini { $\mu_A\ (sx_1\,sy_1), \mu_B(\,sx_2sy_2)\}$
$=$ mini { $\mu_A\ (sy_1\,sx_1), \mu_B(\,sy_2sx_2)\}$
$= \mu_{A\times B}\ (sy_1sx_1,\ sy_2sx_2)$
$= \mu_{A\times B}\ ((sy_1, sy_2)\ (sx_1, sx_2))$
$= \mu_{A\times B}\ (s(y_1, y_2)\ s(x_1, x_2))$
$= \mu_{A\times B}\ (\,sy\,sx)$

Proposition 3.7: If $A_1, A_2, \ldots A_n$ are fuzzy lattice KS operator normal groups of groups $G_1, G_2, \ldots G_n$ respectively, then $A_1\times A_2\times \ldots A_n$ is fuzzy lattice KS operator normal subgroup of $G_1 \times G_2 \times \ldots G_n$.

Proof- The direct product of fuzzy lattice KS operator groups is a fuzzy lattice KS operator group.
Let $A_1, A_2, \ldots A_n$ are fuzzy lattice KS operator normal groups of groups $G_1, G_2, \ldots G_n$ respectively.
Then $A_1\times A_2\times \ldots A_n$ is a fuzzy lattice KS operator group of $G_1 \times G_2 \times \ldots G_n$.
Let $x = (\,x_1, x_2, \ldots x_n\,)$, $y = (\,y_1, y_2, \ldots y_n)\ \epsilon\ G_1 x\, G_2 \times \ldots \times G_n$
$\mu_{A1\times A2 \ldots \times An}\ (\,kx\,ky) = \mu_{A1\times A2 \ldots \times An}\ (kx_1\,ky_1, kx_2ky_2, \ldots kx_ny_n)$

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$= \text{mini} \{ \mu_{A1} (kx_1ky_1), \mu_{A2}(kx_2ky_2),\ldots\ldots\ldots \mu_{An}(kx_nky_n)\}$
$= \text{mini} \{ \mu_{A1} (ky_1kx_1), \mu_{A2}(ky_2kx_2),\ldots\ldots\ldots \mu_{An}(ky_nkx_n)$
$= \mu_{A1\times A2\ldots\ldots\times An}(ky_1kx_1, ky_2kx_2,\ldots\ldots\ldots kynkxn)$
$= \mu_{A1\times A2\ldots\ldots\times An} (kykx)$

$\mu_{A1\times A2\ldots\ldots\times An} (sx\ sy) = \mu_{A1\times A2\ldots\ldots\times An} (sx_1 sy_1, sx_2ky_2,\ldots\ldots\ldots sx_ny_n)$
$= \text{mini} \{ \mu_{A1} (sx_1sy_1), \mu_{A2}(sx_2sy_2),\ldots\ldots\ldots \mu_{An}(sx_nsy_n)\}$
$= \text{mini} \{ \mu_{A1} (sy_1sx_1), \mu_{A2}(sy_2sx_2),\ldots\ldots\ldots \mu_{An}(sy_nsx_n)$
$= \mu_{A1\times A2\ldots\ldots\times An}(sy_1sx_1, sy_2sx_2,\ldots\ldots\ldots synsxn)$
$= \mu_{A1\times A2\ldots\ldots\times An} (sysx)$

Proposition 3.8: If a fuzzy lattice KS operator normal group A is conjugate to fuzzy lattice KS operator normal group P of G_1 & If a fuzzy lattice KS operator normal group B is conjugate to fuzzy lattice KS operator normal group Q of G_2 Then A×B is conjugate to P×Q of $G_1\times G_2$

Proof- $\mu_{A\times B} ((ky)^{-1} (k x_1 , kx_2) ky) = \mu_{A\times B} ((ky)^{-1} k x_1(ky) , (ky)^{-1} kx_2(ky))$
$= \text{mini} (\mu_A((ky)^{-1} k x_1(ky) , \mu_B ((ky)^{-1} kx_2(ky)))$
$= \text{mini} (\mu_P(k x_1) , \mu_Q (kx_2))$
$= \mu_{P\times Q} ((k x_1 , kx_2))$

$\mu_{A\times B} ((sy)^{-1} (s x_1 , sx_2) sy) = \mu_{A\times B} ((sy)^{-1} s x_1(sy) , (sy)^{-1} sx_2(sy))$
$= \text{mini} (\mu_A((sy)^{-1} s x_1(sy) , \mu_B ((sy)^{-1} sx_2(sy)))$
$= \text{mini} (\mu_P(s x_1) , \mu_Q (sx_2))$
$= \mu_{P\times Q} ((sx_1 , sx_2))$

Therefore A×B is conjugate to P×Q.

Proposition 3.9: If A and B are fuzzy lattice KS operator groups of G_1 and G_2 respectively & A×B is a fuzzy lattice KS operator normal group of $G_1\times G_2$ Then the following are true.

i) If $\mu_A(kx) \le \mu_B(ke')$ & $\mu_A(sx) \le \mu_B(se')$ then A is a fuzzy lattice KS operator normal group of G_1
ii) If $\mu_B (kx) \le \mu_A(ke)$ & $\mu_A(sx) \le \mu_B(se')$ then B is a fuzzy lattice KS operator normal group of G_2

Proof- i) $\mu_{A\times B} ((kx,ke') (ky,ke')) = \mu_{A\times B} ((kx\ ky , ke'))$
$= \text{mini} \{\mu_A(kx\ ky), \mu_B(ke'))$
$= \text{mini} \{\mu_A(kxy), \mu_B(ke'))$
$= \mu_A(kxy)$
$= \mu_A(kxky)$

$\mu_{A\times B} ((ky,ke')(kx,ke')) = \mu_{A\times B} ((ky\ kx , ke'))$
$= \text{mini} \{\mu_A(ky\ kx), \mu_B(ke'))$
$= \text{mini} \{\mu_A(kyx), \mu_B(ke'))$
$= \mu_A(kyx)$
$= \mu_A(kykx)$

Therefore $\mu_A(kxky) = \mu_A(kykx)$

$\mu_{A\times B} ((sx,se') (sy,se')) = \mu_{A\times B} ((sx\ sy , se'))$
$= \text{mini} \{\mu_A(sx\ sy), \mu_B(se'))$
$= \text{mini} \{\mu_A(sxy), \mu_B(se'))$
$= \mu_A(sxy)$
$= \mu_A(sxsy)$

$\mu_{A\times B} ((sy,se')(sx,se')) = \mu_{A\times B} ((sy\ sx , se'))$
$= \text{mini} \{\mu_A(sy\ sx), \mu_B(se'))$
$= \text{mini} \{\mu_A(syx), \mu_B(se'))$
$= \mu_A(syx)$
$= \mu_A(sysx)$

Therefore $\mu_A(sxsy) = \mu_A(sysx)$

ii) $\mu_{A\times B} ((ke,kx)(ke,ky)) = \mu_{A\times B} ((ke , kx\ ky))$
$= \text{mini} \{\mu_A(ke), \mu_B(kx\ ky))$
$= \text{mini} \{\mu_A(ke), \mu_B(kxy))$
$= \mu_B(kxy)$
$= \mu_B(kxky)$

$\mu_{A\times B} ((ke,ky)(ke,kx)) = \mu_{A\times B} ((ke , kykx))$
$= \text{mini} \{\mu_A(ke), \mu_B(kykx))$
$= \text{mini} \{\mu_A(ke), \mu_B(kyx))$
$= \mu_B(kyx)$
$= \mu_B(kykx)$

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Therefore $\mu_B(kx\,ky) = \mu_B(ky\,kx)$

$$
\begin{aligned}
\mu_{A\times B}((se,sx)(se,sy)) &= \mu_{A\times B}((se\,,\,sx\,sy\,)) \\
&= \text{mini}\,\{\mu_A(se),\, \mu_B(sx\,sy)) \\
&= \text{mini}\,\{\mu_A(se),\, \mu_B(sxy)) \\
&= \mu_B(sxy) \\
&= \mu_B(sxsy) \\
\mu_{A\times B}((se,sy)(se,sx)) &= \mu_{A\times B}((se\,,\,sysx)) \\
&= \text{mini}\,\{\mu_A(se),\, \mu_B(sysx)) \\
&= \text{mini}\,\{\mu_A(se),\, \mu_B(syx)) \\
&= \mu_B(syx) \\
&= \mu_B(sysx)
\end{aligned}
$$

Therefore $\mu_B(sx\,sy) = \mu_B(sy\,sx)$

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