# Modeling, analysis and simulation of piezoelectromagnetic energy harvester

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***Abstract-*** This work presents a nonlinear piezoelectric energy harvester driven by periodic oscillations. The simulated response to the base excitation is illustrated in terms of harvested power. To study the effect of magnetic forces which are commonly occurring in daily life, a magnetic tip mass is included and is excited by two permanent magnets fixed on either sides laterally. The resulting nonlinear dynamic equations are in both mechanical and electrical parameters and are solved simultaneously using Reduced Order Model (ROM). The time history and frequency response diagrams for the cantilever displacement, voltage and power at the constant load resistance gives a stability picture as well as the amount of energy harvested.

***Keywords-*** *Nonlinear parameter, magnetic coupling, lumped parameter model, distributed parameter model, base excitation, etc.*

## MCCCLXIX.INTRODUCTION

Energy harvesting from environmental sources into electrical form has been recently paid a lot of attention. An energy harvester have been considered as a green resources and include several advantages such as long lifetime, high power density, low maintenance, etc. In this regard various vibration based energy harvesting systems have been developed so as to turn it into electrical form through electromagnetic, piezoelectric and electrostatic conversion systems. But piezoelectric energy harvesting approach is widely accepted because of its relatively high energy density. Most of the early research is based on linear resonant power generation concept. For harvesting with a broadband time dependent frequency characteristics several approaches including oscillator arrays, oscillators with active frequency tuning, etc. were proposed. The dynamic nonlinearities in the harvester systems to enhance the performance can be included as a nonlinear stiffness or as a bistable oscillating system. In the first type of adding nonlinearity the effects of nonlinear stiffness on the energy harvesting ability was considered by several researchers. On the other hand the bistable energy harvesting system has a double well elastic potential which gives different regimes given by the strength of excitation applied. Most of the works focused on first two categories. An energy harvester composed of piezoelectric cantilever beam with a magnetic tip and external magnets of same polarities, so as to induce a repulsion or attraction force causing a bifurcation behavior which leads to cantilever beam to become a bistable system that includes two nontrivial stable equilibrium. So this type of systems exhibit a good broadband harvesting performance and produces large voltage.

## MCCCLXIXMCCCLXIX. LITERATURE REVIEW

There are so many researchers worked on conventional piezoelectric energy harvesters [1-7] however adding nonlinearity in the base excitation is recently focused area. Nonlinearity in the piezoelectromagnetic energy harvester can be incorporated by introducing softening and hardening response. An energy harvester presented by S. C. Stanton et al. [8] is bidirectional with both softening and hardening response within quadratic potential field. Ibrahim et al. [9] worked on mathematical modelling and simulation of lumped parameter piezoelectric energy harvester with tip mass attached at the free end of the cantilever beam and fixed magnet at only one side of the beam. The proposed piezoelectromagnetic harvester is compared with conventional cantilever beam harvester with tip mass. Zheu et al. [10] investigated vertically aligned magnetically coupled nonlinear piezoelectric energy harvester. By altering the angular orientation of its external magnets enhanced broadband frequency response can be achieved and low frequency response is achieved by changing the magnetic orientation. Fan et al. [11] developed two magnetically coupled compact piezoelectric cantilever beams with orthogonal directions of deflection for broadband energy harvesting. The proposed energy harvester shows improved performance in voltage output as compared to linear energy harvesters. The energy harvester presented by Kim and Seak [12] uses the magnetic attraction effect between the soft magnetic tip of the cantilever beam and the two externally fixed permanent magnets arranged in series. Finally nonlinear dynamic and energetic characteristics of the multi-stable energy harvester were examined by utilizing the bifurcation analysis and a series of numerical simulations. Jung et al. [13] modelled cantilever beam nonlinear dynamic piezoelectric energy harvester with tip magnet and two external rotatable external magnets fixed in free space using modified Hamilton's principle.

The present work deals with novel piezoelectromagnetic energy harvesting technology. The proposed energy harvester is mathematically modeled by lumped parameter as well as distributed parameter modeling. Finite element approach is also employed. The proposed models are simulated in Matlab software by using Runge-Kutta solver. The experimental work for proposed energy harvester is also attempted.

## MCCCLXXI. MATHEMATICAL MODELING

### A. LUMPED PARAMETER MODEL

A typical schematic of lumped parameter piezoelectric energy harvester is shown in Fig. 1, with the base excitation $x_0(t)$. The system can be modeled as a SDOF and the equations of motion for piezoelectric energy harvester are derived by applying Newton's law in the mechanical domain as well as Kirchhoff's law in electrical domain.

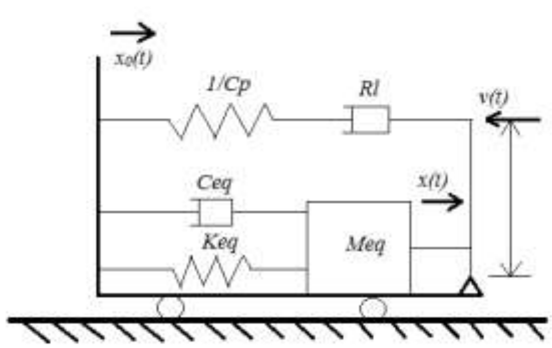


Fig. 1 Single DOF energy harvester

$$M_{eq} \times \ddot{x}(t) + C_{eq} \times \dot{x}(t) + K_{eq} \times x(t) + \theta_b \times v(t) + f_m = -\mu \times M_{eq} \times \ddot{x}_0(t) \quad (1)$$

$$\frac{v(t)}{R_l} + C_p \times \dot{v}(t) - \theta_f \times \dot{x}(t) = 0 \quad (2)$$

$x(t)$ is the relative displacement of the tip mass $M_t$ and $M_{eq}$, $C_{eq}$ and $K_{eq}$ equivalent mass, damping and stiffness of the piezoelectric energy harvester respectively. $v(t)$ is the induced voltage in the harvester due to mechanical vibration. $C_p$ is the clamped capacitance of the piezoelectric transducer and $R_l$ can be considered as the load resistance of the piezoelectric circuit. $\theta_f$ and $\theta_b$ are forward and backward electromechanical effects respectively.

The term $\mu$ is the amplitude correction factor for improving the lumped parameter model and it is calculated by using the equation as

$$\mu = \frac{\left(\frac{M_t}{M_b}\right)^2 + 0.60 \times \left(\frac{M_t}{M_b}\right) + 0.09}{\left(\frac{M_t}{M_b}\right)^2 + 0.46 \times \left(\frac{M_t}{M_b}\right) + 0.06} \quad (3)$$

$M_t / M_b$ is the ratio of tip mass to distributed mass of the beam.The proposed energy harvesting system contains fixed magnets at both the sides of tip magnet. By adding the tip mass displacement of proposed energy harvesting system, the magnetic force expression to the right hand side of the Eq. 1 can be written as

$$f_m(t) = \frac{-3\tau_0 \times m_1 \times m_2}{\pi\left[x(t) + D_0\right]^4} \quad (4)$$

Here, $m_1$ and $m_2$ are magnetic dipole moments of the identical magnets, $D_0$ is the distance between the two magnets. The equations (1) and (2) are electromechanically coupled as both contain displacement and voltage terms. These differential equations should be decoupled by using state variables for solving them simultaneously. Let,

$$z_1(t) = x(t),\ z_2(t) = \dot{x}(t) \text{ and } z_3(t) = v(t)$$

$$\dot{z}_1(t) = z_2(t) \quad (5)$$

$$\dot{z}_2(t) = -\frac{C_{eq}}{M_{eq}} z_2(t) - \frac{K_{eq}}{M_{eq}} z_1(t) - \frac{\theta_b}{M_{eq}} z_3(t) + \frac{3\tau_0 \times m_1 \times m_2}{2\pi \times M_{eq}\left[z_1(t) + D_0\right]^4} - \mu \times \ddot{x}_0(t) \quad (6)$$

$$z_3(t) = \frac{\theta_f}{C_p} z_2(t) - \frac{1}{R_l \times C_p} z_3(t) \quad (7)$$

These decoupled equations are solved by using Runge-Kutta method in Matlab software to get the displacement and voltage responses.

### B. Distributed Parameter Model

Fig. 2 illustrates the schematic diagram of the piezoelectric energy harvester that consists of a cantilever beam with magnetic tip at the end and two fixed magnets as shown. The magnets are fixed at the transverse directions to generate repulsive interaction as shown below.The symmetric bimorph cantilever beam configuration is modelled as a uniform composite beam based on Euler-Bernoulli beam theory.

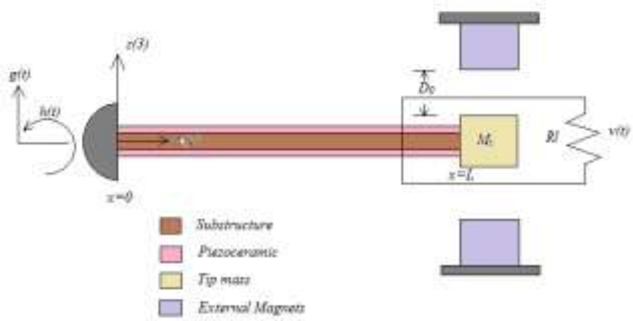


Fig. 2 Bimorph piezoelectric enegy harvester

The base excitation of the cantilever beam is represented by translational displacement $g(t)$ in the transverse direction with superimposed small rotational displacement $h(t)$. Hence, the effective base displacement can be written as

$$w_b(x,t) = g(t) + x \times h(t) \quad (8)$$

The coupled mechanical equation of motion can be obtained for series connection as

$$YI\frac{\partial^4 w(x,t)}{\partial x^4} - \vartheta v(t)\left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx}\right] + CI\frac{\partial^5 w(x,t)}{\partial x^4 \times \partial t} + m\frac{\partial^2 w(x,t)}{\partial t^2} = -m\frac{\partial^2 w_b(x,t)}{\partial t^2} + f_m\frac{\partial^2 w(x,t)}{\partial x^2} \quad (9)$$

Where $w(x,t)$ is the transverse displacement of the beam relative to its base at position *x* and time *t*, $v(t)$ is the voltage across electrodes of piezoceramic layers, m is the mass per unit length of the beam, $M_t$ is the tip mass, *C* is the strain-rate damping coefficient, $\vartheta$ is the coefficient of backward coupling term, $f_m$ is the *x*-directional magnetic force and $\delta(x)$ is the Dirac delta function.

Based on proportional damping assumption, the vibration response relative to the base of the bimorph cantilever can be represented as a convergent series of the Eigen functions as

$$w(x,t) = \phi_r(x) \times \eta(t) \quad (10)$$

Where $\phi_r(x)$ is the mass-normalized Eigen function for $r^{th}$ vibration mode and $\eta(t)$ is modal mechanical coordinate expression of the series connection. Considering $r = 1$, $\phi(x)$ can be calculated as

$$\phi(x) = A\left[\cos\frac{\lambda}{L}x - \cosh\frac{\lambda}{L}x + \varsigma\left(\sin\frac{\lambda}{L}x - \sinh\frac{\lambda}{L}x\right)\right] \quad (11)$$

After substituting equation (9) into equation (1) and applying boundary conditions for undamped problem, the mechanical equation of motion can be obtained as

$$\frac{d^2\eta(t)}{dt^2} + 2\xi \times \omega\frac{d\eta(t)}{dt} + \omega^2 \times \eta(t) - \chi \times v(t) = f_r(t) + f_{mx}(t) \quad (12)$$

The modal mechanical forcing function and magnetic force due to tip magnet and two external fixed magnets can be expressed as

$$f_r(t) = -m\left(\frac{d^2 g(t)}{dt^2}\int_0^L \phi(x)\,dx + \frac{d^2 h(t)}{dt^2}\int_0^L x \times \phi(x)\,dx\right) - M_t \times \phi(L)\left(\frac{d^2 g(t)}{dt^2} + L \times \frac{d^2 h(t)}{dt^2}\right) \quad (13)$$

$$f_{mx}(t) = \frac{-3\tau_0 \times m_1 \times m_2}{2\pi\left[\phi(x) \times \eta(t) + D_0\right]^4} \quad (14)$$

The piezoelectric layers of the bimorph configuration shown in figure 1 are connected in series. Kirchoff's laws can be applied to obtain the coupled electrical circuit equation as given below

$$\frac{C_p}{2}\frac{dv(t)}{dt} + \frac{v(t)}{R_l} - \kappa\frac{d\eta(t)}{dt} = 0 \quad (15)$$

Hence, equation (15) is the electrical circuit equation of the bimorph cantilever for the series connection of the piezoelectric layers. Equation (12) and (15) are coupled with time dependent parameter and voltage parameter. For the

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simulation purpose one can decouple these equations by using state variables.

$z_1(t) = \eta(t)$, $z_2(t) = \frac{d\eta(t)}{dt} = \dot{\eta}(t)$ and $z_3(t) = v(t)$

$$\dot{z}_1(t) = z_2(t) \quad (16)$$

$$\dot{z}_2(t) = f(t) + f_{mx}(t) - 2\xi\omega z_2(t) - \omega^2 z_1(t) + \chi z_3(t) \quad (17)$$

$$\dot{z}_3(t) = \frac{-2}{R_l \times C_p} \times z_3(t) - \frac{2\kappa}{C_p} \times z_3(t) \quad (18)$$

The three equations above represents decoupled form of coupled mechanical and electrical equations. The above equations can be simulated in Matlab for getting time dependent parameter and voltage parameter. Finally by using the mathematical equations stated previously transverse displacement of the beam and corresponding voltage can be calculated.

### C. Finite Element Modeling

The schematic drawing of bimorph piezoelectric energy harvester with tip mass is shown in Fig. 3.

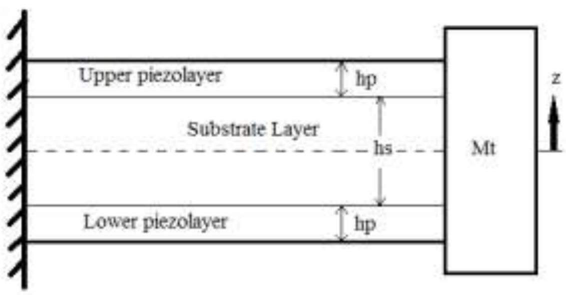


Fig. 3 Bimorph piezoelectric energy harvester with tip mass

Where z is the coordinate through the thickness direction of energy harvester and its origin is assumed to be located at the middle line of harvester as shown in above figure.For the purpose of finite element modeling, the above cantilever beam is divided into discrete number of elements and each elemental length is $l = L/n$. The figure below represents one of the discretized element.

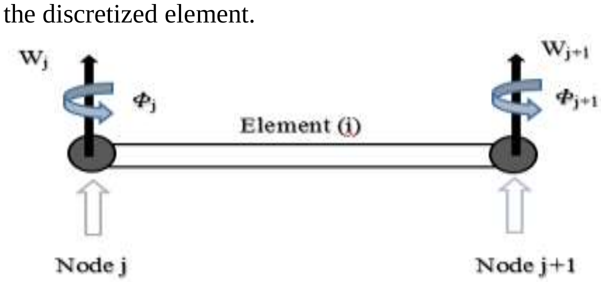


Fig. 4 Discretized element

By using the extended Hamilton's principle the following system of ordinary differential equations is obtained for each element.

$$M_{ij}^e \times \ddot{\Delta}_j^e + C_{ij}^e \times \dot{\Delta}_j^e + K_{ij}^e \times \Delta_j^e - \theta_i^e \times v(t) = F_i^e \quad (19)$$

$$C_p \times \frac{dv}{dt} + \frac{v}{R_l} + \left(\theta_i^e\right)^T \times \dot{\Delta}_i^e = 0 \quad (20)$$

$F^e$ is equivalent force vector and $\Delta^e$ is described as

$$\Delta = \left\{W_j \quad \phi_j \quad W_{j+1} \quad \phi_{j+1}\right\}^T \quad (21)$$

The equivalent mass matrix $M_{ij}^e$ and stiffness matrix $K_{ij}^e$ for an element are defined as follows

$$M_{ij}^e = \int_0^L \left(A_p \psi_i \psi_j + D_p \frac{d\psi_i}{dx}\frac{d\psi_j}{dx}\right) dx \quad (22)$$

$$K_{ij}^e = \int_0^L D \frac{d^2\psi_i}{dx^2}\frac{d^2\psi_j}{dx^2} dx \quad (23)$$

Where $\psi_i$ and $\psi_j$ are Hermitian Shape functions. The base of the proposed piezoelectric energy harvester vibrates in the transverse direction. Therefore, the equivalent forcing term can be given by

$$F_i^e = F_{0i} \times \frac{d^2 w_b}{dt^2} \quad (24)$$

$$F_{0i} = \int_0^L \left(m + M_t \delta(x - L)\right)\psi_i dx \quad (25)$$

All the above equations can be added to Eq. 19 and 20 so that global form of coupled mechanical and coupled electrical equations can be formulated as

$$[M]^G \times [\ddot{\Delta}] + [K]^G \times [\Delta] - [\theta]^G \times v(t) = [F]^G \quad (26)$$

$$C_p \times \frac{dv}{dt} + \frac{v}{R_l} + \left([\theta]^G\right)^T \times [\dot{\Delta}]^G = 0 \quad (27)$$

Finally above equations can be analytically simulated in Matlab software and can be solved by using Runge-Kutta solver technique. The transverse displacement and equivalent induced voltage can be calculated.

## IV. Experimental Work

Aluminum is taken as substrate material along with mild steel as a tip mass. The cantilever beam and tip mass dimensions are precisely obtained by performing various operations like grinding, filing, etc. The geometric dimensions of substrate and tip mass are summarized in Table 1 below.

Table 1. Geometric properties of substrate and tip mass

| Parameter | Substrate (Aluminum) | Tip mass (Mild steel) |
|---|---|---|
| Length (mm) | L=62 | $L_m$ =15.70 |
| Width (mm) | B=14.5 | $B_m$ =7.10 |
| Thickness (mm) | $h_S$ =0.7 | $h_m$ =5.90 |

To find out the first mode resonance frequency of aluminum cantilever beam with tip mass, base excitation is provided by using vibration shaker and additional magnetic force is provided at tip by using fixed external magnets. Fig. 5 shows the cantilever beam with tip mass.


Fig.5 Cantilever beam with tip mass in magnetic field

The complete experimental set-up contains following devices-

1. Vibration shaker
2. Function generator
3. Power amplifier
4. Accelerometer
5. Digital oscilloscope

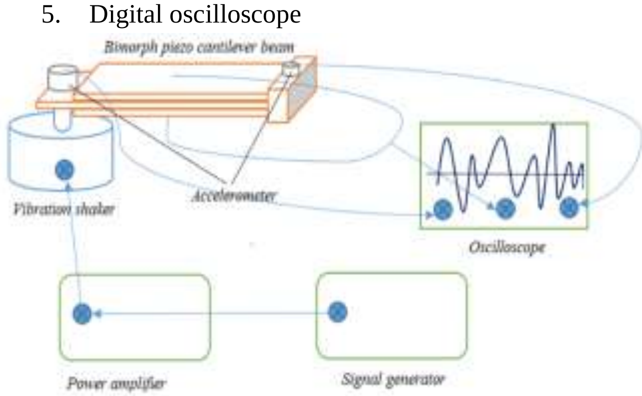


Fig. 6 Experimental Set-up

## V. Results

### A. Lumped Parameter Modeling

The lumped parameter approach is the simplest technique for the simulation of SDOF piezoelectric energy harvester. This approach converts coupled second order differential equations into multiple first order differential form which then can be solved using Runge-Kutta numerical solution. There are two cases considered in lumped parameter model described below.

A.1 Two different Substrate materials with same Piezolayer

Table 2. Material properties of Substrate and piezolayer [18]

| Parameter | Copper | Aluminum | Piezolayer (PZT-5A) |
|---|---|---|---|
| Length (mm) | $l_b$=55 | $l_b$=55 | $l_p$=55 |
| Width (mm) | $w_b$=5 | $w_b$=5 | $w_p$=5 |
| Thickness (mm) | $t_b$=1 | $t_b$=1 | $t_p$=2 |
| Density (kg/m$^3$) | $q_b$=8900 | $q_b$=2700 | $q_p$=7750 |
| Elastic modulus (GPa) | $E_b$=125 | $E_b$=72 | $E_p$=61 |
| Electromechanical coupling coefficient | --- | --- | Tf=1.496×10$^{-5}$ |
| Permittivity constant (nF/m) | --- | --- | $E_{33}$=13.3 |
| Piezoelectric constant (m/V) | --- | --- | $d_{33}$=390×10$^{-12}$ |

A sinusoidal base excitation is applied to the cantilever and equivalent mass of the beam is considered. This model is simulated for two cases namely aluminum as a substrate with PZT-5A as a piezolayer (blue line) and copper as a substrate with PZT-5A as a piezolayer (red line). The Fig. 7 shows time domain history of induced voltage at load resistance 5×10$^6$. It is clear that the aluminum has much sensitivity compared to copper.

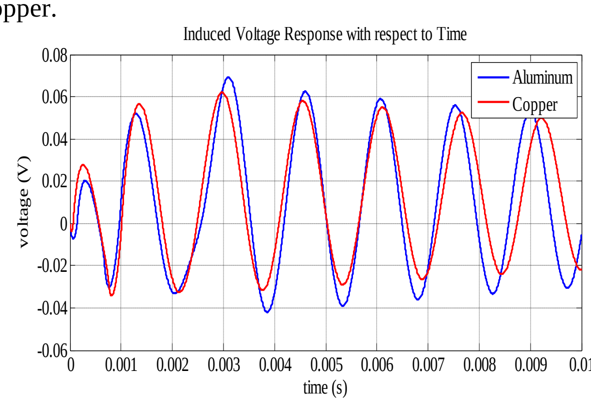


Fig. 7 Induced voltage response

Fig. 8 represents tip mass amplitude response with respect to resonance frequency. For aluminum case the maximum amplitude is 0.007m at resonance frequency of 687.21Hz and in the caseof copper it is about 0.0052m at 646.47Hz.

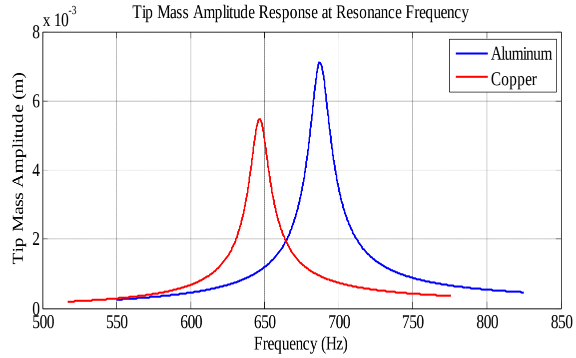


Fig. 8 Tip mass amplitude response

Fig. 9 represents the FFT of induced voltage response. The approximate voltage generated at the maximum amplitude is 1.74 V and 1.5 V respectively.

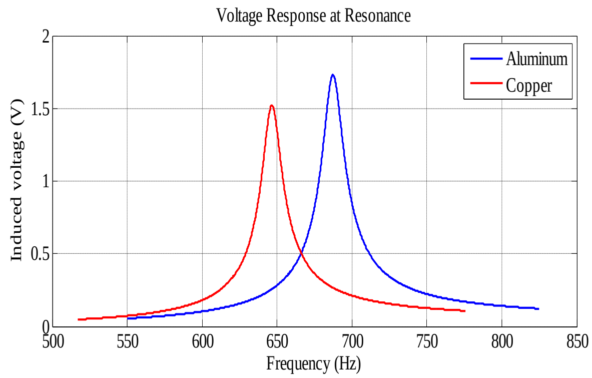


Fig. 9Voltage response at resonance

Fig. 10 shows the variation of amplitude of voltage at resonance frequency with respect to load resistance. It can be seen that after $5\times10^6$ to $6\times10^6$ Ω of load resistance, the induced voltage amplitude becomes constant.

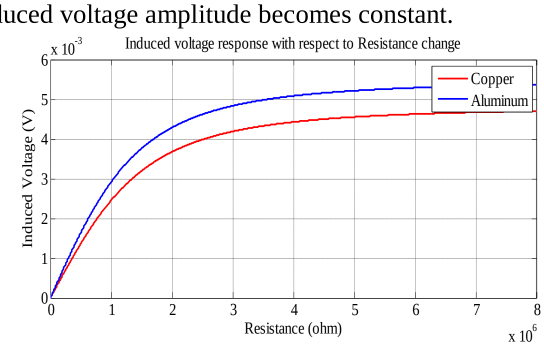


Fig. 10 Amplitude of voltage at resonance frequency

A.2 Two Different Piezoelectric materials with same substrate

The geometric and material properties of PZT-5A and PVDF along with aluminum (Substrate) are summarized in Table 3.

Table 3.Material properties of substrate and piezolayer [18]

| Parameter | Aluminum | PZT 5A | PVDF |
|---|---|---|---|
| Length (mm) | $l_b$=55 | $l_p$=55 | $l_p$=55 |
| Width (mm) | $w_b$=5 | $w_p$=5 | $w_p$=5 |
| Thickness (mm) | $t_b$=1 | $t_p$=2 | $t_p$=2 |
| Density (kg/m3) | qb=2700 | qp=7750 | qp=1750 |
| Elastic modulus (GPa) | $E_b$=72 | $E_p$=61 | $E_p$=2.8 |
| Electromechanical coupling coefficient | --- | $T_f$=1.496 $\times10^{-5}$ | $T_f$ =7.5 $\times10^{-5}$ |
| Permittivity constant (nF/m) | --- | $E_{33}$=13.3 | $E_{33}$=112.2 |
| Piezoelectric constant (m/V) | --- | $d_{33}$=390× $10^{-12}$ | $d_{33}$= -33×$10^{-12}$ |

The two cases considered in this section has aluminum as substrate material and two different independent piezoelectric materials are PZT-5A (red) and PVDF (blue).The resonance frequency by considering PVDF as a piezolayer is 161.8Hz. For PZT-5A case, the resonance frequency is 687.21Hz. The tip mass amplitude response for both the cases is shown in the Fig. 11. It can be seen that the maximum amplitude for PVDF is 0.036m and for PZT-5A is 0.007m and hence former will definitely induce more voltage than later.

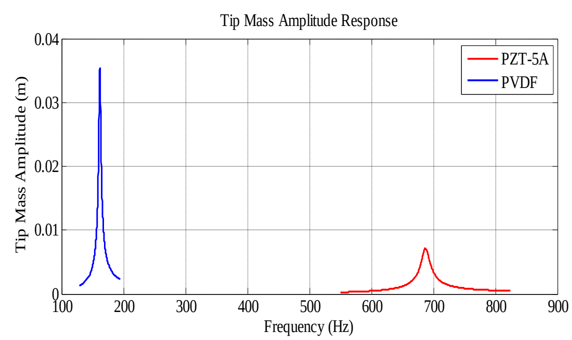


Fig.11 Tip mass amplitude response

Fig. 12 represents induced voltage response at $5\times10^6$in which it can be seen that the maximum voltage induced by PVDF case is about 9 V and PZT-5A case is about 1.74 V.

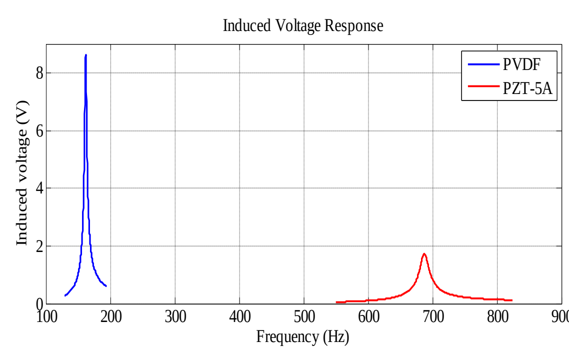

Fig.12 Induced voltage response

Fig. 13 represents the variation of amplitude of voltage at resonance frequency with respect to load resistance. It can be seen that the amplitudes with PVDF are much higher compared to PZT-5A.

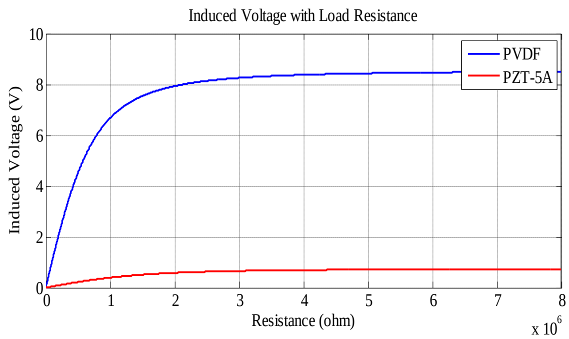


Fig. 13Amplitude of voltage at resonance frequency

B. DISTRIBUTED PARAMETER MODEL

The results obtained by using distributed parameter model are very close to exact solution. The material and geometric properties of substrate and piezolayer are summarized in the Table 3 below.

Table 4. Material properties of substrate and piezolayer [19]

| Parameter | Aluminum | PZT-5A |
|---|---|---|
| Length (mm) | $l_b$=30 | $l_p$=30 |
| Width (mm) | $w_b$=5 | $w_p$=5 |
| Thickness (mm) | $t_b$=0.05 | $t_p$=0.15 |
| Density (kg/m3) | $\rho_b$=2700 | $\rho_p$ =7750 |
| Elastic modulus (GPa) | $E_b$=72 | $E_p$=61 |
| Electromechanical coupling coefficient | --- | $T_f=1.496\times10^{-5}$ |
| Permittivity constant (nF/m) | --- | $E_{33}$=13.3 |
| Piezoelectric constant (C/m$^2$) | --- | $e_{31}$=-10.4 |

In order to reduce the partial differential equations into ODE, single mode approximation is considered and Matlab symbolic mathematics toolbox is employed to integrate and simplify the terms. The Fig. 14 represents the time domain history of tip mass displacement.

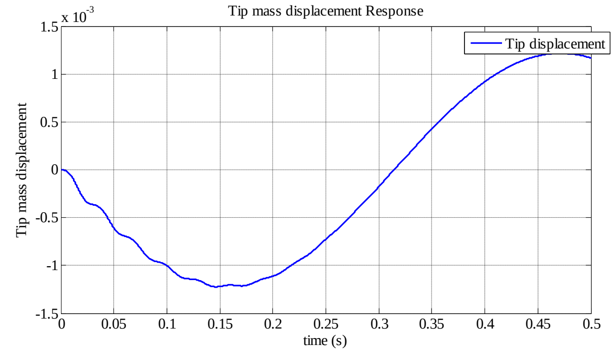


Fig. 14 tip mass displacement Response

The corresponding induced voltage is shown in the Fig. 15. It can be seen that the maximum voltage obtained here is about 0.033 volt or 33 mV.

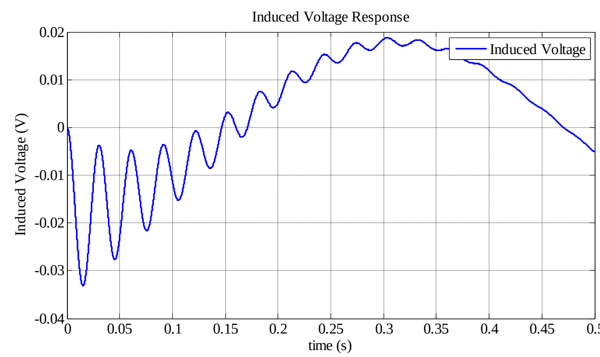


Fig. 15 Induced voltage response

From the Fig. 16, it can be seen that the resonance frequency of the proposed energy harvester is 33.3 Hz and the maximum energy harvested is about $2.201\times10^{-8}$ watt.

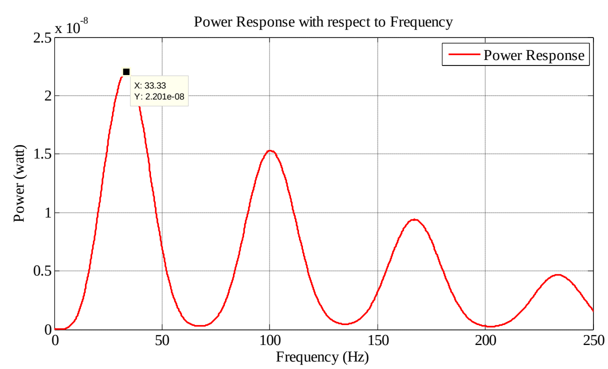


Fig. 16 Power response with frequency

VI. CONCLUSIONS

Adding the nonlinearity like magnetic force to the conventional energy harvester a broadband energy can be harvested.Simulation is performed using 4 GB RAM system with the speed of 3.20 GHz.The results are validated with past data. The mathematical modeling of cantilever beam with magnetic tip mass is done. Finite element formulation is also performed. The additional force due to magnetic coupling reduces the excitation frequency and hence increases the harvester efficiency.

Modeling of distributed parameter cantilever beam by using Timoshenko theory and Rayleigh beam approaches; the analysis of the same through Finite Element technique is the future work of this research. Finally precise fabrication considerations and materials for different piezoelectric energy harvesters are to be discussed.

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