Humanities and Mathematical Sciences

APPLICATION OF SECOND ORDER DIFFERENTIAL EQUATIONSBY USING DIFFERENTIAL TRANSFORM METHOD

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Abstract: Using Differential Transform Method to solve the second order differential equations; some numerical problems are presented to illustrate the efficiency and reliability of the method.
Keywords: Second order differential equation,Differential TransformMethod (DTM), Laplace Transform Method.

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I. INTRODUCTION:

The concept of Differential Transform Method (DTM) was first proposed and applied to solve linear and non linear initial value problems in electric circuit analysis by (1). A variety of numerical methods are available for the solution of differential equations. The Differential Transform Methodis a numerical method that uses Taylor's series for the solving of differential equations.

By applying this method, it is obtain highly accurate exact solutions for differential equations. Ayaz (2) used DTM to find the series solution of a system of differential equations. In recent years, Abdel-Halim Hassan used Differential Transform Method to solve higher order initial value problems. Ayaz used DTM to find series solution of system of differential equations.

The Differential Transform Methodis an iterative method that is described by the transformed equations of original function for solution of differential equations.

In this paper, we solved some numerical examples of second order by using Differential Transform Method. By DTM method, we can solve the problem directly without linearization; it reduces the size of computational work and giving series solution with convergence.

II. DIFFERENTIAL TRANSFORM METHOD

The transformation of the k th derivative of the function with one variable is

$$U(k) = \frac{1}{k!}\left(\frac{d^k u(x)}{dx^k}\right) \; at\; x = x_0 \quad \ldots\ldots\ldots\ldots(1)$$

Where $u(x)$ is the original function and U(k) is the transformed function and the differential inverse transformation $u(x)$ is defined by

$$u(x) = \sum_{0}^{\infty} u(k)\left(x - {x_0}^k\right) \ldots\ldots\ldots\ldots(2)$$

When $x_0 = 0$ the function $u(x)$ defined in (2) is expressed as

$$u(x) = \sum_{0}^{\infty} u(k)x^k \ldots\ldots\ldots\ldots(3)$$

equation (3) gives the similarity between one dimensional differential transform and one dimensional Taylor's series expansion.

The following fundamental theorems on differential transform method are given below [Adio, Vedat]

Theorem 1:

If $u(t) = \alpha\, g(t) \pm \beta\, h(t), then\; U(k) = \alpha G(k) \pm \beta H(k)$.

Theorem 2:

If $u(t) = t^n, then\; U(k) = \delta(k - n)$

Where $\delta(k-n) = \begin{cases} 1 \\ 0 \end{cases} , if \; \begin{matrix} k=n \\ k \neq n \end{matrix}$

Theorem 3:

If $u(t) = e^t, then\; U(k) = \frac{1}{k!}$.

Theorem 4:

If $u(t) = g(t)\, h(t)$, then $U(k) = \sum_0^\infty G(l)H(k-l)$.

Theorem 5:

If $x(t) = x_1(t)x_2(t), then\; X(k) = \sum_0^\infty x_1(k_1)x_2(k - k1$.

Theorem 6:

If x(t) = $\frac{d^m x_1(t)}{dt^m}$, $then\; X(k) = \frac{(k+m)!}{k!} X_1(k+m)$.

Theorem 7:

If $x(t) = sin(\alpha t + \beta),\; then\; X(k) = \frac{\alpha^k}{k!} sin\left(\frac{k\pi}{2} + \beta\right.$ where $\alpha \; and\; \beta$ are constants.

III. EXAMPLES:

Solving by DTMMethod

1. **Consider the second order differential equation:**

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = 20\sin 2t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$with\ initial\ conditions\ x(0) = 1, x'(0) = 2\ .$$

Applying differential transform method with initial conditions u(0) = 1, u(1) = 2 to equation (4), using the above mentioned theorem, we have:

$$u(k+2) = \frac{1}{(k+1)(k+2)}\left[20\left(\frac{2^k}{k!}\sin\left(\frac{k\pi}{2}\right)\right) + (k+1)u(k+1) + 2u(k)\right]$$

Put k = 0, u(2) = 1
k = 1, u(3) = 7
k = 2, u(4) = 23/12
k = 3, u(5) = 29/12

Thus the closed form of the solution when n = 5 (number of terms), using equation (3) can be written easily as;

x(t) = $\sum_0^\infty u(k)t^k$

$=1 + t^2 + 7t^3 + (23/12)t^4 + (29/12)t^5 + \ldots\ldots$

Using Laplace transform method, the exact solution of example 1 is

x(t) = $\frac{-8}{3}e^{-t} + \frac{8}{3}e^{-2t} + \cos 2t - 3\sin 2t$.

2. **Consider the second order differential equation :**

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 4e^{2t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$with\ initial\ conditions\ x(0) = -3,\ x'(0) = 5\ .$$

Applying differential transform method with initial conditions u(0) = -3, u(1) = 5 to equation (5), using the above mentioned theorem, we have :

$$u(k+2) = \frac{1}{(k+1)(k+2)}\left[4.\frac{2^k}{k!} - 2.u(k) + 3(k+1)u(k+1)\right]$$

Put k = 0, u(2) = 25/2
k = 1, u(3) = 73/6
k = 2, u(4) = -1/6
k = 3, u(5) = 3/10

Thus the closed form of the solution when n = 5 (number of terms), using equation (3) can be written easily as;

x(t) = $\sum_0^\infty u(k)t^k$

=-3+5t+ $\left(\frac{25}{2}\right)t^2 + \left(\frac{73}{6}\right)t^3 - \left(\frac{1}{6}\right)t^4 + \left(\frac{3}{10}\right)t^5$

Using Laplace transform method, the exact solution of example 1 is

x(t) = $-7e^t + 4e^{2t} + 4te^{2t}$.

3. **Consider the second order differential equation :**

$$\frac{d^2x}{dt^2} + x = t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$with\ initial\ conditions\ x(0) = 1,\ x'(0) = 0\ .$$

Applying differential transform method with initial conditions u(0) = 1, u(1) = 0 to equation (6), using the above mentioned theorem, we have :

$$u(k+2) = \frac{1}{(k+1)(k+2)}[\delta(k-1) - u(k)]$$

Put k = 0, u(2) = -1/2
k = 1, u(3) = 1/6
k = 2, u(4) = 1/24
k = 3, u(5) = -1/120

Thus the closed form of the solution when n = 5 (number of terms), using equation (3) can be written easily as;

x(t) = $\sum_0^\infty u(k)t^k$

=1+$\left(\frac{-1}{2}\right)t^2 + \left(\frac{1}{6}\right)t^3 + \left(\frac{1}{24}\right)t^4 - \left(\frac{1}{120}\right)t^5$

Using Laplace transform method, the exact solution of example 1 is

x(t) = t+ $\cos t - \sin t$.

IV. CONCLUSION

In this study, the differential transformation method is implemented to the second order differential equations. Two examples are solved and exact solutions are obtained. It is shown that differential transformation method is a very fast convergent, precise and efficient tool for solving the differential equations.

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